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t- Intuitionistic Fuzzy Subgroups

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Abstract

In this paper, the notion of t-intuitionistic fuzzy set and t-intuitionistic fuzzy subgroup (normal subgroup) are defined and discussed. The homomorphic behavior of t-intuitionistic fuzzy subgroup (normal subgroup) and their inverse homomorprhic images has been obtained. Some related result have been derived.

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Introduction
The notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] as a generalization of Zadeh's fuzzy sets. After the introduction of the notion of intuitionistic fuzzy group by Biswas [3]. Many researcher's tried to generalize the notion of intuitionistic fuzzy group, for example Zhan and Tan [11] define the notion of intuitionistic fuzzy M- group , Palaniappan , Naganathan and Arjanan [5] define intuitionistic L-fuzzy Subgroup , Fathi and salleh [4] introduced the notion of intuitionistic fuzzy group based on the notion of fuzzy space. The notion of t-intuitionistic fuzzy coset and t-intuitionistic fuzzy quotient group has already been introduced by the author in [6]. Here in this paper, we introduce the notion of t-intuitionistic fuzzy set and then define t-intuitionistic fuzzy subgroup (normal subgroup) of a group G and study their properties.

Preliminaries
We first recall some definition for the sake of completeness of the topic under study.

Definition 2.1: [1] Let X be a fixed non-empty set. An intuitionistic fuzzy set (IFS) A of X is an object of the form $A = \{< x , \mu_A(x) , \nu_A(x) > : x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Remark 2.2

- When $\mu_A(x) + \nu_A(x) = 1$, i.e. when $\nu_A(x) = 1 - \mu_A(x) = \mu_A^c(x)$. Then A is called fuzzy set.
- We use the notation $A = (\mu_A , \nu_A)$ to denote the IFS A of X.

Definition 2.3: [7] : Let G be a group. An intuitionistic fuzzy subset (IFS) $A = (\mu_A , \nu_A)$ of G is called intuitionistic fuzzy subgroup (IFSG) of G if

- $\mu_A(xy) \geq \min\{\mu_A(x) , \mu_A(y) \}$
- $\nu_A(xy) \leq \max\{\nu_A(x) , \nu_A(y)\}$
- $\mu_A(x^{-1}) = \mu_A(x)$
- $\nu_A(x^{-1}) = \nu_A(x)$, for all $x , y \in G$

Or Equivalently A is IFSG of G if and only if
$\mu_A(xy^{-1}) \geq \min \{\mu_A(x) , \mu_A(y)\}$ and $\nu_A(xy) \leq \max \{\nu_A(x) , \nu_A(y)\}$

Definition 2.4 [7] An IFSG $A = (\mu_A , \nu_A)$ of a group G is said to be intuitionistic fuzzy normal subgroup of G (In short IFNSG) of G if

- $\mu_A(xy) = \mu_A(yx)$
- $\nu_A(xy) = \nu_A(yx)$, for all $x , y \in G$

Or Equivalently A is an IFNSG of a group G is normal if and only if
$\mu_A(y^{-1} x y) = \mu_A(x)$ and $\nu_A(y^{-1} x y) = \nu_A(x)$, for all $x , y \in G$

Definition 2.5: [7] Let G be a group and A be IFSG of group G . Let $x \in G$ be a fixed element. Then for every element $g \in G$, we define

- $(xA)(g) = (\mu_{xA}(g), \nu_{xA}(g))$,where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\nu_{xA}(g) = \nu_A(x^{-1}g)$. Then xA is called intuitionistic fuzzy left coset of G determined by A and x
- $Ax(g) = (\mu_{Ax}(g) , \nu_{Ax}(g))$, where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\nu_{Ax}(g) = \nu_A(gx^{-1})$ Then Ax is called the intuitionistic fuzzy right coset of G determined by A and x .

Definition 2.6: [9] An IFSG A of a group G is IFNSG of G if and only if
$xA = Ax$ for all $x \in G$

Definition 2.7: [7]: Let A be intuitionistic fuzzy set of a universe set X . Then (α , β)-cut of A is a crisp subset $C_{\alpha , \beta}(A)$ of the IFS A is given by
$C_{\alpha , \beta}(A) = \{ x : x \in X$ such that $\mu_A(x) \geq \alpha , \nu_A(x) \leq \beta \}$,

where α , $\beta \in [0 , 1]$ with $\alpha + \beta \leq 1$.

