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# Analytical Study of Reed-Solomon Error Probability

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**Abstract**

The Reed–Solomon codes are very effective for channels that have memory. Particularly for burst-error correction Reed–Solomon codes are very useful. On channels where the set of input symbols is large RS codes can be used efficiently. An interesting feature of the RS code is that we can add as many as two information symbols to an RS code of length n without reducing its minimum distance. Thus this extended RS code has length n + 2 and the same number of parity check symbols as the original code. This paper represents analytical study of Channel symbol error probability p versus Bit Error Probability $P_B$ for t-error-correcting Reed–Solomon coding. Also this paper covers Bit error probability versus $E_b$ /$N_0$ performance of several n = 31, t – error correcting Reed–Solomon coding systems over an AWGN channel.

**Keywords:** Channel Symbol Error, Bit Error, AWGN.

## INTRODUCTION

In the telecommunication industry, it is important to create a smooth noise free system that provides proper experience to the telecommunication service users. In this case, for channel coding or forward error correction, Reed Solomon codes play a very important role in minimizing noise.

Reed Solomon Codes are blunder remedying codes with applications going from information recovery from standardized identifications and Quick Response (QR) codes in our everyday lives to sending transmissions to and from shuttles propelled in profound space missions. The Reed-Solomon (RS) Code was found by Irving Reed and Gus Solomon and was in this way exhibited to the world in their paper "Polynomial Codes over Certain Finite Fields" in the Diary of the Society for Industrial and Applied Mathematics (Kim et al. 2010) [1]. From that point forward, RS Codes have been an essential giver to the information transfers transformation that took place in the last 50% of the twentieth century. Specifically, Reed-Solomon codes

are the most much of the time utilized computerized blunder control codes as a part of the world, due their utilization in PC memory and non-unstable memory applications. A rushed rundown of huge applications incorporates the Digital Sound Disk, Deep Space Telecommunication Systems, and Error Control for Systems with Feedback, Spread-Spectrum Frameworks, and Computer Memory (Bekkali et al. 2010) [2].
No RS codes accomplish the significant conceivable code least separation for any direct code with the unaltered encoder and yield lengths. The separation between two code words is distinct as the quantity of images in which the arrangements vary. RS codes have a huge property that they are competent of remedying any arrangement of N-k images inside of the square. They can be intended to have any excess (Beygi et al. 2010) [3].

## ANALYTICAL STUDY

The R–S decoded symbol error probability, $P_E$, in terms of the channel symbol error probability, p, can be written as given in equation (1) follows:

$$P_E \approx \frac{1}{2^m - 1} \sum_{j=t+1}^{2^m-1} j \binom{2^m - 1}{j} p^j (1-p)^{2^m-1-j} \tag{1}$$

Where t stands for the symbol-error correcting capability of the code, and the symbols are made up of m bits each.
For specific modulation types, the bit error probability can be upper bounded by the symbol error probability. The relationship between $P_B$ and $P_E$ can be given as shown in Equation (2):

$$\frac{P_B}{P_E} = \frac{2^{m-1}}{2^m - 1} \tag{2}$$

Channel symbol error probability p versus Bit Error Probability $P_B$ for t-error-correcting Reed–Solomon coding is tabulated in Table 1 to Table 4

**Table 1:** Channel symbol error probability p versus Bit Error Probability $P_B$ for t = 8

| **t-8** | |
|---|---|
| **X** | **Y** |
| Channel Symbol Error Probability, P | Bit Error Probability, $P_B$ |
| 9.78E-02 | 3.94E-04 |
| 7.97E-02 | 1.02E-04 |
| 5.93E-02 | 1.00E-05 |
| 4.41E-02 | 1.00E-06 |
| 3.36E-02 | 1.00E-07 |

**Table 2:** Channel symbol error probability p versus Bit Error Probability $P_B$ for t = 4

| t-4 | |
|---|---|
| **X** | **Y** |
| **CHANNEL SYMBOL ERROR PROBABILITY, P** | **BIT ERROR PROBABILITY, $P_B$** |
| 7.79E-02 | 9.55E-03 |
| 4.31E-02 | 9.80E-04 |
| 2.62E-02 | 1.08E-04 |
| 1.59E-02 | 9.83E-06 |
| 5.84E-03 | 1.01E-07 |

