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On t-Best Coapproximation in fuzzy anti-2-normed linear spaces

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Abstract

In this paper, we study the concept of t-best coapproximation in fuzzy anti-2-normed linear spaces. We introduce the notion of t-best coapproximation, t-coproximinal sets, t-coChebyshev sets and t-orthogonality and prove some interesting theorems to characterization of t-best coapproximation elements.

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1. Introduction

The concept of best coapproximation was introduced by Franchetti and Furi [3], in order to study some characteristic properties of real Hilbert spaces, and such problems were considered further by Papini and Singer [9] and Rao and Saravanan [10]. The concept of 2-norm on a linear space has been introduced and developed by Gähler in [4, 5] and Gunawan and Mashadi [5]. The idea of fuzzy norm was initiated by Katsaras in [8]. In [7] Iqbal H. Jebril and Samanta introduced fuzzy anti-norm on a linear space depending on the idea of fuzzy anti-norm was introduced by Bag and Samanta [2] and investigated their important properties. In [11] Surender Reddy introduced the notion of convergent sequence and Cauchy sequence in fuzzy anti-2-normed linear space. Further, Surender Reddy studied the set of all t-best approximation and best simultaneous approximation on fuzzy anti-2-normed linear spaces in [12, 13]. Abrishami and Sistani [1] considered the set of all t-best coapproximations in fuzzy 2-normed linear spaces.

In this paper, we consider the set of t-best coapproximation in fuzzy anti-2-normed linear spaces and then prove several theorems pertaining to this set.

2. Preliminaries

Definition 2.1. Let X be a real linear space of dimension greater than one and let $\|\bullet,\bullet\|$ be a real valued function on $X \times X$ satisfying the following conditions

$2N_1$: $\|x,y\| = 0$ if and only if x and y are linearly dependent

$2N_2$: $\|x,y\| = \|y,x\|$

$2N_3$: $\|\alpha x,y\| = |\alpha|\ \|x,y\|$, for every $\alpha \in R$

$2N_4$: $\|x,y+z\| \leq \|x,y\| + \|x,z\|$

then the function $\|\bullet,\bullet\|$ is called a 2-norm on X and the pair $(X,\|\bullet,\bullet\|)$ is called a 2-normed linear space.

Example 2.2. Let $X = R^3$ be a real linear space. Define $\|\bullet,\bullet\| : X \times X \longrightarrow R$ by $\|x,y\| = \max\{|x_1y_2 - x_2y_1|, |x_2y_3 - x_3y_2|, |x_3y_1 - x_1y_3|\}$, where $x = (x_1,x_2,x_3)$, $y = (y_1,y_2,y_3)$ are in R^3. Then $(X,\|\bullet,\bullet\|)$ is a 2-normed linear space.

Definition 2.3. Let X be a linear space over a real field F. A fuzzy subset N of $X \times X \times R$ is called a fuzzy 2-norm on X if the following conditions are satisfied for all $x,y,z \in X$.

$(2-N_1)$: For all $t \in R$ with $t \leq 0$, $N(x,y,t) = 0$,

$(2-N_2)$: For all $t \in R$ with $t > 0$, $N(x,y,t) = 1$ if and only if x,y are linearly dependent

$(2-N_3)$: $N(x,y,t)$ is invariant under any permutation of x,y

$(2-N_4)$: For all $t \in R$ with $t > 0$, $N(x,cy,t) = N\left(x,y,\frac{t}{|c|}\right)$ if $c \neq 0$, $c \in F$

$(2-N_5)$: For all $s,t \in R$, $N(x,y+z,s+t) \geq \min\{N(x,y,s),N(x,z,t)\}$

$(2-N_6)$: $N(x,y,t)$ is a non-decreasing function of $t \in R$ and $\lim_{t\to\infty} N(x,y,t) = 1$.

Then the pair (X,N) is called a fuzzy 2-normed linear space (briefly F-2-NLS).