Theorem 2.8: [7 ,9] : If A is IFS of a group G . Then A is IFSG (IFNSG) of G if and only if $C_{\alpha,\beta}(A)$ is a subgroup (normal) of group G , for all α , $\beta \in [0,1]$ with $\alpha + \beta \leq 1$.

Definition 2.9: [8]: Let X and Y be two non-empty sets and *f* : X → Y be a mapping . Let A and B be IFS's of X and Y respectively. Then the image of A under the map *f* is denoted by *f* (A) and is defined as

$$f(A)(y) = \left(\mu_{f(A)}(y) \, , \, \nu_{f(A)}(y) \right) \text{, where}$$

$$\mu_{f(A)}(y) = \begin{cases} \vee\{ \mu_A(x) : x \in f^{-1}(y)\} \\ 0 \; ; \quad \text{otherwise} \end{cases} \text{ and } \nu_{f(A)}(y) = \begin{cases} \wedge\{ \nu_A(x) : x \in f^{-1}(y)\} \\ 1 \; ; \quad \text{otherwise} \end{cases}$$

i.e. $$f(A)(y) = \begin{cases} \left(\vee\{ \mu_A(x) : x \in f^{-1}(y)\} \, , \wedge\{ \nu_A(x) : x \in f^{-1}(y)\} \right) \\ (0, 1) \; ; \quad \text{otherwise} \end{cases}$$

Also the pre-image of B under *f* is denoted by f^{-1}(B) and is defined as $f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x) \, , \, \nu_{f^{-1}(B)}(x))$ where $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ and $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$ i.e.

$$f^{-1}(B)(x) = (\mu_B(f(x)) \, , \, \nu_B(f(x)))$$

Note (2.10): For any $x \in X$, we have $\mu_{f(A)}(f(x)) \geq \mu_A(x)$ and $\nu_{f(A)}(f(x)) \leq \nu_A(x)$

t - Intuitionistic fuzzy Subgroup

Definition 3.1: Let A be an IFS of a group G. Let $t \in [0,1]$. Then the IFS A^t of G is called the t- intuitionistic fuzzy subset of G w.r.t. IFS A and is defined as $A^t = (\mu_{A^t}, \nu_{A^t})$, where $\mu_{A^t}(g) = \min\{\mu_A(g), t\}$ and $\nu_{A^t}(g) = \max\{\nu_A(g), 1\text{-}t\}$, *for all* $g \in G$

Re*sult*. (3.2). Let $A^t = (\mu_{A^t}, \nu_{A^t})$ and $B^t = (\mu_{B^t}, \nu_{B^t})$ be two t-IFS of a group G. Then $(A \cap B)^t = A^t \cap B^t$.

Proof. Let $x \in G$ be any element , then

$$\begin{aligned} \mu_{(A\cap B)^t}(x) &= \min\{\mu_{A\cap B}(x), \, t\} \\ &= \min[\min\{\mu_A(x), \, \mu_B(x)\}, t] \\ &= \min[\, \min\{\mu_A(x), t\}, \min\{\mu_B(x), t\}] \\ &= \min\{\, \mu_{A^t}(x), \mu_{B^t}(x)\} \\ &= \mu_{A^t \cap B^t}(x) \end{aligned}$$

Similarly, we can show that $\nu_{(A\cap B)^t}(x) = \nu_{A^t \cap B^t}(x)$

Hence $(A \cap B)^t = A^t \cap B^t$.

Re*sult*(3.3) : *Let* $f : X \longrightarrow Y$ be a mapping and A and B are two IFS of X and Y respectively, then (i) $f^{-1}(B^t) = (f^{-1}(B))^t$ (ii) $f(A^t) = (f(A))^t$, for all t $\in [0,1]$

Proof. (i) $f^{-1}(B^t)(x) = B^t(f(x)) = (\mu_{B^t}(f(x)),\ \nu_{B^t}(f(x)))$

$$= \left(\min\{\mu_B(f(x)),\ t\},\ \max\{\nu_B(f(x)), 1-t\}\right)$$

$$= \left(\min\{\mu_{f^{-1}(B)}(x),\ t\},\ \max\{\nu_{f^{-1}(B)}(x), 1-t\}\right)$$

$$= \left(\mu_{(f^{-1}(B))^t}(x),\ \nu_{(f^{-1}(B))^t}(x)\right)$$

$$= (f^{-1}(B))^t(x)$$

Hence $f^{-1}(B^t) = (f^{-1}(B))^t$.

