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Common Fixed Points of Pseudo Compatible Mappings in Intuitionistic Fuzzy Metric Spaces

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Abstract:

The aim of this paper is to obtain common fixed point theorems by employing the recently introduced notion of g -reciprocal continuity and pseudo-compatible mappings in Intuitionistics fuzzy metric spaces. Results proved in this paper do not require the conditions of continuity of maps and closedness of any range.

Keywords: Fixed Point, Intuitionistic fuzzy metric space, occasionally weakly compatible mappings, pseudo compatible, g - reciprocal continuity, reciprocal continuity.

Mathematics Subject Classification: 47H10, 54H25.

1. INTRODUCTION

Atanassov [4] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [14]. In 2004, Park [11] introduced a notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [6]. Further, Alaca et. Al [1] studied new properties of intuitionistic fuzzy metric space and obtained the fuzzy version of Banach and Edelstein fixed point theorem by a strong definition of Cauchy sequence.

The study of common fixed points of compatible mappings has emerged as an area of vigorous research activity ever since Jungck [5] introduced the notion of compatible mappings. Compatibility is useful mainly in the study of common fixed points of contractive type mapping pairs and then requires the assumption of continuity and completeness. However, the study of common fixed points of non-compatible mapping is also equally interesting which extents to the class of Lipschitz type mappings pair without assuming continuity and completeness.

In the study of fixed points of metric space, Pant [9] has initiated the concept of non-compatible maps in metric spaces. Using the concept of noncompatible and pointwise R weakly commuting mappings, Muralisankar and Kalpana [7] obtained a fixed point theorem for pair of mappings in Intuitionistic fuzzy metric space.

Recently, Pant. et. al [8] have introduced two more generalized concepts. Firstly, g-reciprocal continuity which is a generalization of continuity, but independent of reciprocal continuity. Secondly, Pseudo compatible, a proper generalization of occasionally weakly compatible. By using these two newly introduced concepts Pant. et. al [8] have proved some common fixed point theorems. Our results extend and generalize the results of Pant [8] and more results in the literature.

The present paper, the notion of pseudo compatible and g-reciprocal continuous mappings in intuitionistic fuzzy metric space is introduced and prove a common fixed point theorem for a pair of non-compatible mapping without assuming either the completeness of the space or the continuity of the mappings involved and also find an answer to the open problem of Rhoades [12] in intuitionistic fuzzy metric space.

2. PRELIMINARIES

Definition 2.1. [14] Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2. [4] Let a set E be fixed. An intuitionistic fuzzy set (IFS) A of E is an object having the form,

$$A = \{< x, \mu_A(x), V_A\ (x) > \mid x\ \in E\};$$

where the function $\mu_A: E\ \to [0, 1]$, $V_A: E\ \to [0, 1]$, define respectively, the degree of membership and degree of non-membership of the element $x\ \in E$ to the set A, which is a subset of E, and for every $x\ \in E$, $0\ \leq\ \mu_A(x) +\ V_A\ (x)\ \leq 1$.

Definition 2.3. [13] A binary operation $* : [0,\ 1] \times [0,\ 1]$ is a continuous t-norm if it satisfies the following conditions:

(a) $*$ is commutative and associative;

(b) $*$ is continuous;

(c) $a * 1 = a$ for all $a \in [0,\ 1]$;

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0, 1]$.

Definition 2.4. [13] A binary operation $\lozenge : [0,\ 1] \times [0,\ 1]$ is a continuous t-conorm if it satisfies the following conditions:

(a) $\lozenge$ is commutative and associative;

(b) $\lozenge$ is continuous;

(c) $a \lozenge 0 = a$ for all $a \in [0,\ 1]$;

(d) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0, 1]$.