**Table 3:** Channel symbol error probability p versus Bit Error Probability $P_B$ for t = 2

| t-2 | |
|---|---|
| **X** | **Y** |
| **CHANNEL SYMBOL ERROR PROBABILITY, P** | **BIT ERROR PROBABILITY, $P_B$** |
| 4.41E-02 | 9.35E-03 |
| 1.78E-02 | 1.03E-03 |
| 7.85E-03 | 1.03E-04 |
| 3.54E-03 | 9.65E-06 |
| 1.67E-03 | 1.06E-06 |
| 7.39E-04 | 1.04E-07 |

**Table 4:** Channel symbol error probability p versus Bit Error Probability $P_B$ for t = 1

| t-1 | |
|---|---|
| **X** | **Y** |
| **CHANNEL SYMBOL ERROR PROBABILITY, P** | **BIT ERROR PROBABILITY, $P_B$** |
| 2.87E-02 | 1.03E-02 |
| 7.85E-03 | 1.08E-03 |
| 2.30E-03 | 1.04E-04 |
| 7.06E-04 | 9.26E-06 |
| 2.48E-04 | 1.09E-06 |
| 1.00E-04 | 1.56E-07 |

Figure 1 shows the plot of $P_B$ versus the channel symbol error probability p (used graph plotting tool from the website http://www.chartgo.com). It is plotted for various t-error-correcting Reed–Solomon codes with n = 31 (thirty-one 5-bit symbols per code block).

