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Intuitionistic Fuzzy $I(\beta, \delta)$-Mappings

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Abstract

The aim of this paper is to introduce intuitionistic fuzzy $I(\beta, \delta)$-map, intuitionistic fuzzy $M(\mu, \gamma)$-map, intuitionistic fuzzy $\overline{M}(\mu, \gamma)$-map, intuitionistic fuzzy (β, δ)-homeomorphism and investigate their properties in intuitionistic fuzzy topological spaces.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh[9] in the year 1965. After that the notion of intuitionistic fuzzy sets as a generalization of fuzzy set was introduced by Atanassov [1] in the year 1986. Recently, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Sunder Lal and Pushpendra Singh [6] introduced $I(\alpha, \beta)$-mapping in fuzzy topology, as a generalization of fuzzy continuity and fuzzy irresolute. In this paper we extend this concept to intuitionistic fuzzy topology by introducing intuitionistic fuzzy $I(\beta, \delta)$-map, intuitionistic fuzzy$M(\mu, \gamma)$-map, intuitionistic fuzzy$\overline{M}(\mu, \gamma)$-map and intuitionistic fuzzy(β, δ)-homeomorphism, as a generalisation of intuitionistic fuzzy continuity and irresolute,intuitionistic fuzzy open mapping, intuitionistic fuzzy closed mapping and intuitionistic fuzzy homeomorphism respectively.

2. Preliminaries

Definition 2.1. [1] An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be IFS's of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle | x \in X\}$. Then

(a) $A \subseteq B$ (simply $A \leq B$) if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) The complement of A is denoted by $\bar{A}$ and is defined by $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle | x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle | x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle | x \in X\}$.

For the sake of simplicity, we will use the notation $A = \langle x, \mu_A, \gamma_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$. The IFS's $0_\sim$ and $1_\sim$ are defined to be $0_\sim = \{\langle x, \underline{0}, \underline{1}\rangle | x \in X\}$ and $1_\sim = \{\langle x, \underline{1}, \underline{0}\rangle | x \in X\}$ respectively.

Definition 2.3. [5] Let $a, b \in [0,1]$ and $a + b \leq 1$. An intuitionistic fuzzy point (IFP for short) $x_{(a,b)}$ of X is an IFS of X defined by

$$x_{(a,b)}(y) = \begin{cases} (a,b) & \text{if} \quad y = x, \\ (0,1) & \text{if} \quad y \neq x\ . \end{cases}$$

In this case, x is called the support of $x_{(a,b)}$ and a and b are called the value and the nonvalue of $x_{(a,b)}$, respectively. An IFP $x_{(a,b)}$ is said to belong to an IFS $A = (\mu_A, \gamma_A)$ of X, denoted by $x_{(a,b)} \in A$, if $a \leq \mu_A(x)$ and $b \leq \gamma_A(x)$.

Definition 2.4. [5] Let $x_{(a,b)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $x_{(a,b)}$ if there is an IFOS B in X such that $x_{(a,b)} \in B \subseteq A$.

Definition 2.5. [1] Let X and Y be IFTS's and $f : X \to Y$ be a function. If $B = \{\{\langle y, \mu_B(y), \gamma_B(y)\rangle | y \in Y\}$ is an IFS in Y, then the pre image of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle \mid x \in X\}$

and the image of A under f, denoted by $f(A) = \langle y, f(\mu_A), f(\gamma_A) \rangle$ is an IFS of Y, where for each $y \in Y$.

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi \ , \\ \\ 0 & \text{otherwise} \ , \end{cases}$$

$$f(\gamma_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \phi, \\ \\ 1 & \text{otherwise} \ . \end{cases}$$

Definition 2.6. [2] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFS's in X satisfying the following axioms:

(1) $0_\sim, 1_\sim \in \tau$,

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(3) $\cup G_i \in \tau$ for any family $\{ G_i \mid i \in J\} \subseteq \tau$

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS) in X. The complement $\bar{A}$ of an IFOS A in IFTS(X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.7. [2] Let (X, τ)be an IFTS and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by
$Int(A) = \bigcup\{G|G$ is an IFOS in X and $G \subseteq A\}$,
$Cl(A) = \bigcap\{K|K$ is an IFCS in X and $A \subseteq K\}$.

Theorem 2.8. [2] For any IFS's A and B in IFTS (X, τ),

(a) $\overline{Int(A)} = Cl(\bar{A})$;

(b) $\overline{Cl(A)} = Int(\bar{A})$;

(c) $A \leq B \Rightarrow Int(A) \leq Int(B)$ and $Cl(A) \leq Cl(B)$.