Example 2.4. Let $(X,\|\bullet,\bullet\|)$ be a 2-normed linear space. Define

$$\begin{aligned} N(x,y,t) &= \frac{t}{t+\|x,y\|}, \text{ if } t > 0, \ t \in R, \ x,y \in X \\ &= 0, \text{ if } t \leq 0, \ t \in R, \ x,y \in X. \end{aligned}$$

Then (X,N) is a fuzzy 2-normed linear space.

Definition 2.5. Let X be a linear space over a real field F. A fuzzy subset N of $X \times X \times R$ is called a fuzzy anti-2-norm on X if the following conditions are satisfied for all $x, y, z \in X$

$(a-2-N_1)$: For all $t \in R$ with $t \leq 0$, $N(x, y, t) = 1$,

$(a-2-N_2)$: For all $t \in R$ with $t > 0$, $N(x, y, t) = 0$ if and only if x, y are linearly dependent

$(a-2-N_3)$: $N(x, y, t)$ is invariant under any permutation of x, y

$(a-2-N_4)$: For all $t \in R$ with $t > 0$, $N(x, cy, t) = N\left(x, y, \frac{t}{|c|}\right)$ if $c \neq 0$, $c \in F$

$(a-2-N_5)$: For all $s, t \in R$, $N(x, y+z, s+t) \leq \max\{N(x, y, s), N(x, z, t)\}$

$(a-2-N_6)$: $N(x, y, t)$ is a non-increasing function of $t \in R$ and $\lim_{t\to\infty} N(x, y, t) = 0$.

Then the pair (X, N) is called a fuzzy anti-2-normed linear space (briefly Fa-2-NLS).

Remark 2.6. From $(a-2-N_3)$, it follows that in Fa-2-NLS,

$(a-2-N_4)$: For all $t \in R$ with $t > 0$, $N(cx, y, t) = N\left(x, y, \frac{t}{|c|}\right)$ if $c \neq 0$, $c \in F$

$(a-2-N_5)$: For all $s, t \in R$, $N(x+z, y, s+t) \leq \max\{N(x, y, s), N(z, y, t)\}$.

Example 2.7. Let $(X, \|\bullet, \bullet\|)$ be a 2-normed linear space. Define

$$\begin{aligned} N(x, y, t) &= \frac{\|x, y\|}{t + \|x, y\|}, \ \text{if } t > 0, \ t \in R, \ x, y \in X \\ &= 1, \ \text{if } t \leq 0, \ t \in R, \ x, y \in X. \end{aligned}$$

Then (X, N) is a Fuzzy anti-2-normed linear space.

Definition 2.8. A sequence $\{x_k\}$ in a fuzzy anti-2-normed linear space (X, N) is said to be t-converges to $x \in X$ if given $t > 0$, $0 < r < 1$, there exists an integer $n_0 \in N$ such that $N(x_1, x_k - x, t) < r$, for all $k \geq n_0$.

Theorem 2.9. In a fuzzy anti-2-normed linear space (X, N), a sequence $\{x_k\}$ is t-converges to $x \in X$ if and only $\lim_{k\to\infty} N(x_1, x_k - x, t) = 0$, $\forall\, t > 0$.

Definition 2.10. Let (X, N) be a fuzzy anti-2-normed linear space. Let $\{x_k\}$ be a sequence in X then $\{x_k\}$ is said to be t-Cauchy sequence if $\lim_{k\to\infty} N(x_1, x_{k+p} - x_k, t) = 0$, $\forall\, t > 0$ and $p = 1, 2, 3, \ldots$.