(*ii*) $f(A^t)(y) = \left(\vee\{\mu_{A^t}(x) : f(x) = y\},\ \wedge\{\nu_{A^t}(x) : f(x) = y\}\right)$

$$= \left(\vee[\min\{\mu_A(x),\ t\}:\ f(x) = y\}],\ \wedge[\max\{\nu_A(x),\ 1\text{-}t\}:\ f(x) = y\}]\right)$$

$$= \left(\min[\vee\{\mu_A(x) : f(x) = y\}, t],\ \max[\wedge\{\nu_A(x) : f(x) = y\}, 1-t]\right)$$

$$= \left(\min\{\mu_{f(A)}(y),\ t\}\ ,\ \max\{\nu_{f(A)}(y),\ 1\text{-}t\}\right)$$

$$= \left(\mu_{(f(A))^t}(y),\ \nu_{(f(A))^t}(y)\right)$$

$$= (f(A))^t(y)$$

Hence $f(A^t) = (f(A))^t$.

Definition 3.4: Let A be an IFS of a group G. Let t $\in$ [0,1]. Then A is called t-intuitionistic fuzzy subgroup (In short t- IFSG) of G if A^t is IFSG of G i.e. if the following conditions hold

(i) $\mu_{A^t}(xy) \geq \min\{\mu_{A^t}(x),\ \mu_{A^t}(y)\}$ (ii) $\nu_{A^t}(xy) \leq \max\{\nu_{A^t}(x),\ \nu_{A^t}(y)\}$

(*iii*) $\mu_{A^t}(x^{-1}) = \mu_{A^t}(x)$ (iv) $\nu_{A^t}(x^{-1}) = \nu_{A^t}(x)$, for all x , y $\in$ G

Proposition 3.5: If A is IFSG of a group G , then A is also t- IFSG of G.

Proof: Let x, y $\in$ G be any element of the group G.

$$\mu_{A^t}(xy) = \min\{\mu_A(xy),\ t\}$$

$$\geq \min[\min\{\mu_A(x),\ \mu_A(y)\}, t]$$

$$= \min[\ \min\{\mu_A(x),\ t\},\ \min\{\mu_A(x),\ t\}]$$

$$= \min\{\ \mu_{A^t}(x),\ \mu_{A^t}(y)\}$$

Thus $\mu_{A^t}(xy) \geq \min\{\mu_{A^t}(x),\ \mu_{A^t}(y)\}$

Similarly, we can show that $\nu_{A^t}(xy) \le \max\{\nu_{A^t}(x),\ \nu_{A^t}(y)\}$

Also, $\mu_{A^t}(x^{-1}) = \min\{\mu_A(x^{-1}),\ t\} = \min\{\mu_A(x),\ t\} = \mu_{A^t}(x)$

Similarly, we can show that $\nu_{A^t}(x^{-1}) = \nu_{A^t}(x)$

Hence A is t-IFSG of G.

Remark 3.6: The converse of above proposition (3.5) need not be true

Example 3.7: Let G = { e , *a* , *b* , *ab* }, where $a^2 = b^2 = e$ and *ab* = *ba* be the Klein four group. Define the IFS A = {< e , 0.1 , 0.1> , < *a* , 0.3 , 0.3> , < *b* , 0.3 , 0.4> , < *ab* , 0.2 , 0.4> } of G. Clearly , A is not IFSG of G . Take t = 0.05 . Then

$\mu_A(x) > t$ for all x $\in$G also $\nu_A(x) < 1 - t$ for all x $\in$G

$\mu_{A^t}(x) = \min\{\mu_A(x),\ t\} = t$ and $\nu_{A^t}(x) = \max\{\nu_A(x),\ 1\text{-}t\} = 1 - t$, for all $x \in G$

Therefore, $\mu_{A^t}(xy) \ge \min\{\mu_{A^t}(x),\ \mu_{A^t}(y)\}$ and $\nu_{A^t}(xy) \le \max\{\nu_{A^t}(x),\ \nu_{A^t}(y)\}$ hold

Further, as $a^{-1} = a$, $b^{-1} = b$, $(ab)^{-1} = (ab)$. So $\mu_{A^t}(x^{-1}) = \mu_{A^t}(x)$ and $\nu_{A^t}(x^{-1}) = \nu_{A^t}(x)$ hold

Hence A is t- IFSG of G.