Theorem 2.9. [7] For any IFS A in IFTS (X, τ),

(i) $Int(A) \leq Int(Cl(Int(A))) \leq Int(Cl(A)) \leq Cl(Int(Cl(A))) \leq Cl(A)$;

(ii) $Int(A) \leq Int(Cl(Int(A))) \leq Cl(Int(A)) \leq Cl(Int(Cl(A))) \leq Cl(A)$.

Proof. Follows from Theorem 2.8(c). ∎

Definition 2.10. Intuitionistic fuzzy nearly open sets: An IFS A in an IFTS (X, τ) is called.

(a) [3] an intuitionistic fuzzy semi-open set(IFSOS) if $A \leq cl(int(A))$;

(b) [3] an intuitionistic fuzzy α-open set(IFαOS)if $A \leq int(cl(int(A)))$;

(c) [3] an intuitionistic fuzzy pre-open set(IFPOS) if $A \leq int(cl(A))$;

(d) [3] an intuitionistic fuzzy regular open set(IFROS) if $A = int(cl(A))$;

(e) [8]an intuitionistic fuzzy semi-pre open set(IFSPOS) if there exists $B \in IFPO(X)$ such that $B \subseteq A \subseteq cl(B)$ (or) if $A \leq cl(int(cl(A)))$.

An IFS A is called an intuitionistic fuzzy semi closed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy pre closed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-pre closed set (resp. IFSCS, IFαCS, IFPCS, IFRCS, IFSPCS), if the complement of A is an IFSOS, IFαOS, IFPOS, IFROS and IFSPOS respectively.

3. Intuitionistic fuzzy (β, δ)-closed sets

Throughout this paper $\beta, \delta, \gamma, \ldots$ denote the collection of intuitionistic fuzzy sets in X containing $0_\sim$ & $1_\sim$. They need not be intuitionistic fuzzy topologies on X. Let M denote the family of all such collection of intuitionistic fuzzy sets in X.

Definition 3.1. [7] Let (X, τ) be a intuitionistic fuzzy topological space and let $\gamma \in M$. For each intuitionistic fuzzy sets A and B in X, define the two operators relative to γ as follows:

(a) $cl_\gamma(A) = \bigwedge\{B \geq A : \bar{B} \in \gamma\}$;

(b) $int_\gamma(A) = \bigvee\{B \leq A : B \in \gamma\}$.

Proposition 3.2. [7] Let A and B be intuitionistic fuzzy sets in a IFTS (X, τ) and $\gamma \in M$. Then

(a) $A \leq B \Rightarrow int_\gamma(A) \leq int_\gamma(B)$ and $cl_\gamma(A) \leq cl_\gamma(B)$;

(b) $int_\gamma(A) \leq A$ and $A \leq cl_\gamma(A)$;

(c) $\overline{int_\gamma(A)} = cl_\gamma(\bar{A})$;

(d) $\overline{cl_\gamma(A)} = int_\gamma(\bar{A})$;

(e) $cl_\gamma(A) = A$ if $\bar{A} \in \gamma$;

(f) $int_\gamma(A) = A$ if $A \in \gamma$;

(g) $cl_\gamma(cl_\gamma(A)) = cl_\gamma(A)$ and $int_\gamma(int_\gamma(A)) = int_\gamma(A)$.

where $Cl_\gamma(A)$ is intuitionistic fuzzy γ-closed set and $Int_\gamma(A)$ is intuitionistic fuzzy γ-open set.

Reverse statement of (a), (e) and (f) of the above proposition are need not be true.

Definition 3.3. Let (X,τ) be a IFTS and $\beta \in M$. Then

(a) an intuitionistic fuzzy set $A \in \beta$ is said to be IFβ-open set in X if $Int_\beta(A) = A$

(b) an intuitionistic fuzzy set $A \in \beta$ is said to be IFβ-closed set in X if $Cl_\beta(A) = A$.

Definition 3.4. An IFS A in IFTS(X,τ) is said to be an intuitionistic fuzzy δ-neighborhood (simply IFδ-neighborhood) of an IFP $x_{(a,b)}$ in (X,τ) if there is an IFδ-open set B in (X,τ) such that $x_{(a,b)} \in B \subseteq A$.