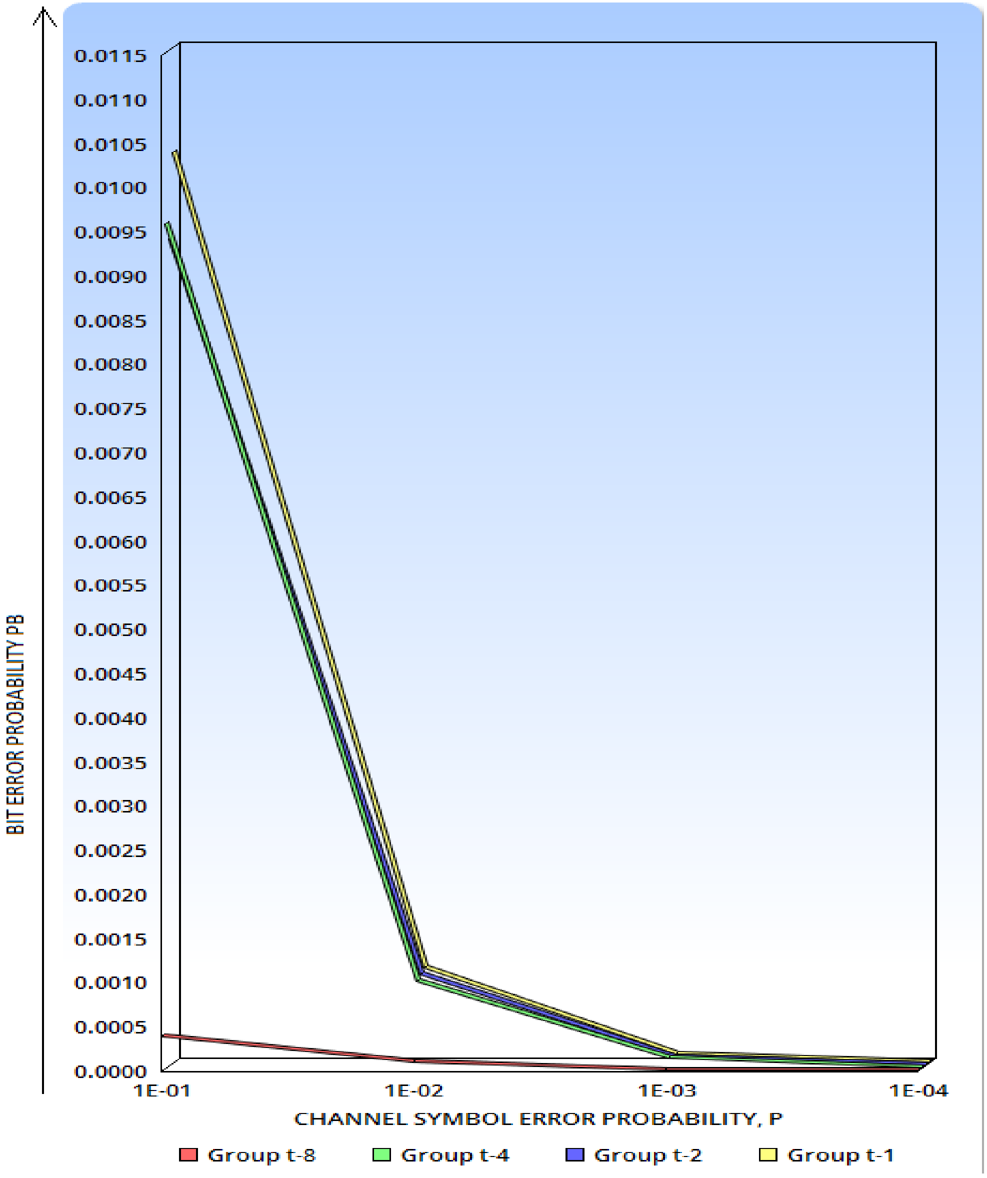


**Figure 1:** Plot of the channel symbol error probability P versus Bit Probability $P_B$

Figure 1 represents a decoder transfer function, which provides no information about the channel and demodulation. Thus Figure 1 transfer function only presents the output verses input benefits of the decoder, and displays nothing about the loss of energy as a function of lower code rate. Bit error probability versus $E_b/N_0$ performance of several n = 31, t –error correcting Reed–Solomon coding systems over an AWGN channel is tabulated in Table 5 to Table 8

**Table 5:** $E_b$ /$N_0$ (dB) versus Bit Error Probability $P_B$ **for t = 1**

| **t-1** | |
|---|---|
| **X** | **Y** |
| **Eb/N0 (dB)** | **Bit Error Probability, PB** |
| 3.70E+00 | 1.02E-02 |
| 3.74E+00 | 9.12E-03 |
| 3.97E+00 | 5.28E-03 |
| 4.10E+00 | 3.71E-03 |
| 4.74E+00 | 5.40E-04 |
| 5.49E+00 | 4.43E-05 |
| 6.02E+00 | 7.27E-06 |
| 6.58E+00 | 9.59E-07 |
| 6.79E+00 | 4.36E-07 |
| 6.84E+00 | 3.72E-07 |
| 6.89E+00 | 3.18E-07 |
| 6.93E+00 | 2.72E-07 |

**Table 6:** $E_b$ /$N_0$ (dB) versus Bit Error Probability $P_B$ **for t = 2**

| **t-2** | |
|---|---|
| **X** | **Y** |
| **Eb/N0 (dB)** | **Bit Error Probability, PB** |
| 3.64E+00 | 9.99E-03 |
| 4.27E+00 | 1.14E-03 |
| 4.50E+00 | 4.15E-04 |
| 4.81E+00 | 1.22E-04 |
| 5.01E+00 | 5.29E-05 |
| 5.32E+00 | 1.30E-05 |
| 5.49E+00 | 5.78E-06 |
| 5.80E+00 | 1.42E-06 |
| 5.95E+00 | 6.90E-07 |
| 6.09E+00 | 3.50E-07 |
| 6.19E+00 | 2.02E-07 |
| 6.27E+00 | 1.42E-07 |

**Table 7:** $E_b$ /$N_0$ (dB) versus Bit Error Probability $P_B$ **for t = 4**

| **t-4** | |
|---|---|
| **X** | **Y** |
| **Eb/N0 (dB)** | **Bit Error Probability, PB** |
| 3.64E+00 | 1.01E-02 |
| 4.02E+00 | 2.24E-03 |
| 4.51E+00 | 1.93E-04 |
| 4.86E+00 | 2.85E-05 |
| 5.01E+00 | 1.27E-05 |
| 5.28E+00 | 2.46E-06 |
| 5.44E+00 | 9.16E-07 |
| 5.57E+00 | 4.01E-07 |
| 5.68E+00 | 2.06E-07 |
| 5.70E+00 | 1.75E-07 |
| 5.75E+00 | 1.24E-07 |
| 5.79E+00 | 1.08E-07 |

**Table 8:** $E_b$ /$N_0$ (dB) versus Bit Error Probability $P_B$ **for t = 8**

| **t-8** | |
|---|---|
| **X** | **Y** |
| **Eb/N0 (dB)** | **Bit Error Probability, PB** |
| 4.27E+00 | 9.81E-03 |
| 4.60E+00 | 2.69E-03 |
| 4.90E+00 | 6.64E-04 |
| 5.00E+00 | 4.36E-04 |
| 5.11E+00 | 2.31E-04 |
| 5.26E+00 | 1.04E-04 |
| 5.49E+00 | 2.89E-05 |
| 5.69E+00 | 9.14E-06 |
| 6.01E+00 | 1.24E-06 |
| 6.12E+00 | 6.36E-07 |
| 6.22E+00 | 3.24E-07 |
| 6.39E+00 | 1.07E-07 |