Definition 2.5. [1] A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t -norm, $\lozenge$ is a continuous t -conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$;

(IFM-1) $M(x,y,t) + N(x,y,t) \leq 1$;

(IFM-2) $M(x,y,t) = 0$;

(IFM-3) $M(x,y,t) = 1$ *if and only if* $x = y$;

(IFM-4) $M(x,y,t) = M(y,x,t)$;

(IFM-5) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;

(IFM-6) $M(x,y,\ .)$: $[0,\ \infty) * [0,1]$ is left continuous;

(IFM-7) $\lim_{t \to \infty} M(x,\ y, t) = 1$;

(IFM-8) $N(x,y,0) = 1$;

(IFM-9) $N(x,y,t) = 0$ *if and only if* $x = y$;

(IFM-10) $N(x,y,t) = N(y,x,t)$;

(IFM-11) $N(x,y,t) \lozenge N(y,z,s) \leq N(x,z,t+s)$;

(IFM-12) $N(x,y,\ .)$: $[0,\ \infty)\ \to [0,1]$ is right continuous;

(IFM-13) $\lim_{t\to\infty} N(x,\ y,t\ ) = 0$;

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Definition 2.6. [1] Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is called:

(a) A sequence $\{x_n\}$ in X is Convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n,x,t\ ) = 1$ and $\lim_{n\to\infty} N(x_n,x,t\ ) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence if for each $t > 0\ and\ p > 0$ if $\lim_{n\to\infty} M(x_{n+p},\ x_n,\ t\ ) = 1$ and $\lim_{n\to\infty} N(x_{n+p},\ x_n,t\ ) = 0$; .

(c) An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Lemma. 2.7. [11] In an intuitionistic fuzzy metric space X, $M(x,y,.)$ is non decreasing and $N(x,y,.)$ is non-increasing for all $x,y \in X$.

Lemma 2.8. [2] Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0,1)$ such that

$M(x,y,kt) \geq M(x,y,t)$,

$N(x,y,kt) \leq N(x,y,t)$

for $x,\ y \in X$. Then $x = y$.

Definition 2.9. [2] Let f and g be mappings from an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ into itself. Then the mappings are said to be compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$, $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$, for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.

Definition 2.10. [7] Let f and g be a mappings from an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ into itself. Then the mappings are said to be non-compatible if whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$ $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$, $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or non-existent.

Definition 2.11. [7] Let f and g be a mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ into itself. Then the mappings are said to be reciprocally continuous if $\lim_{n\to\infty} fgx_n = fz$ and $\lim_{n\to\infty} gfx_n = fz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.

Remark 2.12. If f and g are both continuous then they are obviously reciprocally continuous. But the converse need not be true.

Definition 2.13. [8] Let f and g be self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$. Then for a sequence $\{y_n\}$ in X satisfying $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n$ a sequence $\{z_n\}$ will be called a associated sequence if $fy_n = gz_n$ or $gy_n = fz_n$ and $\lim_{n\to\infty} fz_n = \lim_{n\to\infty} gz_n$.

Al-Thagafi and shahzad [3] generalized the notion of weak compatibility by introducing the notion of occasionally weak compatible.

Definition 2.14. [3] A Pair (f, g) of self mappings of a non empty set X is said to be occasionally weakly compatible (owc) if the pair (f, g) commutes at least on one coincidence point. i.e there exists at least one point x in X such that $fx = gx$ and $fgx = gfx$.

Now, we introduce a new notion of commutativity in intuitionistic fuzzy metric space which is a proper generalization of occasionally weakly compatible mappings (owc).

Definition 2.15. Two self mappings f and g of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ will be defined to be pseudo-compatible iff whenever the set of sequences $\{x_n\}$ satisfying $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n$ is non empty, there exists a sequence $\{y_n\}$ in X satisfying $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = t$, (Say)

$$\lim_{n\to\infty} M(fgy_n, gfy_n, t) = 1, \lim_{n\to\infty} N(fgy_n, gfy_n, t) = 0;$$

$$\lim_{n\to\infty} M(fgz_n, gfz_n, t) = 1, \lim_{n\to\infty} N(fgz_n, gfz_n, t) = 0;$$

for any associated sequence $\{z_n\}$ of $\{y_n\}$.