Definition 3.5. [7] Let (X,τ) be a IFTS and $\beta,\delta \in M$. Then,

(a) an intuitionistic fuzzy set B in X is called a intuitionistic fuzzy (β,δ)-closed set in X, if $B \leq A \Rightarrow cl_\beta(B) \leq A$, whenever A is δ- open.

(b) an intuitionistic fuzzy set B in X is called a intuitionistic fuzzy (β,δ)-open set in X, if $\bar{B}$ is a intuitionistic fuzzy (β,δ)-closed set in X.

4. Intuitionistic fuzzy $I(\beta,\delta)$-Mappings

In this section, we introduce intuitionistic fuzzy $I(\beta,\delta)$-map, intuitionistic fuzzy $M(\mu,\gamma)$-map, intuitionistic fuzzy $\overline{M}(\mu,\gamma)$-map and intuitionistic fuzzy (β,δ)-homeomorphism on intuitionistic fuzzy topological spaces and investigate some of their properties.

Definition 4.1. Let (X,τ) and (Y,σ) be IFTS's. Then a function $f : X \to Y$ is said to be a

(a) Intuitionistic fuzzy $I(\beta,\delta)$-map(simply $I(\beta,\delta)$-map) if $f^{-1}(B)$ is an IFβ-open set of X for each IFδ-open set B of Y.

(b) Intuitionistic fuzzy $M(\mu,\gamma)$-map (simply $M(\mu,\gamma)$-map) if $f(A)$ is an IFγ-open set of Y for each IFμ-open set A of X.

(c) Intuitionistic fuzzy$\overline{M}(\mu,\gamma)$-map (simply $\overline{M}(\mu,\gamma)$-map) if $f(A)$ is an IFγ-closed set of Y for each IFμ-closed set A of X.

(d) Intuitionistic fuzzy (β,δ)-homeomorphism (simply (β,δ)-homeomorphism) if f is bijective,$I(\beta,\delta)$-map and $M(\beta,\delta)$-map.

Equality of $M(\mu,\gamma)$-map and $\overline{M}(\mu,\gamma)$-map is established in the following theorem.

Theorem 4.2. Let (X,τ) and (Y,σ) be IFTS's. Let $f : X \to Y$ be one-to-one and onto. Then f is a $M(\mu,\gamma)$-map if and only if f is $\overline{M}(\mu,\gamma)$-map.

Proof. Let f be a $M(\mu,\gamma)$-map. Let A be a IFμ-closed set in X. Then $\overline{A}$ is IFμ-open set in X and hence $\overline{f(A)} = f(\overline{A})$ is IFγ-open set in Y. Thus f(A) is IFγ-closed set in Y and hence f is $\overline{M}(\mu,\gamma)$-map.

Reversing the steps converse follows.

Theorem 4.3. Let $f : (X,\tau) \to (Y,\sigma)$ be a map. Then f is $I(\beta,\delta)$-map if and only if for any IFP $x_{(a,b)}$in X and any IFδ-neighborhood B of $f(x_{(a,b)})$, there is an IFβ-neighborhood A of $x_{(a,b)}$ such that $x_{(a,b)} \in A$ and $f(A) \subseteq B$.

Proof. Let $x_{(a,b)}$ be any IFP in X and B be any IFδ-neighborhood of $f(x_{(a,b)})$.

Then there is an IFδ-open set C in Y such that $f(x_{(a,b)}) \in C \subseteq B$. Since f is $I(\beta,\delta)$-map, $f^{-1}(C)$ is an IFβ-open set and $x_{(a,b)} \in f^{-1}f(x_{(a,b)}) \subseteq f^{-1}(C) \subseteq f^{-1}(B)$. Put $A = f^{-1}(C)$. Then A is an IFβ-neighborhood of $x_{(a,b)}$ and $x_{(a,b)} \in A \subseteq f^{-1}(B)$. Thus $x_{(a,b)} \in A$ and $f(A) \subseteq ff^{-1}(B) \subseteq B$.

Conversely, Let B be an IFδ-open set in Y. If $f^{-1}(B) = 0_\sim$, then it is obvious. Suppose $x_{(a,b)} \in f^{-1}(B)$. Then B is an IFδ-neighborhood of $f(x_{(a,b)})$. By hypothesis, there is an IFβ-neighborhood $A_{x_{(a,b)}}$ of $x_{(a,b)}$ such that $x_{(a,b)} \in A_{x_{(a,b)}}$ and $f(A_{x_{(a,b)}}) \subseteq B$. Since $A_{x_{(a,b)}}$ is an IFβ-neighborhood of $x_{(a,b)}$, there is an IFβ-open set $C_{x_{(a,b)}}$ in X such that $x_{(a,b)} \in C_{x_{(a,b)}} \subseteq A_{x_{(a,b)}} \subseteq f^{-1}f(A_{x_{(a,b)}}) \subseteq f^{-1}(B)$.