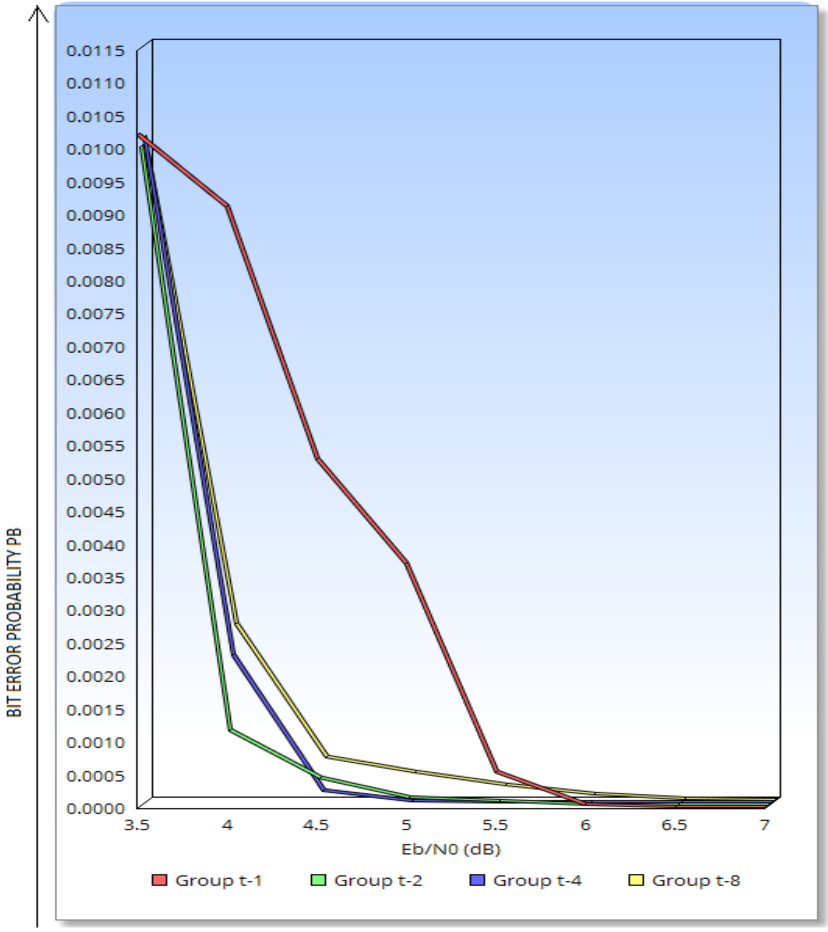


**Figure 2**: Plot of Bit error probability $P_B$ versus Eb /$N_0$ performance

Figure 2 shows plot of $P_B$ versus $E_b/N_0$ for such a coded system over an AWGN channel. For R–S codes, decoding complexity is proportional to a small power of the block length and error probability is an exponentially decreasing function of block length, n. Sometimes for a concatenated arrangement the R–S codes can be used.
Figure 2 above shows that the error performance over an AWGN channel improves as the symbol error correcting capability, t, increases from t = 1 to t = 4. Here t=1 corresponds to R-S (31, 29) and t = 4 corresponds to R-S (31, 23). However, at t = 8, which corresponds to R-S (31, 15), the error performance at $P_B = 10^{-5}$ degrades by about 0.5 dB of Eb/$N_0$ compared to the t = 4 case.

**CONCLUSION**

The analytical study of Reed-Solomon code shows that RS codes are have great power and utility. Reed Solomon codes are non-binary cyclic codes that performs well against burst noise. In (n, k) = (255,247) R-S code, where each symbol is made up of m= 8. Since n-k =8, it indicates that this code can correct any 4 symbol errors in a block of 255. Imagine the presence of a noise burst, lasting for 25-bit durations and disturbing one block of data during transmission. The R-S decoder for the (255,247) code will correct any 4-symbol errors without regard to the type of damage suffered by the symbol. In other words, when a decoder corrects a byte, it replaces the incorrect byte with the correct one, whether the error was caused by one bit being corrupted or all 8 bits being corrupted. Thus, if a symbol is wrong, it might as well be wrong in all of its bit positions. This gives a R-S code a tremendous burst-noise advantage over binary codes, even allowing for the interleaving of binary codes

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