Proposition 3.8: Let A be a IFS of group G such that $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ hold for all x $\in$G. Let t < min{ p , 1- q } , where p = min{ $\mu_A(x)$: for all x$\in$G} and q = max{ $\nu_A(x)$: for all x$\in$G}. Then A is t-IFSG of G.

Proof: Since t < min{ p , 1- q } implies that p > t and q < 1- t

$\Rightarrow$ min{ $\mu_A(x)$: for all x$\in$G} > t and max{ $\nu_A(x)$: for all x$\in$G} < 1 – t

$\Rightarrow$ $\mu_A(x) > t$ for all x $\in$G also $\nu_A(x) < 1 - t$ for all x $\in$G

Therefore, $\mu_{A^t}(xy) \ge \min\{\mu_{A^t}(x),\ \mu_{A^t}(y)\}$ and $\nu_{A^t}(xy) \le \max\{\nu_{A^t}(x),\ \nu_{A^t}(y)\}$ hold

Further , as $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ holds for all x $\in$G. So $\mu_{A^t}(x^{-1}) = \mu_{A^t}(x)$ and $\nu_{A^t}(x^{-1}) = \nu_{A^t}(x)$ hold . Hence A is t- IFSG of G.

Proposition 3.9: Intersection of two t-IFSG's of a group G is also t-IFSG of G.

Proof: Let x, y $\in$ G be any element of the group G. Then

$$\begin{aligned}
\mu_{(A\cap B)^t}(xy) &= \min\{\mu_{A\cap B}(xy),\ t\} \\
&= \min[\min\{\mu_A(xy),\ \mu_B(xy)\}, t] \\
&= \min[\ \min\{\mu_A(xy),\ t\},\ \min\{\mu_B(xy),\ t\}] \\
&= \min\{\ \mu_{A^t}(xy),\ \mu_{B^t}(xy)\} \\
&\geq \min[\min\{\ \mu_{A^t}(x), \mu_{A^t}(y)\}, \min\{\mu_{B^t}(x), \mu_{B^t}(y)\}] \\
&= \min[\ \min\{\ \mu_{A^t}(x), \mu_{B^t}(x)\}, \min\{\mu_{A^t}(y), \mu_{B^t}(y)\}] \\
&= \min\{\ \mu_{A^t\cap B^t}(x), \mu_{A^t\cap B^t}(y)\} \\
&= \min\{\ \mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\}
\end{aligned}$$

Thus $\mu_{(A\cap B)^t}(xy) \geq \min\{\mu_{(A\cap B)^t}(x),\ \mu_{(A\cap B)^t}(y)\}$

Similarly, we can show that $\nu_{(A\cap B)^t}(xy) \leq \max\{\nu_{(A\cap B)^t}(x),\ \nu_{(A\cap B)^t}(y)\}$

$$\begin{aligned}
\textit{Also},\quad \mu_{(A\cap B)^t}(x^{-1}) &= \min\{\mu_{A\cap B}(x^{-1}),\ t\} \\
&= \min[\min\{\mu_A(x^{-1}),\ \mu_B(x^{-1})\}, t] \\
&= \min[\ \min\{\mu_A(x^{-1}),\ t\},\ \min\{\mu_B(x^{-1}),\ t\}] \\
&= \min\{\ \mu_{A^t}(x^{-1}),\ \mu_{B^t}(x^{-1})\} \\
&= \min\{\ \mu_{A^t}(x),\ \mu_{B^t}(x)\} \\
&= \mu_{A^t\cap B^t}(x) \\
&= \mu_{(A\cap B)^t}(x)
\end{aligned}$$

Similarly, we can show that $\nu_{(A\cap B)^t}(x^{-1}) = \nu_{(A\cap B)^t}(x)$

Hence $A \cap B$ is t- IFSG of G.

Corollary 3.10: Intersection of a family of t-IFSG's of a group G is again t-IFSG of G.

Definition 3.11: [6]. Let A be t- IFSG of a group G, where t $\in$ [0,1]. For any x$\in$G Define an IFS $A^t{}_x$ of G called t – intuitionistic fuzzy right coset of A in G as follows

$A^t{}_x(g) = (\mu_{A^t{}_x}(g)\ , \nu_{A^t{}_x}(g)\ )$, where

$\mu_{A^t{}_x}(g) = \min\{\mu_A(gx^{-1}),\ t\}$ and $\nu_{A^t{}_x}(g) = \max\{\nu_A(gx^{-1}),\ 1\text{-}\ t\}$ for all x , g $\in$ G.

