

Taylor Series Solution of Multiobjective Linear Fractional Programming Problem by Vague Set

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Abstract

In this paper, we have proposed a solution to Multiobjective Linear Fractional Programming problem (MOLFPP) by expanding the 1st order Taylor polynomial series. These objective functions at optimal point of each linear fractional objective functions in feasible region, reduce to an equivalent Multiobjective Linear Programming Problem (MOLPP). The resulting MOLPP is solved assuming that weight of these linear objective functions are equal and considering the sum of the these linear objective function. The proposed solution to MOLFPP always yields efficient solution even a strong efficient solution. Therefore the complexity in solving MOLFPP, has reduced to easy computation. To show the ability of proposed solution numerical example has been presented. The given example are solved using optimization software "TORA".

AMS subject classification:

Keywords: Multiobjective Linear Fractional Programming Problem (MOLFPP), Multiobjective Linear Programming problem (MOLPP), Taylor Series, TORA, Membership Function, Non Membership Function, Vague Goal Programming.

1. Introduction

In the modeling of real word problems like financial and corporate planning, production planning, marketing and media selection, university planning and student admission, health care and hospital planning, air force maintenance units, bank branches etc may be faced up frequently with the decisions to optimize debt-equity ratio, profit/cost ratio inventory/ sales, actual cost/standard cost output/employee, student/cost, nurse/patient ratio etc. with respect to some constraints. In the literature, different approaches is appeared to solve different models of Linear Fractional Programming Problem (LFPP). Since fractional programming solves more efficiently the above problems in respect to Linear Programming Problem (LPP). There exist several methodologies to solve multi objective linear fractional programming problem (MOLFPP) in the literature. Most of these methodologies are computationally burdensome (Chakraborty and Gupta [1], 2002), Kornbluth and Steuer [6] (1981), Y.J.Lai and C.L.Hwang [7] (1996) have developed an algorithm for solving the MOLFPP for the all weak-efficient vertices of the feasible region. Nykowski [10], Z. Aolkiewski(1978) and Dutta [4] et al. (1992) have proposed a compromise procedure for MOLFPP. Choo and Atkins [3] (1982) have given an analysis of the bicriteria LFPP, M.K.Luhandjula [8] (1984) solved MOLFPP using a fuzzy approach (Lai and Hwang [7] , 1996). He used linguistic approach to solve MOLFPP by introducing linguistic to represent linguistic aspiration of the DM.Dutta et all (1992) modified the linguistic approach of Luhandjula to solve MOLFPP. Goal programming (GP) is a multi-criteria decision making technique which applies to many real world problems in a precise manner. Goal programming is a extension of linear programming to include multiple objectives. In FGP, each objective function should be substantially less than or equal to same value, called aspiration level. Often, in real world problems, aspiration levels and/or priority factors of the DM, some time even the weights to be assigned to the goals are not assigned in precise manner. To overcome this ambiguity, Fuzzy set theory plays an important role. Robin and Narasimhan [14] (1984) and Tiwari R.N. [17] (1986), Dharmar S. and Rao J.R. [18] (1987), Hannan [13] (1981) are those persons which use Fuzzy set theory in Goal programming and investigated various aspects of decision problems using FGP.

In this paper, we have proposed a solution to MOLFPP using the 1st order Taylor polynomial series at optimal point of each linear fractional function in feasible region . LFPP has transformed into a more simplified structure MOLPP by using Taylor polynomial series and a very strong efficient solution has been proposed by using the approach.

2. Linear Fractional Programming Problem (LFPP)

The general format of the linear fractional programming problem can be defined as:

$$\text{Max } \frac{c^T x + \alpha}{d^T x + \beta} \quad (1)$$

subject to

$$Ax = b \quad (2)$$

$$\text{where } x \geq 0, \quad x, c^T, d^T \in R^n \quad (3)$$

$$\text{and } A \in R^{m \times n}, \alpha, \beta \in R \quad (4)$$

Definition : The two mathematical programming problems :

1. $Max F(x)$ subject to $x \in \Delta$,
2. $Max G(x)$, subject to $x \in \Delta$, will be said to be equivalent if and only if there is a one-to-one map $q(\cdot)$ of the feasible set of onto the feasible set of such that $F(x) = G(q(x))$ for all $x \in \Delta$