A fuzzy anti-2-normed linear space (X, N) is said to be complete if every Cauchy sequence in X is convergent. A complete fuzzy anti-2-normed linear space (X, N) is called a fuzzy anti-2-Banach space. The open ball $B(x, r, t)$ and the closed ball $B[x, r, t]$ with the center $x \in X$ and radius $0 < r < 1$, $t > 0$ are defined as follows:

$$B(x, r, t) = \{y \in X : N(x_1, x - y, t) < r\}$$
$$B[x, r, t] = \{y \in X : N(x_1, x - y, t) \leq r\}.$$

A subset A of X is said to be t-open if there exists $r \in (0,1)$ such that $B(x,r,t) \subset A$ for all $x \in A$ and $t > 0$. A subset A of X is said to be t-closed if for any sequence $\{x_k\}$ in A converges to $x \in A$. i.e., $\lim_{k \to \infty} N(x_1, x_k - x, t) = 0$, for all $t > 0$ implies that $x \in A$. A subset A of X is said to be t-compact if for every sequence $\{x_k\}$ in A has a subsequence $\{x_{n_k}\}$ which t-converges to an element $x_0 \in A$.

3. t-Best Coapproximation

Definition 3.1. Let A be a nonempty subset of fuzzy anti-2-normed linear space (X, N) and $t > 0$. For $x \in X$, an element $y_0 \in A$ is said to be a t-best coapproximation of x from A if $N(x_1, y_0 - y, t) \leq N(x_1, x - y, t)$, for all $y \in A$.

The set of all elements of t-best coapproximation of x from A is denoted by $R_A^t(x)$ and is defined as

$$R_A^t(x) = \{y_0 \in A : N(x_1, y_0 - y, t) \leq N(x_1, x - y, t), \forall\, y \in A\}$$

For $t > 0$ putting

$$\check{A}_x^t = \{x \in X : N(x_1, y, t) \leq N(x_1, y - x, t), \forall\, y \in A\} = (R_A^t)^{-1}(\{0\}).$$

It is clear $y_0 \in R_A^t(x)$ if and only $x - y_0 \in \check{A}_x^t$.

Definition 3.2. Let A be a non empty subset of a fuzzy anti-2-normed linear space (X, N). If for $t > 0$ and each $x \in X$ has at least (respectively exactly) one t-best coapproximation in A, then A is called a t-coproximinal (respectively t-coChebyshev) set. Also A is called t-quasi-coChebyshev set if $R_A^t(x)$ is a t- compact set.

Theorem 3.3. Let (X, N) be a fuzzy anti-2-normed linear space. and A be a subspace of X and $t > 0$. Then for each $x \in X$

(i) A is a t-coproximinal if and only if $X = A + \check{A}_x^t$.

(ii)A is a t-coChebyshev subspace if and only if $X = A \oplus \check{A}_x^t$.

Proof. (i) ($\Rightarrow$) Assume that A is t-coproximinal, $x \in X$ and $y_0 \in R_A^t(x)$. Then $x - y_0 \in \check{A}_x^t$. Now, $x = y_0 + (x - y_0) \in A + \check{A}_x^t$. Hence $X = A + \check{A}_x^t$.
($\Leftarrow$) Let $x \in X = A + \check{A}_x^t$. Then $x = y_0 + \check{y}$, $y_0 \in A$, $\check{y} \in \check{A}_x^t$ and so $0 \in R_A^t(\check{y}) = R_A^t(x - y_0)$. Since, $N(x_1, 0 - (x - y_0), t) \leq N(x_1, y - (x - y_0), t)$, so $N(x_1, y_0 - x, t) \leq N(x_1, (y + y_0) - x, t)$, where $y + y_0 \in A$; hence $y_0 \in R_A^t(x)$. Therefore A is t-coproximinal.
(ii) ($\Rightarrow$) Suppose that A is t-coChebyshev subspace, $x \in X$, and $x = y_1 + \check{y}_1 = y_2 + \check{y}_2$, where $y_1, y_2 \in A$ and $\check{y}_1, \check{y}_2 \in \check{A}_x^t$. We show that $y_1 = y_2$ and $\check{y}_1 = \check{y}_2$. Since $x = y_1 + \check{y}_1 = y_2 + \check{y}_2$, then $x - y_1 = \check{y}_1$, $x - y_2 = \check{y}_2$, this implies that $y_1, y_2 \in R_A^t(x)$. Therefore $y_1 = y_2$, it follows that $\check{y}_1 = \check{y}_2$. Thus $X = A \oplus \check{A}_x^t$.
($\Leftarrow$) Let $X = A \oplus \check{A}_x^t$, and suppose for $x \in X$, there exists $y_1, y_2 \in R_A^t(x)$. Then $x - y_1$,