Similarly, we can define the t – intuitionistic fuzzy left coset xA^t of A in G as follows

$xA^t(g) = (\mu_{xA^t}(g)\ , \nu_{xA^t}(g)\ )$, where

$\mu_{xA^t}(g) = \min\{\mu_A(x^{-1}g),\ t\}$ and $\nu_{xA^t}(g) = \max\{\nu_A(x^{-1}g),\ 1\text{-}\ t\}$ for all x , g $\in$ G.

Definition 3.12: Let A be t-IFSG of a group G, where t $\in$ [0,1]. Then A is called t – intuitionistic fuzzy normal subgroup (t- IFNSG) of G if and only if $xA^t = A^t{}_x$ for all x $\in$ G.

Note: Clearly , 1- IFNSG is ordinary IFNSG of G.

Remark 3.13: If A is IFNSG of a group G , then A is also t- IFNSG of G.

Proof: Let A be IFNSG of G. Then for any x $\in$ G , we have xA = Ax i.e.,

$\mu_A(x^{-1}g) = \mu_A(gx^{-1})$ and $\nu_A(x^{-1}g) = \nu_A(gx^{-1})$, for all g $\in$ G.

So $\min\{\mu_A(x^{-1}g),\ t\} = \min\{\mu_A(gx^{-1}),\ t\}$ and $\max\{\nu_A(x^{-1}g),\ 1\text{-}\ t\} = \max\{\nu_A(gx^{-1}),\ 1\text{-}\ t\}$

i.e., $\mu_{xA^t}(g) = \mu_{A^t_x}(g)$ and $\nu_{xA^t}(g) = \nu_{A^t_x}(g)$ for all g $\in$ G

Thus, $xA^t = A^t_x$.

The converse of above result need not be true .

Example 3.14: Let G = D_3 = < *a* , *b* : $a^3 = b^2$ = e , $ba = a^2b$ > be the dihedral group with six elements . Define the IFSG A = (μ_A , ν_A) of D_3 by

$$\mu_A(x) = \begin{cases} 0.8 & \text{if } x \in <b> \\ 0.7 & \text{if otherwise} \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0.1 & \text{if } x \in <b> \\ 0.2 & \text{if otherwise} \end{cases}$$

$$i.e.,\ A = \{<e, 0.8, 0.1>, <a, 0.7, 0.2>, <a^2, 0.7, 0.2>, <b, 0.8, 0.1>, <ab, 0.7, 0.1>, <ba, 0.8, 0.1>\}$$

Note that A is not IFNSG of G , for $\mu_A(ab) = 0.7 \neq 0.8 = \mu_A(ba)$.

Now, take t = 0.6 , we get

$\min\{\mu_A(x^{-1}g),\ t\} = t = \min\{\mu_A(gx^{-1}),\ t\}$ and $\max\{\nu_A(x^{-1}g),\ 1\text{-}\ t\} = 1\text{-}\ t = \max\{\nu_A(gx^{-1}),\ 1\text{-}\ t\}$, for all x , g $\in$ G. Hence A is t- IFNSG of G.

Proposition 3.15: Let A be t- IFNSG of a group G. Then

$\mu_{A^t}(xy) = \mu_{A^t}(yx)$ and $\nu_{A^t}(xy) = \nu_{A^t}(yx)$ hold for all x , y $\in$ G

Proof: Since A be t- IFNSG of a group G. Therefore $xA^t = A^t_x$ hold for all x$\in$G

$\Rightarrow xA^t(y^{-1}) = A^t_x(y^{-1})$ hold for all $y^{-1} \in$ G

$\Rightarrow \min\{\mu_A(x^{-1}y^{-1}),\ t\} = \min\{\mu_A(y^{-1}x^{-1}),\ t\}$

$\Rightarrow \mu_{A^t}(x^{-1}y^{-1}) = \mu_{A^t}(y^{-1}x^{-1}) \Rightarrow \mu_{A^t}((yx)^{-1}) = \mu_{A^t}((xy)^{-1})$

$\Rightarrow \mu_{A^t}(yx) = \mu_{A^t}(xy)$ [as A is t- IFSG of G so $\mu_{A^t}(g^{-1}) = \mu_{A^t}(g)$, for all g $\in$ G]

Similarly, we can show that $\nu_{A^t}(xy) = \nu_{A^t}(yx)$ hold for all x ,y $\in$G.