So $f^{-1}(B) = \cup\{x_{(a,b)} : x_{(a,b)} \in f^{-1}(B)\} \subseteq \cup\{C_{x_{(a,b)}} : x_{(a,b)} \in f^{-1}(B)\} \subseteq f^{-1}(B)$.

Hence $f^{-1}(B) = \cup\{C_{x_{(a,b)}} : x_{(a,b)} \in f^{-1}(B)\}$. Thus $f^{-1}(B)$ is a IFβ-open set of X. Therefore f is $I(\beta,\delta)$-map. ■

Theorem 4.4. Let $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following statements are equivalent:

(a) f is $I(\beta,\delta)$-map.

(b) $f(Cl_\beta(A)) \subseteq Cl_\delta(f(A))$ for each IFS A of X.

(c) $Cl_\beta(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$ for each IFS B of Y.

(d) $f^{-1}(Int_\delta(C)) \subseteq Int_\beta(f^{-1}(C))$ for each IFS C of Y.

Proof. (a)⇒(b): Let f be a $I(\beta,\delta)$-map and A be any IFS of X. Then,
$Cl_\beta(A) \subseteq Cl_\beta(f^{-1}f(A)) \subseteq Cl_\beta(f^{-1}(Cl_\delta(f(A)))) = f^{-1}(Cl_\delta(f(A)))$,
since $Cl_\delta(f(A))$ is IFδ-closed set in Y and $f^{-1}(Cl_\delta(f(A)))$ is IFβ-closed set in X. Hence $Cl_\beta(A) \subseteq f^{-1}(Cl_\delta(f(A)))$. Thus $f(Cl_\beta(A)) \subseteq ff^{-1}(Cl_\delta(f(A))) \subseteq Cl_\delta(f(A))$ and (b) follows.

(b)⇒(c): Let B be any IFS of Y. Replace A by $f^{-1}(B)$ in (b) we get, $f(Cl_\beta(f^{-1}(B))) \subseteq$

$Cl_\delta(ff^{-1}(B)) \subseteq Cl_\delta(B)$.
Thus, $Cl_\beta(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$and (c) follows.
(c)⇒(d): Let C be any IFS of Y. Replace B by $\overline{C}$ in (c) we get,
$Cl_\beta(f^{-1}(\overline{C})) \subseteq f^{-1}(Cl_\delta(\overline{C}))$, then $\overline{Cl_\beta(f^{-1}(\overline{C}))} \supseteq \overline{f^{-1}(Cl_\delta(\overline{C}))}$.
Hence $Int_\beta(\overline{f^{-1}(\overline{C})}) \supseteq f^{-1}(\overline{Cl_\delta(\overline{C})})$. Thus $f^{-1}(Int_\delta(\overline{\overline{C}})) \subseteq Int_\beta(f^{-1}(\overline{\overline{C}})$. Therefore $f^{-1}(Int_\delta(C)) \subseteq Int_\beta(f^{-1}(C)$and (d) follows.
(d)⇒(a): Let C be an IFδ-open set in Y. Then $Int_\delta(C) = C$.
Using (d), $f^{-1}(C) \subseteq Int_\beta(f^{-1}(C)$. Hence,$f^{-1}(C) = Int_\beta(f^{-1}(C)$. Therefore $f^{-1}(C)$ is a IFβ-open set in X. Thus f is $I(\beta,\delta)$-map and (a) follows. ■

Example 4.5. Let $X = \{a,b\}$ and A and B be IFS's of X defined as $A = \langle x, \frac{0.3}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.4}{b}\rangle$, $B = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle$. Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\sigma = \{0_\sim, 1_\sim, B\}$ are IFT's on X. Define a mapping $f : (X,\tau) \to (X,\sigma)$ by $f(a) = a$, $f(b) = a$. Take $\beta = \tau$ and $\delta = \sigma$. Then,
$f(A) = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle$ and $f(B) = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle = B$,
$f^{-1}(B) = \langle x, \frac{0.3}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.4}{b}\rangle$. Since B is IF$\delta$-open set in $(X,\sigma)$and $f^{-1}(B) = A$ is IFβ-open set of (X,τ). Hence f is $I(\beta,\delta)$-map. But, $Int_\delta(f(B)) = Int_\delta(B) = B$ in (X,σ) and $f(Int_\beta(B)) = f(0_\sim) = 0_\sim$ in (X,σ). Thus $Int_\delta(f(B)) = B \nsubseteq 0_\sim = f(Int_\beta(B))$.