$x - y_2 \in \check{A}_x^t$ and therefore $x = y_1 + \check{y_1} = y_2 + \check{y_2}$, where $\check{y_1} = x - y_1$ $\check{y_2} = x - y_2$. It follows that $y_1 = y_2$ and $\check{y_1} = \check{y_2}$. ■

Theorem 3.4. Let A be a non empty subset of a fuzzy anti-2-normed linear space (X, N). Then for $t > 0$ and each $x \in X$,

(i) $R^t_{A+y}(x + y) = R^t_A(x) + y$, for every $x, y \in X$.

(ii) $R^{|\alpha|t}_{\alpha A}(\alpha x) = \alpha R^t_A(x)$, for every $x \in X$ and $\alpha \in R\backslash\{0\}$.

(iii) A is t-coproximinal (respectively t-coChebyshev) if and only if $A + y$ is t-coproximinal (respectively t-coChebyshev), for any $y \in X$.

(iv) A is t-coproximinal (respectively t-coChebyshev) if and only if αA is $|\alpha|t$-coproximinal (respectively $|\alpha|t$-coChebyshev), for any given $\alpha \in R\backslash\{0\}$.

Proof. (i) For any $x, y \in X, t > 0, y_0 \in R^t_{A+y}(x+y)$ if and only if $N(x_1, y_0-(a+y), t) \leq N(x_1, x + y - (a + y), t)$ for all $(a + y) \in A + y$ if and only if, $N(x_1, (y_0 - y) - a, t) \leq N(x_1, x - a, t)$ for all $a \in A$, if and only if, $(y_0 - y) \in R^t_A(x)$, i.e., $y_0 \in R^t_A(x) + y$.
(ii) For any $x \in X$, $\alpha \in R\backslash\{0\}$ and $t > 0$, $y_0 \in R^{|\alpha|t}_{\alpha A}(\alpha x)$ if and only if, $N(x_1, (y_0 - \alpha a), |\alpha|t) \leq N(x_1, (\alpha x - \alpha a), \alpha t)$ for all $a \in A$ if and only if

$$N\left(x_1, \left(\frac{1}{\alpha}y_0 - a\right), |\alpha|t\right) \leq N(x_1, (x - a), t)$$

for all $a \in A$ if and only if $\frac{1}{\alpha}y_0 \in R^t_A(x)$ if and only if $y_0 \in \alpha R^t_A(x)$. Therefore $R^{|\alpha|t}_{\alpha A}(\alpha x) = \alpha R^t_A(x)$
(iii) The proof is an immediate consequence of (i).
(iv) The proof is an immediate consequence of (ii). ■

Corollary 3.5. Let M be a nonempty subspace of a fuzzy anti-2-normed linear space (X, N). Then for $t > 0$ and each $x \in X$,

(i) $R^t_M(x + y) = R^t_M(x) + y$, for every $x, y \in X$,

(ii) $R^{|\alpha|t}_M(\alpha x) = \alpha R^t_M(x)$, for every $x \in X$ and $\alpha \in R\backslash\{0\}$.

Proof. The proof is an immediate consequence of theorem 3.4 and this fact that $M + y = M$ and $\alpha M = M$ for all $y \in M$ and $\alpha \in R\backslash\{0\}$. ■

Definition 3.6. For $x \in X, a \in A, 0 < r < 1$, and $t > 0$, define $e^t_a(x) = N(x_1, x - a, t)$.