Next, we show that for some specific values of t , every IFSG A of G will always be t-IFNSG of G. In this direction , we have the following:

Proposition 3.16: Let A be an t-IFSG of a group G such that t < Min{ p , 1- q } , where p = min{ $\mu_A(x)$: for all x∈G} and q = max{ $\nu_A(x)$: for all x∈G}. Then A is t-IFNSG of G.

Proof: Since t < min{ p , 1- q } implies that p > t and q < 1- t
⇒ min{ $\mu_A(x)$: for all x∈G} > t and max{ $\nu_A(x)$: for all x∈G} < 1 – t
⇒ $\mu_A(x)$ > t for all x ∈G also $\nu_A(x)$ < 1- t for all x ∈G

$$\mu_{A^t_x}(g) = \min\{\mu_A(gx^{-1}),\ t\} = t \ \text{ and } \ \nu_{A^t_x}(g) = \max\{\nu_A(gx^{-1}),\ 1\text{-}\ t\} = 1-t$$

$$similarly,\ \mu_{xA^t}(g) = \min\{\mu_A(x^{-1}g),\ t\} = t \ \text{ and } \ \nu_{xA^t}(g) = \max\{\nu_A(x^{-1}g),\ 1\text{-}\ t\} = 1-t$$

$$\text{i.e., } \mu_{A^t_x}(g) = \mu_{xA^t}(g) \ \text{ and } \ \nu_{A^t_x}(g) = \nu_{xA^t}(g) \ , \text{ for all } g \in G$$

Thus $A^t_x = {}_xA^t$, for all x ∈ G . Hence A is t-IFNSG of G.

Proposition 3.17: Let G/ A^t denote the collection of all t – intuitionistic fuzzy cosets of an t-IFNSG A of G. i.e. G/ A^t = { A^t_x : x∈G }. Then the binary operations ⊗ defined on the set G/ A^t as follows:

$$A^t_x \otimes A^t_y = A^t_{xy} \quad , \text{ for all } x , y \in G$$

is a well defined operation .

Proof: Let $A^t_x = A^t_{x'}$ and $A^t_y = A^t_{y'}$, for some x , y , x′ , y′ ∈ G
Let g∈G be any element, then

$$[A^t_x \otimes A^t_y](g) = (A^t_{xy})(g) = (\mu_{A^t_{xy}}(g)\ ,\nu_{A^t_{xy}}(g))$$

$$\begin{aligned} Now\ \ \mu_{A^t_{xy}}(g) &= \min\{\mu_A g(xy)^{-1},\ t\} = \min\{\mu_A(gy^{-1})x^{-1},\ t\} = \mu_{A^t_x}(gy^{-1}) = \mu_{A^t_{x'}}(gy^{-1}) \\ &= \min\{\mu_A(gy^{-1})x'^{-1},\ t\} = \min\{\mu_A(x'^{-1}g)y^{-1},\ t\} = \mu_{A^t_y}(x'^{-1}g) = \mu_{A^t_{y'}}(x'^{-1}g) \\ &= \min\{\mu_A(x'^{-1}g)y'^{-1},\ t\} = \min\{\mu_A y'^{-1}(x'^{-1}g),\ t\} = \min\{\mu_A(y'^{-1}x'^{-1})g,\ t\} \\ &= \min\{\mu_A(x'y')^{-1}g,\ t\} = \min\{\mu_A g(x'y')^{-1},\ t\} \\ &= \mu_{A^t_{x'y'}}(g) \end{aligned}$$

Similarly , we can show that $\nu_{A^t_{xy}}(g) = \nu_{A^t_{x'y'}}(g)$, ∀ g ∈ G.

Therefore ⊗ is well defined operation on G/ A^t.

Lemma 3.18: If A is t-IFNSG of a group G. Then

$$A^t_x = A^t_{x'} \quad \Leftrightarrow \quad Nx = Nx' , \ \text{ for } x , x' \in G , \ \text{ wherer } \ N = C_{t,1-t}(A)$$

Proof: It can be outline similarly as lemma (3.3) in [6]

Proposition 3.19: The set G/ A^t of all t- Intuitionistic fuzzy cosets of t-IFNSG A of a group G , form a group under the well-defined operations ⊗.