Example 4.6. Let $X = \{a,b\}$ and A and B be IFS's of X defined as $A = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle$, $B = \langle x, \frac{0.4}{a} + \frac{1}{b}, \frac{0.2}{a} + \frac{0}{b}\rangle$. Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\sigma = \{0_\sim, 1_\sim, B\}$ are IFT's on X. Define a mapping $f : (X,\tau) \to (X,\sigma)$ by $f(a) = a$, $f(b) = a$. Take $\beta = \tau$ and $\delta = \sigma$.
Then, $f(A) = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle = A$, $Int_\delta(f(A)) = 0_\sim$ in (X,σ) and $f(Int_\beta(A)) = f(A) = A$. Thus $Int_\delta(f(A)) \subseteq f(Int_\beta(A))$.
But $f^{-1}(B) = \langle x, \frac{0.4}{a} + \frac{0.4}{b}, \frac{0.2}{a} + \frac{0.2}{b}\rangle$ is not an IFβ-open set of (X,τ). Hence f is not$I(\beta,\delta)$-map.

These examples show that generally the two conditions, f is $I(\beta,\delta)$-map and $Int_\delta(f(A)) \subseteq f(Int_\beta(A))$ are independent. But, they behave nicely when f is bijective as shown in the following theorem.

Theorem 4.7. Let $f : (X,\tau) \to (Y,\sigma)$ be a bijection. Then the following statements are equivalent:

(a) f is $I(\beta,\delta)$-map.

(b) $f(Cl_\beta(A)) \subseteq Cl_\delta(f(A))$ for each IFS A of X.

(c) $Cl_\beta(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$ for each IFS B of Y.

(d) $f^{-1}(Int_\delta(C)) \subseteq Int_\beta(f^{-1}(C))$ for each IFS C of Y.

(e) $Int_\delta(f(A)) \subseteq f(Int_\beta(A))$ for each IFS A of X.

Proof. By Theorem 4.4 it suffices to show that (d) is equivalent to (e).
Let A be any IFS of X. Then f(A) is IFS of Y.
Then by (d), $f^{-1}(Int_\delta(f(A))) \subseteq Int_\beta(f^{-1}f(A)) = Int_\beta(A)$.
Thus $ff^{-1}(Int_\delta(f(A))) \subseteq f(Int_\beta(A))$. Hence $Int_\delta(f(A)) \subseteq f(Int_\beta(A))$ and (e) follows.
Conversely, Let C be any IFS of Y. Then $f^{-1}(C)$ is an IFS of X.
Replacing A by $f^{-1}(C)$ in (e), $Int_\delta(C) = Int_\delta(ff^{-1}(C)) \subseteq f(Int_\beta(f^{-1}(C)))$. $f^{-1}(Int_\delta(C)) \subseteq f^{-1}f(Int_\beta(f^{-1}(C))) = Int_\beta(f^{-1}(C))$ and (d) follows. Hence the theorem. ■

Theorem 4.8. Let $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following statements are equivalent:

(a) f is $M(\mu,\gamma)$-map.

(b) $f(Int_\mu(A)) \subseteq Int_\gamma(f(A))$ for each IFS A of X.

(c) $Int_\mu(f^{-1}(B)) \subseteq f^{-1}(Int_\gamma(B))$ for each IFS B of Y.

Proof. (a)⇒(b): Let A be any IFS of X. Clearly $Int_\mu(A)$ is an IFμ-open set of X. Since f is $M(\mu,\gamma)$-map, $f(Int_\mu(A))$ is an IFγ-open set in Y.
Thus, $f(Int_\mu(A)) \subseteq Int_\gamma(f(Int_\mu(A))) \subseteq Int_\gamma(f(A))$ and (b) follows.
(b)⇒(c): Let B be any IFS of Y. Then $f^{-1}(B)$ is an IFS of X. Replacing A by $f^{-1}(B)$ in (b), $f(Int_\mu(f^{-1}(B))) \subseteq Int_\gamma(ff^{-1}(B)) = Int_\gamma(B)$.
Thus, $Int_\mu(f^{-1}(B)) \subseteq f^{-1}(Int_\gamma(B))$ and (c) follows.
(c)⇒(a): Let A be any IFμ-open set of X. Then $Int_\mu(A) = A$ and $f(A)$ is an IFS of Y.
Replacing B by $f(A)$ in (c), $Int_\mu(f^{-1}f(A)) \subseteq f^{-1}(Int_\gamma(f(A)))$.
Thus $A = Int_\mu(A) \subseteq f^{-1}(Int_\gamma(f(A)))$ and hence $f(A) \subseteq f^{-1}(Int_\gamma(f(A))) \subseteq Int_\gamma(f(A)) \subseteq f(A)$.
Thus $f(A) = Int_\gamma(f(A))$ and hence $f(A)$ is an IFγ-open set in Y.
Therefore f is a $M(\mu,\gamma)$-map. ■