2.1. Multiple Objective Linear Fractional Programming Problem (MOLFPP)

A MOLFPP can be written as followed as:

$$Max(Z_i x) = \{Z_1(x), Z_2(x), Z_3(x), \dots, Z_n(x)\} \quad (5)$$

$$\text{s.t. } x \in X = \{x \in R^n, Ax \leq b, x \geq 0\}, \text{ with } b \in R^n, A \in R^{m \times n}$$

$$\text{and } Z_i(x) = Max\{Z_1(x), Z_2(x), Z_3(x), \dots, Z_k(x)\} \quad (6)$$

$$= \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)} \text{ where } c_i, d_i \in R^n \text{ and } \alpha_i, \beta_i \in R \quad (7)$$

Let us suppose that

$$Max Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)} = Z_i^{max} \quad (8)$$

$$Min Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)} = Z_i^{min} \quad (9)$$

3. Model Development

In this paper, we consider the Multiobjective Linear Fractional Programming Problem (MOLFPP)

$$Max(Z_i x) = \{Z_1(x), Z_2(x), Z_3(x), \dots, Z_n(x)\} \quad (10)$$

$$\text{s.t. } x \in X = \{x \in R^n, Ax \leq b, x \geq 0\}, \text{ with } b \in R^n, A \in R^{m \times n}$$

$$\text{and } Z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)}$$

where $c_i, d_i \in R^n$ and $\alpha_i, \beta_i \in R, D_i(x) > 0 \forall i$ (11)

Let the maximum value of the i^{th} objective function $Max Z_i(x) = Z_i^* \forall i$ on the feasible region, it occurs when $x_i^* = (x_{i1}^*, x_{i2}^*, \dots, x_{im}^*)$ for $i = 1, 2, 3, \dots, k$.

Suppose that $Z(x)$ and all of its partial derivatives of order less than and equal to $n + 1$ are continuous on the feasible region X , $x^* \in X$. By expanding the 1st order Taylor polynomial series for objective function $Z_i(x)$ about x_i^* , the objective function $Z_i(x)$ is obtained from $Z_i(x) = P_{i1}(x) + R_{i1}(x)$, where function $P_{i1}(x)$ is called the 1st Taylor polynomial in n variables about x_i^* and $R_{i1}(x)$ is the remainder term associated with $P_{i1}(x)$, we have

$$Z_i(x) \cong P_n(x) = Z_i(x^*) + \left[(x_1 - x_{i1}^*) \left(\frac{\partial Z_i(x_i^*)}{\partial x_1} \right) + (x_2 - x_{i2}^*) \frac{\partial Z_i(x_i^*)}{\partial x_2} \right. \\ \left. + \dots + (x_n - x_{in}^*) \frac{\partial Z_i(x_i^*)}{\partial x_n} \right] + O(h^2) \quad (12)$$

$$Z_i(x) \cong P_n(x) = Z_i(x_i^*) + \sum_{j=1}^n \left[(x_j - x_{ij}^*) \frac{\partial Z_i(x_i^*)}{\partial x_j} \right] + O(h^2) \\ \text{for } i = 1, \dots, k \quad (13)$$

where $O(h^2)$ is order of the maximum error. This polynomial gives an accurate approximation to $Z_i(x)$ when x is close to x^* . By replacing $Z_i(x) = \frac{c_i(x) + \alpha_i}{d_i(x) + \beta_i} = \frac{N_i(x)}{D_i(x)}$ by $Z_i(x) \cong P_n(x) = Z_i(x_i^*) + \sum_{j=1}^n \left[(x_j - x_{ij}^*) \frac{\partial Z_i(x_i^*)}{\partial x_j} \right]$ in above MOLFPP, all of the objective functions $Z_i(x)$, $i = 1, \dots, k$ become the 1st order linear functions as $e_i + \sum_{j=1}^n a_j(x_j)$, where $e_i, a_j \in R$, and $i = 1, 2, \dots, k$, therefore the above MOLFPP reduces to the following MOLPP:

$$\text{Max } \left\{ Z_1(x) = e_1 + \sum_{j=1}^n a_{1j}x_j, Z_2(x) = e_2 + \sum_{j=1}^n a_{2j}x_j, \dots, \right. \\ \left. Z_k(x) = e_k + \sum_{j=1}^n a_{kj}x_j \right\} \quad (14)$$

where $x \in X = \{x \in R^n, Ax \leq b, x \geq o\}$ with $b \in R^n, A \in R^{m \times n}$.

3.1. Vague Goal Programming Formulation

A linear membership function μ_i for each i^{th} fuzzy goal of the type $G_i(x) \geq g_i$ can be expressed as

$$\mu_i = \begin{cases} 1 & : G_i(x) \geq g_i \\ \frac{G_i(x) - L_i}{g_i - L_i} & : L_i \leq G_i(x) \leq g_i \\ 0 & : G_i(x) \leq L_i \end{cases} \quad (15)$$

where L_i is the lower tolerance limit for the fuzzy goal $G_i(x)$.