Remark 2.16: If f and g are occasionally weakly compatible then they are obviously pseudo compatible, but the converse is not be true.

Definition 2.17. Let f and g be a mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ into itself. Then the mappings are said to be g-reciprocally continuous iff $\lim_{n\to\infty} \mathrm{ffx_n} = \mathrm{ft}$, and $\lim_{n\to\infty} \mathrm{gfx_n} = \mathrm{gt}$ whenever $\{x_n\}$ is a sequence in X such $\lim_{n\to\infty} \mathrm{fx_n} = \lim_{n\to\infty} \mathrm{gx_n} = \mathrm{t}$, for some $t \in X$.

Example 2.18. Let $X = [2, 20]$ and defined the self mappings f and g of X by

$$f(x) = \begin{cases} 2, & if\ x = 2\ and > 5, \\ \frac{x+7}{3}, & if\ 2 < x \le 5. \end{cases}$$

$$g(x) = \begin{cases} 2, & x = 2. \\ 6, & if\ 2 < x \le 5, \\ \frac{x+5}{5}, & if\ x > 5. \end{cases}$$

$M(fx, gy, t) = \frac{t}{t+|fx-gy|}$, $N(fx, gy, t) = \frac{|fx-gy|}{t+|fx-gy|}$ for all $x, y \in X$ and $t > 0$; $M(x, y, 0) = 0$; and $N(x, y, 0) = 1$;

let $\{x_n\} = \left\{5 + \frac{1}{n} : n \ge 1\right\}$ be a sequence in X. Then $\mathrm{fx_n} \to 2$ and $\mathrm{gx_n} = 2 + \frac{1}{5n} \to 2$. $\mathrm{fgx_n} = \mathrm{f}\left(2 + \frac{1}{5n}\right) \to 3$, $\mathrm{ffx_n} = \mathrm{f2} = 2$. $\mathrm{gfx_n} = \mathrm{g}(2) = 2$ and $\mathrm{ggx_n} = \mathrm{g}\left(2 + \frac{1}{5n}\right) \to 6$. Thus, $\lim_{n\to\infty} \mathrm{ffx_n} = \mathrm{f2}$ and $\lim_{n\to\infty} \mathrm{gfx_n} = \mathrm{g2}$. Hence f and g are g-reciprocally continuous mappings but not reciprocally continuous.

Remark 2.19. If f and g are both continuous then they are obviously g −reciprocally continuous. But the converse need not be true.

Theorem 2.20. [8] Let f and g be a g- reciprocally continuous self mappings of a complete metric space (X, d) such that

(i) $f(x) \subseteq g(x)$;

(ii) $d(fx, f y) \le k\, d(gx, gy)$, $k \in [0, 1)$;

If f and g are pseudo compatible, then f and g have a unique common fixed point.

Theorem 2.21. [8] Let f and g be a g- reciprocally continuous non-compatible self mappings of a complete metric space (X, d) such that

(i) $f(x) \subseteq g(x)$;

(ii) $d(fx, fy) < \; max \begin{Bmatrix} d(gx, gy), \frac{k[\, d(fx,gx)+d(fy,gy)]}{2}, \\ \frac{[\, d(fx,gy)+d(fy,gx)]}{2} \end{Bmatrix}$; where $1 \le k < 2$.

(iii) $d(x, fx) \neq \max\{d(x, gx), d(fx, gx)\}$

whenever right-hand side is non zero. If f and g are pseudo compatible then f and g have a unique common fixed point.

3 MAIN RESULT

Theorem 3.1. Let f and g be a g- reciprocally continuous self mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ such that

(i) $f(X) \subseteq g(X)$;

(ii) $M(fx, f\, y, qt) \ge M(gx, gy, t)$,
$N(fx, f\, y, qt) \le N(gx, gy, t)$; where $q \in (0, 1)$;

If f and g are pseudo compatible, then f and g have a unique common fixed point.