Theorem 3.7. Let (X, N) be a fuzzy anti-2-normed linear space, A be a subset of X, $x \in X\backslash\overline{A}$ and $t > 0$. Then we have $R^t_A(x) = [\bigcap_{a\in A} B[a, e^t_a(x), t]\ ] \bigcap A$.

Proof. By definition of $R_A^t(x)$, for each $a \in A$ we have $R_A^t(x) \subseteq [B[a, e_a^t(x), t]\,] \bigcap A$. Therefore $R_A^t(x) \subseteq [\bigcap_{a\in A} B[a, e_a^t(x), t]\,] \bigcap A$. Conversely, let $y \in [\bigcap_{a\in A} B[a, e_a^t(x), t]\,] \bigcap A$, then we have $y \in A$, and for each $a \in A$, $N(x_1, a - y, t) \leq e_a^t(x) = N(x_1, x - a, t)$, which implies that $y \in R_A^t(x)$. So $[\bigcap_{a\in A} B[a, e_a^t(x), t]\,] \bigcap A \subseteq R_A^t(x)$, which completes the proof. ■

Corollary 3.8. Let (X, N) be fuzzy anti-2-normed linear space, A be a subset of X, $x \in X\backslash \bar{A}$ and $t > 0$. Then

(i) The set $R_A^t(x)$ is t-bounded.

(ii) If A is t-closed then $R_A^t(x)$ is t-closed.

Theorem 3.9. Let (X, N) be fuzzy anti-2-normed linear space. For each $x \in X$ and $t > 0$, if A is a convex subset of X, then $R_A^t(x)$ is a convex subset of A (for $R_A^t(x) \neq \emptyset$).

Proof. Let $z_1, z_2 \in R_A^t$, then for $t > 0$ and each $x \in X$, $N(x_1, y - z_1, t) \leq N(x_1, x - y, t)$ and $N(x_1, y - z_2, t) \leq N(x_1, x - y, t)$ for all $y \in A$. Now for each $\lambda \in (0, 1)$ we have

$$\begin{aligned} N(x_1, y - (\lambda z_1 + (1-\lambda)z_2), t) &= N(x_1, \lambda y - \lambda z_1 + y - \lambda y - z_2 + \lambda z_2, t) \\ &= N(x_1, \lambda(y - z_1) + (1-\lambda)(y - z_2), \lambda t + (1-\lambda)t) \\ &\leq \max\left\{N\left(x_1, y - z_1, \frac{\lambda t}{\lambda}\right), N\left(x_1, y - z_2, \frac{(1-\lambda)t}{1-\lambda}\right)\right\} \\ &\leq \max\left\{N\left(x_1, x - y, \frac{\lambda t}{\lambda}\right), N\left(x_1, x - y, \frac{(1-\lambda)t}{1-\lambda}\right)\right\} \\ &\leq N(x_1, x - y, t). \end{aligned}$$

So $\lambda z_1 + (1-\lambda)z_2 \in R_A^t(x)$ and $R_A^t(x)$ is convex. ■

Theorem 3.10. For $t > 0$ and each $x \in X$. Let A be a t-coproximinal subspace of fuzzy anti-2-normed linear space (X, N). Then

(i) If $\check{A}_x^t$ is a t-compact set then A is t-quasi-coChebyshev.

(ii) If $\check{A}_x^t$ is a t-closed set then $R_A^t(x)$ is t-closed, for every $x \in X$.

Proof. (i) Suppose $x \in X$ and $\{y_n\}$ is a sequence in $R_A^t(x)$. Since $x - y_n \in \check{A}_x^t$ and $\check{A}_x^t$ is a t-compact set, there exists a subsequence $\{x - y_{n_k}\}$ that t-convergent to $x - y_0 \in \check{A}_x^t$. Consequently, $\{y_n\}$ has a subsequence $y_{n_k} \longrightarrow y_0 \in R_A^t(x)$ and hence A is t-quasi-coChebyshev.
(ii) The proof is similar to (i). ■

Definition 3.11. A subset A of a fuzzy anti-2-normed linear space (X, N) is said to be t-boundedly compact if every t-bounded sequence in A has a subsequence t-converging to an element of X.

