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Eigen Values of Complete Fuzzy Graphs

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Abstract

In this paper we investigate the eigen values of complete fuzzy graphs. With examples we show that some of the results corresponding to crisp graphs do not carry over to fuzzy graphs.

Introduction

There are many useful connections between eigen values of a graph and its combinatorial properties. Tremendous literature is available for theoretical results about graphs and their spectra. However, the Eigen values of fuzzy graphs has not been studied extensively so far. In this paper we have compared the eigen value properties of complete fuzzy graphs and their underlying crisp graphs.

Preliminaries

Definition 1 : A fuzzy graph G=(V, μ, ρ) is a non empty set V together with a pair of functions

μ : V→ [0, 1] and ρ : VxV →[0, 1] such that for all x, y in V, $\rho(x,y) \leq \mu(x) \Lambda \mu(y)$.

Here μ is a fuzzy subset of V and it is called membership function and ρ is a fuzzy relation on μ.

The underlying crisp graph of G=(V, μ, ρ) is denoted by $G^*(V, E)$ where E⊆VxV.

Definition 2: A fuzzy graph G:(μ, ρ) is strong graph if

$\rho(u, v) = \mu(u) \Lambda\, \mu(v)\ \forall\ (u, v) \in E$

Definition 3: A fuzzy graph G:(μ, ρ) is complete graph if

ρ(u, v) = μ(u)Λ μ(v) ∀ u, v ∈ V

Every complete fuzzy graph is strong but not conversely [3].

Definition 2: Adjacency matrix A of a graph G^*(V, E) is defined as A=[a_{ij}] where

a_{ij} = 1 if v_i and v_j are adjacent and 0 otherwise.

Definition 3: Adjacency matrix Mρ of fuzzy graph G(V, μ, ρ) is represented by a matrix where the rows and columns are indexed by the vertex set V and the x, y entry is ρ(x, y).

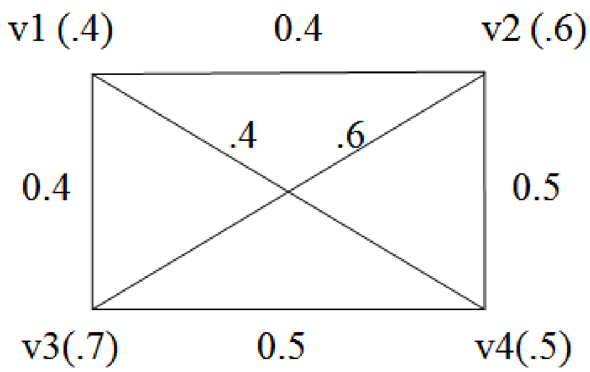
Definition 4: Eigen values of a graph are the Eigen values of its adjacency matrix.

Definition 5: Spectrum of a graph is the set of all eigen values of its adjacency matrix.

$\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots\ldots \geq \lambda_n$. Here λ1 is also called λmax.

Theorem 1: The complete graph K_n has eigen values n-1 and -1 with multiplicities 1 and n-1.

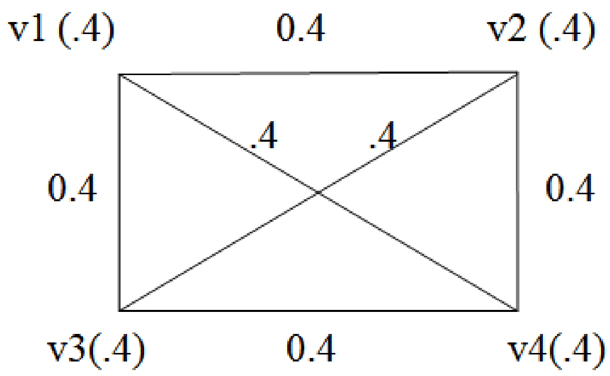
Example 1: The above theorem does not hold for fuzzy graphs, for eg. Consider the following complete fuzzy graph with vertex set V={v_1, v_2, v_3, v_4} with membership function $\mu(v_1) = .4$ $\mu(v_2) = .6$ $\mu(v_3) = .7$ $\mu(v_4) = .5$



Its eigen values are $1.4086, -.3, -.4, -.6$ which are distinct.

Lemma 1: If G(V, μ, ρ) is a complete fuzzy graph and μ (v_i) = k (constant) then G has eigen values (n-1)k and –k with multiplicity 1 and n-1 but the converse is not true.

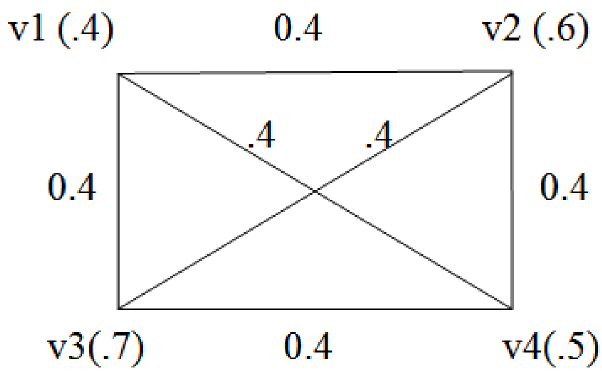
Example 2: If V=(v_1, v_2, v_3, v_4) and k=0.4 then the eigen values are $1.2, -.4, -.4, -.4$.



The underlying crisp graph of above graph is a complete graph K_4 and by theorem 1 has eigen values 3 and -1 with multiplicity 1 and 3.

Conversely, If a graph has eigenvalues (n-1)k and $-$k for some k then G need not be a complete fuzzy graph.

Example 3:



The eigen values are $1.2, -.4, -.4, -.4$ but the graph is not a complete fuzzy graph.

Lemma 2: If in a complete fuzzy graph μ (v) $\neq$ k, $\forall$ v $\in$ V then its eigen values are distinct.

In Example 1, μ (v) $\neq$ k, $\forall$ v $\in$ V and the graph has distint eigen values 1.4086, $-.3, -.4, -.6$

Theorem 2: For a crisp graph G the chromatic number χ(G) is at least $1 - (\frac{\lambda_1}{\lambda_n})$

The above theorem does not hold for fuzzy graphs if μ (v) $\neq$ k, $\forall$ v $\in$ V.In the example 1, λ_1=1.4086 and $\lambda_n = -.6$ but $1 - (\frac{\lambda_1}{\lambda_n})$ =3.347 where as the chromatic number of complete fuzzy graph is 4.

Lemma 3: Let G be a connected graph on n≥2 vertices.Then $\lambda_{max} \geq 1$

Proof: The above result does not hold for fuzzy graphs. For eg. if μ (v) =.3 for all v in V then by Lemma 1 the eigenvalues are 0.9, $-.3, -.3, -.3$

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