Proof: It is easy to check that the identity element of G/ A^t is ${A^t}_e$, where e is the identity element of group G, and the inverse of an element ${A^t}_x$ is $A^t_{x^{-1}}$.

Proposition 3.20: A mapping f : G → G/ A^t , where G is a group and G/ A^t is the set of all t-intuitionistic fuzzy cosets of the t-IFNSG A of G defined by f (x) = ${A^t}_x$, is an onto homomorphism with ker f = N (= $C_{t,1-t}$(A)), where t ∈[0,1])

Proof: It can be outline similarly as Proposition (3.6) in [6]

Homomorphism of t-Intuitionistic fuzzy groups

Theorem (4.1): Let f : G_1 → G_2 be homomorphism of group G_1 into a group G_2. Let B be t-IFSG of group G_2. Then f^{-1}(B) is t-IFSG of group G_1.

Proof: Let B be t-IFSG of group G_2. Let x_1 , x_2 ∈ G_1 be any element. Then

$$f^{-1}(B^t)(x_1x_2) = (\mu_{f^{-1}(B^t)}(x_1x_2)\ ,\ \nu_{f^{-1}(B^t)}(x_1x_2))$$

$$\begin{aligned}\mu_{f^{-1}(B^t)}(x_1x_2) &= \mu_{B^t}(f(x_1x_2)) = \mu_{B^t}(f(x_1)f(x_2))\\ &\geq \min\{\mu_{B^t} f(x_1),\ \mu_{B^t} f(x_2)\}\\ &= \min\{\ \mu_{f^{-1}(B^t)}(x_1),\ \mu_{f^{-1}(B^t)}(x_2)\}\end{aligned}$$

Thus $\mu_{f^{-1}(B^t)}(x_1x_2) \geq \min\{\ \mu_{f^{-1}(B^t)}(x_1),\ \mu_{f^{-1}(B^t)}(x_2)\}$

Similarly, we can show that $\nu_{f^{-1}(B^t)}(x_1x_2) \leq \max\{\ \nu_{f^{-1}(B^t)}(x_1),\ \nu_{f^{-1}(B^t)}(x_2)\}$

Further, $\mu_{f^{-1}(B^t)}(x^{-1}) = \mu_{B^t}(f(x^{-1})) = \mu_{B^t}(f(x)^{-1}) = \mu_{B^t}(f(x)) = \mu_{f^{-1}(B^t)}(x)$

Simiarly, we can show that $\nu_{f^{-1}(B^t)}(x^{-1}) = \nu_{f^{-1}(B^t)}(x)$

Thus $f^{-1}(B^t) = (f^{-1}(B))^t$ is IFSG of G_1 and hence $f^{-1}(B)$ is t-IFSG of G_1.

Theorem 4.2: Let f : G_1 → G_2 be surjective homomorphism and A be t-IFSG of group G_1. Then f (A) is t-IFSG of group G_2.

Proof: Since A is t-IFSG of group G_1. Let y_1 , y_2 ∈ G_2 be any element. Then there exists some x_1 , x_2 ∈ G_1 such that f (x_1) = y_1 and f (x_2) = y_2. [Note that x_1 , x_2 need not be unique]

$$f(A^t)(y_1y_2) = (\mu_{f(A^t)}(y_1y_2)\ ,\ \nu_{f(A^t)}(y_1y_2))$$

$$\mu_{f(A^t)}(y_1y_2) = \mu_{(f(A))^t}(y_1y_2) = \min\{\ \mu_{f(A)}(f(x_1)f(x_2)),\ \text{t}\ \}$$
$$= \min\ \{\ \mu_{f(A)}(f(x_1x_2)),\ \text{t}\ \}$$
$$\geq \min\{\ \mu_A(x_1x_2)\ ,\text{t}\ \} = \mu_{A^t}(x_1x_2)$$
$$\geq \min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}\ ,\ \text{for all } x_1\ , x_2 \in G_1 \text{ such that } f(x_1) = y_1 \text{ and } f(x_2) = y_2$$
$$= \min\{\ \vee\{\mu_{A^t}(x_1) : f(x_1) = y_1\},\ \vee\{\mu_{A^t}(x_2) : f(x_2) = y_2\}\}$$
$$= \min\ \{\ \mu_{f(A^t)}(y_1),\ \mu_{f(A^t)}(y_2)\}$$