Theorem 3.12. Suppose for some $t > 0$ and each $x \in X$, A is a t-boundedly compact and t-closed subset of a fuzzy anti-2-normed linear sapce (X, N) then A is t-quasi-coChebyshev.

Proof. Let $\{y_n\}$ be any sequence in $R^t_A(x)$. Then
$N(x_1, y_n - y, t) \leq N(x_1, x - y, t)$ for every $y \in A$. Since $R^t_A(x)$ is t-bounded, $\{y_n\}$ is a t-bounded sequence in A, and so $\{y_n\}$ has a t-convergent subsequence $\{y_{n_k}\}$, let $y_{n_k} \longrightarrow y_0 \in A$, as A is t-closed. Consider

$$N(x_1, y_0 - y, t) = \lim_k N(x_1, y_{n_k} - y, t) \leq N(x_1, x - y, t), \quad \text{for every } \ y \in A,$$

So $y_0 \in R^t_A(x)$, which implies that A is t-quasi-coChebyshev. ■

Definition 3.13. Let (X, N) be a fuzzy anti-2-normed linear space and A be a subset of X. For $t > 0$ and an element $x \in X$ is said to be t-orthogonal to an element $y \in X$ and we denote it by $x \perp^t_x y$, if $N(x_1, x + \lambda y, t) \geq N(x_1, x, t)$ for all scalar $\lambda \in R$, $\lambda \neq 0$. We say $A \perp^t_x y$ if $x \perp^t_x y$ for every $x \in A$.

Theorem 3.14. For $t > 0$ and each $x \in X$ and $y_0 \in A$, let (X, N) be a fuzzy anti-2-normed linear space and A be a subspace of X. If $A \perp^t_x x - y_0$ then $y_0 \in R^t_A(x)$.

Proof. Suppose $t > 0, x \in X$ and $A \perp^t_x x - y_0$. Then $N(x_1, a + \lambda(x - y_0), t) \geq N(x_1, a, t)$ for all $a \in A$ and all scalar $\lambda \in R$, $\lambda \neq 0$. Then $N\left(x_1, x - y_0 + \lambda^{-1}a, \frac{t}{|\lambda|}\right) \geq N\left(x_1, \lambda^{-1}a, \frac{t}{|\lambda|}\right)$. Hence $N\left(x_1, x - a', \frac{t}{|\lambda|}\right) \geq N\left(x_1, y_0 - a', \frac{t}{|\lambda|}\right)$, where $a' = y_0 - \lambda^{-1}a$. Now if $\lambda = 1$ then, $N(x_1, y - y_0, t) \geq N(x_1, x - y, t)$ for all $y \in A$ and so $y_0 \in R^t_A(x)$. ■

4. F-Best coapproximation

Definition 4.1. Let A be a nonempty subset of a fuzzy anti-2-normed linear space (X, N). An element $y_0 \in A$ is said to be an F-best coapproximation of $x \in X$ from A if it is a t-best coapproximation of x from A, for every $t > 0$, i.e., $y_0 \in \bigcap_{t \in (0,\infty)} R^t_A(x)$.

The set of all elements of F-best coapproximation of X from A is denoted by $FR^t_A(x)$, i.e., $FR^t_A(x) = \bigcap_{t \in (0,\infty)} R^t_A(x)$.

If each $x \in X$ has at least (respectively exactly) one F-best coapproximation in A, then A is called F-coproximinal (respectively F-coChebyshev) set.