Proof:

Let x_0 be any point in X. Then since $f(X) \subseteq g(X)$; there exists a sequence of points $x_0, x_1, x_2, \dots\, x_{n\dots}\; in\, X$ such that $fx_0 = gx_1,\; fx_1 = gx_2, \dots, fx_n = gx_{n+1} \dots$

Now, define a sequence in $\{S_n\}$ in X as $S_n = f\, x_n = gx_{n+1}$, $for\; n = 0, 1,2 \dots$

We claim that $\{s_n\}$ is a Cauchy Sequence. Using (ii), we obtain,

$$\begin{aligned} M(s_n, s_{n+1}, qt) &= M(f\,x_n, fx_{n+1}, qt) \\ &\ge M(g\,x_n, gx_{n+1}, t) \\ &\ge M(s_{n-1}, s_n, t) \end{aligned}$$

$$\begin{aligned} N(s_n, s_{n+1}, qt) &= N(f\,x_n, fx_{n+1}, qt) \\ &\le N(g\,x_n, gx_{n+1}, t) \\ &\le N(s_{n-1}, s_n, t) \end{aligned}$$

By simple induction and condition (ii) we have for all $t > 0$,

$M(s_n, s_{n+1}, t) \geq M(s_0, s_1, \frac{t}{q^n})$,

$N(s_n, s_{n+1}, t) \leq N(s_0, s_1, \frac{t}{q^n})$

Also for every integer $p > 0$, we get

$M(s_n, s_{n+p}, t) \geq M\left(s_n, s_{n+1}, \frac{t}{p}\right) * M\left(s_{n+1}, s_{n+2}, \frac{t}{p}\right) * \ldots * M\left(s_{n+p-1}, s_{n+p}, \frac{t}{p}\right)$,

$N(s_n, s_{n+p}, t) \leq N\left(s_n, s_{n+1}, \frac{t}{p}\right) \Diamond N\left(s_{n+1}, s_{n+2}, \frac{t}{p}\right) \Diamond \ldots \Diamond N\left(s_{n+p-1}, s_{n+p}, \frac{t}{p}\right)$.

using (3.1) we get $M(s_n, s_{n+p}, t) \geq M\left(s_0, s_1, \frac{t}{pq^n}\right) * M\left(s_0, s_1, \frac{t}{pq^{n+1}}\right) \ldots * M\left(s_0, s_1, \frac{t}{pq^{n+p-1}}\right)$, $N(s_n, s_{n+p}, t) \leq N\left(s_0, s_1, \frac{t}{pq^n}\right) \Diamond N\left(s_0, s_1, \frac{t}{pq^{n+1}}\right) \Diamond \ldots \Diamond N\left(s_0, s_1, \frac{t}{pq^{n+p-1}}\right)$

Letting $n \to \infty$, we have

$\lim_{n\to\infty} M(s_n, s_{n+p}, t) \geq 1 * 1 * 1 * \ldots * 1 = 1$,

$\lim_{n\to\infty} N(s_n, s_{n+p}, t) \leq 0 \Diamond 0 \Diamond 0 \Diamond \ldots \Diamond 0 = 0$.

Hence $\{s_n\}$ is a Cauchy sequence. Since X is a complete, there exists a point u in X such that the sequence $\{s_n\}$ is convergent.

Therefore, pseudo compatibility of f and g implies there exists $\{y_n\}$ such that

$\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = u$, and $\lim_{n\to\infty} M(fgy_n, gfy_n, t) = 1$, and $\lim_{n\to\infty} N(fgy_n, gfy_n, t) = 0$.

Since $f(x) \subseteq g(x)$, for each y_n there exists a z_n in X such that $fy_n = gz_n \; \forall n$. This implies $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} gz_n = z$.

By virtue of this and using (ii) we obtain $fz_n \to u$ which in turns implies that $\{y_n\}$ and $\{z_n\}$ are associated sequences and $\lim_{n\to\infty} M(fgz_n, gfz_n, t) = 1$, $\lim_{n\to\infty} N(fgz_n, gfz_n, t) = 0$.