The non-membership function v_i for the goal is

$$v_i = \begin{cases} 0 & : G_i(x) \geq g_i \\ \frac{g_i - G_i(x)}{g_i - L_i^*} & : L_i^* \leq G_i(x) \leq g_i \\ 1 & : G_i(x) \leq L_i^* \end{cases} \quad \text{where } L_i^* \leq L_i \quad (16)$$

and for the goal $G_i(x) \leq g_i$, we have

$$\mu_i = \begin{cases} 1 & : G_i(x) \leq g_i \\ \frac{U_i - G_i}{U_i - g_i} & : g_i \leq G_i(x) \leq U_i \\ 0 & : G_i(x) \geq U_i \end{cases} \quad (17)$$

The non membership function is

$$v_i = \begin{cases} 0 & : G_i(x) \leq g_i \\ \frac{G_i - g_i}{U_i^* - g_i} & : g_i \leq G_i(x) \leq U_i^* \\ 1 & : G_i(x) \geq U_i^* \end{cases} \quad \text{where } U_i \leq U_i^* \quad (18)$$

And for each type problem i.e either $G_i(x) \geq g_i$ or $G_i(x) \leq g_i$ sum of the membership and non membership function is always less than and equal to one i.e $\mu_i + v_i \leq 1$.

Then vague goal programming problem for membership problem is followed as:

$$\text{Max } V(\mu) = \sum_{i=1}^k \mu_i \quad (19)$$

subject to $\mu_k = \frac{G_k(x) - L_k}{g_k - L_k}$ or $\mu_k = \frac{u_k - G_k(x)}{u_k - g_k}$ according to \geq or \leq type restriction, and $\mu_i \leq 1$.

For non membership function $\text{Max } V(v) = \sum_{i=1}^k (1 - v_i)$.

subject to $v(k) = \frac{g_k - G_k(x)}{g_k - L_k^*}$ or $v_k = \frac{G_k(x) - g_k}{U_k^* - g_k}$ according to \geq or \leq type restriction and $v_i \geq 0$.

In the above problem, the sum of the membership and non membership function is less than or equal to one in each case.

3.2. Numerical Example

Example 3.1. Let us consider a MOLFPF with two objective as follows:

$$\text{Max } \{Z_1(x) = \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, Z_2(x) = \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}\} \quad (20)$$

subject to the constraints:

$$x_1 - x_2 \geq 1 \quad (21)$$

$$2x_1 + 3x_2 \leq 15 \quad (22)$$

$$x_1 \geq 3 \quad (23)$$

$$\text{where } x_1, x_2 \geq 0 \quad (24)$$

It is observed that $Z_1 < 0$, $Z_2 \geq 0$, for each x in the feasible region.

$$\text{On solving, we get } Z_1^{\max}(3.6, 2.6) = \frac{-14}{23}, Z_2^{\max}(7.5, 0) = \frac{15}{11}.$$

By expanding the 1st order Taylor polynomial series for the objective function $Z_1(x)$ and $Z_2(x)$ about point (3.6, 2.6) and (7.5, 0) in feasible region respectively, we obtain

$$Z_1(x) = \frac{-14}{23} + (x_1 - 3.6) \frac{\partial Z_1(3.6, 2.6)}{\partial x_1} + (x_2 - 2.6) \frac{\partial Z_1(3.6, 2.6)}{\partial x_2} \quad (25)$$

$$Z_1(x) = -0.2599x_1 + 0.2835x_2 - 0.41 \quad (26)$$

$$\text{and } Z_2(x) = \frac{15}{11} + (x_1 - 7.5) \frac{\partial Z_2(7.5, 0)}{\partial x_1} + (x_2 - 0) \frac{\partial Z_2(7.5, 0)}{\partial x_2} \quad (27)$$

$$Z_2(x) = 0.004722x_1 - 0.04486x_2 + 1.3282 \quad (28)$$

As we observe that all the fractional objectives are transformed into the linear objectives.

Now it is possible to give aspirant level for both goal. Hence the corresponding Fuzzy Goal programming is as follows:

Find X satisfying the Fuzzy Goal

$$Z_1(x) = -0.2599x_1 + .2835x_2 - .41 \geq -0.5 \quad (29)$$

$$Z_2(x) = .004722x_1 - .04486x_2 + 1.3282 \leq 1 \quad (30)$$

subject to the constraints:

$$x_1 - x_2 \geq 1 \quad (31)$$

$$2x_1 + 3x_2 \leq 15 \quad (32)$$

$$x_1 \geq 3 \quad (33)$$

Let us considering the tolerance limit of 2 goal to be (- 1, 2) respectively.