Thus $\mu_{f(A^t)}(y_1y_2) \geq \min\{\mu_{f(A^t)}(y_1),\ \mu_{f(A^t)}(y_2)\}$

Similarly, we can show that $\nu_{f(A^t)}(y_1y_2) \leq \max\{\nu_{f(A^t)}(y_1),\ \nu_{f(A)^t)}(y_2)\}$

Further, $f(A^t)(y^{-1}) = (\mu_{f(A^t)}(y^{-1})\ ,\ \nu_{f(A^t)}(y^{-1}))$

$$\mu_{f(A^t)}(y^{-1}) = \vee\{\mu_{A^t}(x^{-1}) :\ f(x^{-1}) = y^{-1}\}$$
$$= \vee\{\mu_{A^t}(x) :\ f(x) = y\}$$
$$= \mu_{f(A^t)}(y)$$

Similarly, we can show that $\nu_{f(A^t)}(y^{-1}) = \nu_{f(A^t)}(y)$

Thus $f(A^t) = (f(A))^t$ is IFSG of G_2 and hence $f(A)$ is t-IFSG of G_2.

Theorem 4.3: Let f : G_1 → G_2 be homomorphism of group G_1 into a group G_2. Let B be t-IFNSG of group G_2. Then f^{-1}(B) is t-IFNSG of group G_1.

Proof: Let B be t-IFNSG of group G_2. Let x_1 , x_2 ∈ G_1 be any element. Then $f^{-1}(B^t)(x_1x_2) = (\mu_{f^{-1}(B^t)}(x_1x_2)\ ,\ \nu_{f^{-1}(B^t)}(x_1x_2))$

$$\mu_{f^{-1}(B^t)}(x_1x_2) = \mu_{B^t}(f(x_1x_2)) = \mu_{B^t}(f(x_1)f(x_2))$$
$$= \mu_{B^t}(f(x_2)f(x_1)) = \mu_{B^t}(f(x_2x_1))$$
$$= \mu_{f^{-1}(B^t)}(x_2x_1)$$

Thus $\mu_{f^{-1}(B^t)}(x_1x_2) = \mu_{f^{-1}(B^t)}(x_2x_1)$

Similarly, we can show that $\nu_{f^{-1}(B^t)}(x_1x_2) = \nu_{f^{-1}(B^t)}(x_2x_1)$

Hence $f^{-1}(B^t) = (f^{-1}(B))^t$ is IFNSG of G_1 and hence $f^{-1}(B)$ is t-IFNSG of G_1.

Theorem 4.4: Let f : G_1 → G_2 be bijective homomorphism and A be t-IFNSG of group G_1. Then f (A) is t-IFNSG of group G_2.

Proof: Since A is t-IFNSG of group G_1. Let y_1 , y_2 ∈ G_2 be any element. Then there exists unique x_1 , x_2 $\in$ G_1 such that f (x_1) = y_1 and f (x_2) = y_2.

$$(f(A))^t(y_1y_2) = (\mu_{(f(A))^t}(y_1y_2)\ ,\ \nu_{(f(A))^t}(y_1y_2))$$

$$\mu_{(f(A))^t}(y_1y_2) = \min\{\ \mu_{f(A)}(f(x_1)f(x_2)),\ t\ \}$$

$$= \min\ \{\ \mu_{f(A)}(f(x_1x_2)),\ t\ \}$$

$$= \min\{\ \mu_A(x_1x_2)\ ,\ t\ \} = \mu_{A^t}(x_1x_2) = \ \mu_{A^t}(x_2x_1)$$

$$= \ \min\{\ \mu_A(x_2x_1)\ ,\ t\ \} = \min\ \{\ \mu_{f(A)}(f(x_2x_1)),\ t\ \}$$

$$= \min\ \{\ \mu_{f(A)}(f(x_2)f(x_1)),\ t\ \} = \min\ \{\ \mu_{f(A)}(y_2y_1),\ t\ \}$$

$$= \ \mu_{(f(A))^t}(y_2y_1)$$

Thus $\mu_{(f(A))^t}(y_1y_2) = \mu_{(f(A))^t}(y_2y_1)$

Similarly, we can show that $\nu_{(f(A))^t}(y_1y_2) = \nu_{(f(A))^t}(y_2y_1)$

Hence $(f(A))^t$ is IFNSG of G_2 and hence $(f(A))$ is t- IFNSG of G_2.

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