Now, g - reciprocal continuity of f and g implies that $\lim_{n\to\infty} ffy_n = fu$; $\lim_{n\to\infty} gfy_n = \lim_{n\to\infty} ggz_n = gu$. Similarly, $\lim_{n\to\infty} ffz_n = fu$; $\lim_{n\to\infty} gfz_n = gu$.

Pseudo compatibility of f and g implies that $\lim_{n\to\infty} fgy_n = gu$. Similarly,

$$\lim_{n\to\infty} fgy_n = \lim_{n\to\infty} fgz_n = gu.$$ Hence $gfz_n \to gu$.

Using (ii) we get,

$$M(fu, fgz_n, qt) \geq M(gu, ggz_n, t),$$

$$N(fu, fgz_n, qt) \leq N(gu, ggz_n, t)$$

Letting $n \to \infty$, we get

$$M(fu, gu, qt) \geq M(gu, gu, t)$$

$$N(fu, gu, qt) \leq N(gu, gu, t)$$

By using Lemma 2.8, $fu = gu$. Again Using (ii) we get

$$M(fu, fz_n, qt) \geq M(gu, gz_n, t)$$

$$N(fu, fz_n, qt) \leq N(gu, gz_n, t)$$

Letting $n \to \infty$, we get

$$M(fu, u, qt) \geq M(gu, u, t) = M(fu, u, t),$$

$$N(fu, u, qt) \leq N(gu, u, t) = N(fu, u, t).$$

this implies $fu = u$. Hence $fu = gu = u$ and u is a common fixed point of f and g.

Uniqueness of the common fixed point easily follows by using (ii). Hence the result.

Example 3.2. Let $X = [2, 20]$ and define the self mappings f and on g on X as follows:

$$f(x) = \begin{cases} x + 2, & if\ x \in (2, 3] \\ 5, & otherwise, \end{cases}$$

$$g(x) = \begin{cases} 2x, & if\ x \in (2, 3], \\ 20, & otherwise. \end{cases}$$

$M(fx, gy, t) = \frac{t}{t+|fx-gy|}$, $(fx, gy, t) = \frac{|fx-gy|}{t+|fx-gy|}$, for all $x,\ y \in X$ and $t > 0$; , $\mathrm{M(x, y, 0)} = 0$ and $\mathrm{N(x, y, 0)} = 1$.

In this example we can see that $f(X) = (4, 5] \subseteq (4, 6] \cup \{20\} = g(X)$ and the pair (f, g) is $g -$ reciprocally continuous. It can be verified that

$M(fx, f\, y, qt) \geq M(gx, gy, t)$, $N(fx, f\, y, qt) \leq N(gx, gy, t)$; for all $x,\ y \in X$. Thus, f and g satisfy all the conditions of the Theorem **3.1** except pseudo compatibility.

For the pseudo compatibility, consider the only existent sequence $x_n = y_n = 2 + \frac{1}{n}$, then $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = 4$, but $\lim_{n\to\infty} fgy_n = \lim_{n\to\infty} f\left(4 + \frac{2}{n}\right) = 5$;

$\lim_{n\to\infty} gfy_n = \lim_{n\to\infty} f\left(4 + \frac{2}{n}\right) = 20$; $\lim_{n\to\infty} M(fgy_n,\ gfy_n\ , t) \neq 1$, $\lim_{n\to\infty} N\big(fgx_n,$ $gyx_{n,}\ t\big) \neq 0$. Also note that the pair (f, g) is not compatible. Here, (f, g) has no coincide point therefore it is also not an occasionally weakly compatible, but vacuously weakly compatible pair.

This suggests that pseudo- compatible is stronger than weakly compatible (and occasionally weakly compatible).