Example 4.2. Let $X = R^3$. Define $N : X \times X \times X \times [0,\infty) \longrightarrow [0,1]$ by

$$\begin{aligned} N(x_1,x_2,x_3,t) &= \frac{\|x_1,x_2,x_3\|}{t}, \text{ if } t>0,\ t\in R,\ x_1,x_2,x_3\in X \\ &= 1, \qquad\qquad \text{if } t\le 0,\ t\in R,\ t\in R,\ x_1,x_2,x_3\in X, \end{aligned}$$

where $\|x_1,x_2,x_3\| = \min\limits_{1\le i\le 3}\sum\limits_{j=1}^{3}|x_{ij}|$. Then (X,N) is a fuzzy anti-3-normed linear space.

Let

$$A = \left\{(a,b,c)\in R^3 : a^2+b^2\le 1,\ 0\le c\le a^2+b^2\right\}$$

and $x_1 = (1,0,0)$, $x_2 = (0,1,0)$, $x = (0,0,4)$ are in X. Let $a_0 = (0,-1,1)$ and $a_1 = (0,1,1)$ are in A. Hence $a_0 = (0,-1,1)$ and $a_1 = (0,1,1)$ are F-best coapproximations of $x = (0,0,4)$ from A. Then $(0,-1,1),(0,1,1) \in FR_A^t(0,0,4)$. So, A is not a F-coChebyshev set.

Theorem 4.3. Let $\left\{\|\bullet,\bullet\|_\alpha^* : \alpha\in(0,1]\right\}$ be a descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X. Then $y_0 \in A$ is a best coapproximation to $x \in X$ in the descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X if and only if y_0 is a F-best coapproximation to x in the fuzzy anti-2-normed linear space (X,N).

Proof. For each $x \in X$, y_0 is a best coapproximation to $x \in X$ in the descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X if and only if $\|x_1, y-y_0\|_\alpha^* \le \|x_1, x-y\|_\alpha^*$, for every $y \in A$, if and only if

$$\frac{t}{t+\|x_1,y-y_0\|_\alpha^*} \ge \frac{t}{t+\|x_1,x-y\|_\alpha^*}$$

for every $y \in A$ and $t \in (0,\infty)$, if and only if $N(x_1, y-y_0, t) \le N(x_1, x-y, t)$ for every $y \in A$ and $t \in (0,\infty)$ if and only if $y_0 \in FR_A^t(x)$. ■

Definition 4.4. Let (X,N) be a fuzzy anti-2-normed linear space and A be a subset of X. For each element $x \in X$ is said to be F-orthogonal to an element $y \in X$ and we denote it by $x\perp_x^F y$, if for every $t>0$, $x\perp_x^t y$. We say $A\perp_x^F y$ if $x\perp_x^F y$ for every $x \in A$.

Theorem 4.5. Let $\left\{\|\bullet,\bullet\|_\alpha^* : \alpha\in(0,1]\right\}$ be a descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X. Then $x \in X$ is Brikhoff orthogonal to $y \in X$ in the descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X if and only if x is a F-orthogonal to y in the fuzzy anti-2-normed linear space (X,N).

Proof. For each $x \in X$, x is a Brikhoff orthogonal to $y \in X$ in the descending family of α-2-norm on X corresponding to the fuzzy anti-2-norm on X if and only if $\|x_1,x\|_\alpha^* \le \|x_1, x+\lambda y\|_\alpha^*$, for every $\lambda \in R\backslash\{0\}$, if and only if

$$\frac{t}{t+\|x_1,x\|_\alpha^*} \ge \frac{t}{t+\|x_1,x+\lambda y\|_\alpha^*}$$

for every $\lambda \in R\backslash\{0\}$ and $t > 0$, if and only if $N(x_1, x, t) \leq N(x_1, x + \lambda y, t)$ for every $\lambda \in R\backslash\{0\}$ and $t > 0$ if and only if $x \perp_x^F y$. ∎

Remark 4.6. The converse of theorem 3.14 is true, if we replace t-orthogonality with F-orthogonality.

5. Conclusion

In this paper, we introduced the concept of t-best coapproximation and F-best coapproximation in fuzzy anti-2-normed linear spaces and then prove several theorems pertaining to this sets.

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