Define membership function $\mu_1(x)$ for the 1st Fuzzy Goal $Z_1(x) \geq -0.5$

$$\mu_1(x) = \begin{cases} 1 & : \text{if } Z_1(x) \geq -0.5 \\ \frac{Z_1(x) + 1}{0.5} & : \text{if } -1 \leq Z_1(x) \leq -0.5 \\ 0 & : \text{if } Z_1(x) \leq -1 \end{cases} \quad (34)$$

for goal satisfying the inequality $Z_2(x) \leq 1$.

The membership function can be defined as

$$\mu_2(x) = \begin{cases} 1 & \text{if } Z_2(x) \leq 1 \\ \frac{2 - Z_2(x)}{1} & \text{if } 1 \leq Z_2(x) \leq 2 \\ 0 & \text{if } Z_2(x) \geq 2 \end{cases} \quad (35)$$

where 2 is the upper tolerance limit.

Fuzzy Goal Programming problem can be formulated as:

$$\text{Max } v = \sum_{i=1}^2 u_i \quad (36)$$

subject to

$$\mu_1 = \frac{Z_1(x) + 1}{0.5} \quad (37)$$

$$\text{i.e. } 0.5\mu_1 + 0.2599x_1 - 0.2835x_2 = 0.59 \quad (38)$$

$$\mu_2 = \frac{2 - Z_2(x)}{1} \quad (39)$$

$$\text{i.e. } \mu_2 + 0.004722x_1 - 0.04486x_2 = 0.6718 \quad (40)$$

$$x_1 - x_2 \geq 1 \quad (41)$$

$$2x_1 = 3x_2 \leq 15 \quad (42)$$

$$x_1 \geq 3 \quad (43)$$

$$\mu_1 \leq 1 \quad (44)$$

$$\mu_2 \leq 1 \quad (45)$$

$$\text{and } x_1, x_2, \mu_1, \mu_2 \geq 0 \quad (46)$$

The solution of the above problem using (TORA) software is $x_1 = 3.60$, $x_2 = 2.60$. The achieved goal value along with the corresponding membership grades are as given below:

$$Z_1 = -.61, \quad Z_2 = 1.23, \quad \mu_1 = .78, \quad \mu_2 = .77 \quad (47)$$

For non-membership function,

$$\text{Max } V^* = \sum_{i=1}^2 (1 - v_i) \quad (48)$$

subject to

$$-v_1 + 0.2599x_1 - 0.2835x_2 = 0.09 \quad (49)$$

$$1.5v_2 - 0.004722x_1 + 0.4486x_2 = 0.3282 \quad (50)$$

$$x_1 - x_2 \geq 1 \quad (51)$$

$$2x_1 + 3x_2 \leq 15 \quad (52)$$

$$x_1 \geq 3 \quad (53)$$

$$v_1 \leq 1 \quad (54)$$

$$v_2 \leq 1 \quad (55)$$

The solution of the above problem using (TORA) software is $x_1 = 3, x_2 = .76$.

The achieves goal values along with the corresponding non-membership function grades are obtain as follows:

$$z_1 = -.97, z_2 = 1, v_1 = .17, v_2 = .00. \quad (56)$$

4. Conclusion

In this paper, we have proposed a solution to Multiobjective Linear Fractional programming problem (MOLFPP) using Taylor polynomial series. With the help of the 1st Taylor polynomial series at optimal point of each linear fractional objective function in the feasible region, Multiobjective Linear Fractional Programming Problem reduces to an equivalent Multiobjective Linear Programming Problem(MOLPP). To obtain the solution for MOLPP, we assume the tolerance limit for goals and then we define the membership function for each goals, then we get new constraints with the help of membership function, and Objective function for new MOLPP is equal to the sum of the membership function. Similarly, we obtain a MOLPP corresponding to non-membership function and also sum of membership function and non-membership function is always less than and equal to one. The proposed solution to MOLPP always yield efficient solution. The complexity in solving MOLFPP has reduced to easy computation. The solution obtained from this method is very near to the solution of MOLFPP. Hence this method gives more accurate solution as compare to Nuran Guzel, Mustafa Sivri method to Taylor series solution of Multi-Objective Linear Fractional Programming Problem.

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