Example 3.3. Let $\mathrm{X} = [2,\ 20\ ]$ and define the self mappings f and on g on X as follows:

$$f(x) = \begin{cases} 2, & if\ x\ = 2\ or\ \geq 5 \\ 6, & if\ 2 < x < 5. \end{cases}$$

$$g(x) = \begin{cases} 2, & x = 2, \\ 12, & if\ 2 < x < 5, \\ \frac{(x+1)}{3}\ , & if\ x\ \geq 5. \end{cases}$$

$M(fx, gy, t) = \frac{t}{t+|fx-gy|}$, $(fx, gy, t) = \frac{|fx-gy|}{t+|fx-gy|}$, for all $x,\ y \in X$ and $t > 0$; , $\mathrm{M(x, y, 0)} = 0$ and $\mathrm{N(x, y, 0)} = 1$. Then, f and g satisfy all the conditions of the Theorem **3.1** and have a unique common fixed point at $x = 2$.

To see this, f and g are non-compatible, let $\{x_n\}$ be the sequence in X such that $x_n = 5 + \frac{1}{n}$. Then $\lim_{n\to\infty} fx_n = 2$, $\lim_{n\to\infty} gx_n = 2$, $\lim_{n\to\infty} fgx_n = 6 \neq f(2)$ and $\lim_{n\to\infty} gfx_n = g2 \to 2$.

Hence f and g are neither compatible nor reciprocally continuous. On the other hand, for the constant sequences $\{y_n\}$ given by $y_n = 2$ we have $\lim_{n\to\infty} fy_n = 2$, $\lim_{n\to\infty} gy_n = 2$,

$\lim_{n\to\infty} fgy_n = 2$, $\lim_{n\to\infty} gfy_n = 2$ and $\lim_{n\to\infty} M(fgy_n, gfy_n, t) = 1$, $\lim_{n\to\infty} N(fgy_n, gfy_n, t) = 0$. Moreover, if $\{z_n\}$ is associated sequence of $\{y_n\}$ such that $fy_n = gz_n$ and $\lim_{n\to\infty} fz_n = \lim_{n\to\infty} gz_n$ then $z_n = 2$ for each n and $\lim_{n\to\infty} M(fgz_n, gfz_n, t) = 1$, $\lim_{n\to\infty} N(fgz_n, gyz_n, t) = 0$. This shows that f and g are pseudo-compatible.

To see this, f and g are g reciprocally continuous, let $\{x_n\}$ be the sequence in X such that

$\lim_{n\to\infty} fx_n = t = \lim_{n\to\infty} gx_n$ for some t in X. Then $t = 2$, $x_n = 2$ or $x_n = 5 + \frac{1}{n}$. $\lim_{n\to\infty} ffx_n = f2 = 2$ and $\lim_{n\to\infty} gfx_n = g2 = 2$. Hence f and g are g reciprocally continuous.

Theorem.3.4: Let f and g be g - reciprocally continuous non-compatible self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ such that

(i) $f(x) \subseteq g(x)$;

(ii) $M(fx, fy, t) \geq$

$$min\left\{M(gx, gy, t), \frac{k\,[M(fx,gx,t) * M(fy,gy,t)]}{2}, \frac{[M(fx,gy,t) * M(fy,gx,t)]}{2}\right\}$$

$N(fx, fy, t)$

$$\leq max\left\{N(gx, gy, t), \frac{k\,[N(fx, gx, t) \lozenge N(fy, gy, t)]}{2}, \frac{[N(fx, gy, t) \lozenge N(fy, gx, t)]}{2}\right\}$$

(iii) $M(x, fx, t) \neq \min\left\{M(x, gx, t), M(fx, gx, t)\right\}$

(iv) $\quad N(x, fx, t) \neq max \left\{ N(x, gx, t),\ N(fx, gx, t) \right\}$

where $1 \leq k \leq 2$, whenever $f(x) \neq x$. If f and g are pseudo compatible then f and g have a unique common fixed point.

Proof: Since f and g are non-compatible maps, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$ but either $\lim_{n\to\infty} M\big(fgx_n,$ $gfx_n,\ t\big) \neq 1$, or $\lim_{n\to\infty} N(fgx_n,\ gfx_n, t) \neq 0$ or the limit does not exist.

Pseudo compatibility of f and g implies there exists a sequence $\{y_n\}$ such that $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = u$ (say) and $\lim_{n\to\infty} M(fgy_n,\ gfy_n, t) = 1$ and

$\lim_{n\to\infty} N(fgy_n,\ gfy_n, t) = 0.$

Since $fX \subseteq gX$, for each y_n there exists a z_n in X such that $fy_n = gz_n$ for all n.

This implies that $\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} gz_n = u$ using this and equation (ii) we obtain $fz_n \to u$. Therefore, $\{y_n\}$ and $\{z_n\}$ are associated sequences and consequently $\lim_{n\to\infty} M(fgz_n,\ gfz_n, t) = 1$, and $\lim_{n\to\infty} N(fgz_n,\ gfz_n, t) = 0$. Moreover, g -reciprocal continuity of f and g implies that $ffy_n \to fu$ and $gfy_n = ggz_n \to gu$. Similarly $ffz_n \to fu$ and $gfz_n \to gu$. In view of this and pseudo compatibility we show that f and g we get $fgy_n \to gu$, $fgz_n = ffy_n \to gu$. Hence $fu = gu$.

Again, if $u \neq fu$, then using (iii) we have

$$M(u, fu, t) \neq \min\{M(u, gu, t),\ M(fu, gu, t)\}$$

$$= M\ (u, fu, t)$$

$$N(u, fu, t) \neq \max\{N(u, gu, t),\ N(fu, gu, t)\}$$

$$= N\ (u, fu, t)$$

a contradiction. Hence $fu = gu = u$ and u is a common fixed point of f and g. Uniqueness of the common fixed point easily.

We now show that f and g are discontinuous at the common fixed point u. If possible, suppose f is continuous at u. Then considering the sequence $\{x_n\}$ of the present theorem, we have $ffx_n \to fu = u$ and $fgx_n \to gu = u$. Now, g - reciprocal continuity implies that

$\lim_{n\to\infty} ffx_n = fu = u \quad \lim_{n\to\infty} gfx_n = gu = u$. This, yields

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$, and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ which contradicts that either

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$, and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or the limit does not exists.

Hence f is discontinuous at the fixed point. Next, suppose that g is continuous at u. Then,

for the sequence $\{x_n\}$, we get $gfx_n \to gu = u$ and $ggx_n \to gu = u$. Now by using (ii),

$$M(fu, fgx_n, t) \geq min\left\{ M(gu, ggx_n, t), \quad \left(\frac{k\,[M(fu, gu, t) * M(fgx_n, ggx_n, t)]}{2}\right), \left(\frac{[M(fu, ggx_n, t) * M(fgx_n, gu, t)]}{2}\right) \right\}$$

$$N(fu, fgx_n, t) \leq max\left\{ N(gu, ggx_n, t), \left(\frac{k\,[N(fu, gu, t) \lozenge N(fgx_n, ggx_n, t)]}{2}\right), \left(\frac{[N(fu, ggx_n, t) \lozenge N(fgx_n, gu, t)]}{2}\right) \right\}$$

a contradiction implies $fgx_n \to fu = u$ but $fgx_n \to u$ and $gfx_n \to u$ contradicts the fact that either $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$, and $\lim_{n\to\infty} N(fgz_n, gfz_n, t) \neq 0$ or the limit does not exists. Thus, both f and g both are discontinuous at their common fixed point. Hence the results.

CONCLUSION

In all the results proved in this paper, we have not assumed any mappings to be continuous. In fact the mappings assumed by us become discontinuous at their common fixed point. Thus we provide more answer to the open problem posed by Rhoades [12] regarding existence a contractive condition which is strong enough to generate a fixed point, but which does not force the map to be continuous at the fixed point.

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