Theorem 4.9. Let $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following statements are equivalent:

(a) f is $\overline{M}(\mu,\gamma)$-map.

(b) $Cl_\gamma(f(A)) \subseteq f(Cl_\mu(A))$ for each IFS A of X.

Proof. (a)⇒(b): Let A be an IFS of X. Then f(A) is IFS of Y. Clearly $Cl_\mu(A)$ is an IFμ-closed set in X. Since f is an $\overline{M}(\mu,\gamma)$-map, $f(Cl_\mu(A))$ is an IFγ-closed set in Y. Thus, $Cl_\gamma(f(A)) \subseteq Cl_\gamma(f(Cl_\mu(A))) = f(Cl_\mu(A))$.
(b)⇒(a): Let A be any IFμ-closed set of X. Then $Cl_\mu(A) = A$. By (b),
$Cl_\gamma(f(A)) \subseteq f(Cl_\mu(A)) = f(A) \subseteq Cl_\gamma(f(A))$. Thus $f(A) = Cl_\gamma(f(A))$ and hence $f(A)$ is an IFγ-closed set in Y. Therefore f is $\overline{M}(\mu,\gamma)$-map.

The conditions in Theorem 4.9 are not equivalent to the condition that $f^{-1}(Cl_\gamma(B)) \subseteq Cl_\mu(f^{-1}(B))$ for each IFS B of Y. This is shown by the following examples. ■

Example 4.10. Let $X = \{a,b\}$ and A and B be IFS's of X defined as $A = \langle x, \frac{0.3}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.4}{b}\rangle$, $B = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle$. Then $\tau = \{0_\sim, 1_\sim, \overline{A}\}$ and $\sigma = \{0_\sim, 1_\sim, \overline{B}\}$ are IFT's on X. Define a mapping $f : (X,\tau) \to (X,\sigma)$ by $f(a) = a$, $f(b) = a$. Take $\mu = \tau$ and $\gamma = \sigma$.
Then, $f(A) = \langle x, \frac{0.3}{a} + \frac{0}{b}, \frac{0.4}{a} + \frac{1}{b}\rangle = B$ is IFγ-closed set of (X,σ).
$f^{-1}(B) = \langle x, \frac{0.3}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.4}{b}\rangle = A$ is IFμ-closed set of (X,τ).
Hence f is $\overline{M}(\mu,\gamma)$-map. $f^{-1}(Cl_\gamma(A)) = f^{-1}(1_\sim) = 1_\sim$ in (X,σ) and $Cl_\mu(f^{-1}(A)) = Cl_\mu(A) = A$ in (X,τ). Thus $f^{-1}(Cl_\gamma(A)) \nsubseteq Cl_\mu(f^{-1}(A))$.

Example 4.11. Let $X = \{a,b\}$ and A and B be IFS's of X defined as $\overline{A} = \langle x, \frac{0}{a} + \frac{1}{b}, \frac{1}{a} + \frac{0}{b}\rangle$, $B = \langle x, \frac{0}{a} + \frac{0.6}{b}, \frac{1}{a} + \frac{0.3}{b}\rangle$. Then $\tau = \{0_\sim, 1_\sim\}$ and $\sigma = \{0_\sim, 1_\sim, A\}$ are IFT's on X. Define a mapping $f : (X,\tau) \to (X,\sigma)$ by $f(a) = a$, $f(b) = a$. Take $\mu = \tau$ and $\gamma = \sigma$. Then, $f^{-1}(Cl_\gamma(B)) = f^{-1}(A) = 0_\sim$ and $Cl_\mu(f^{-1}(B)) = 1_\sim$. Thus $f^{-1}(Cl_\gamma(B)) \subseteq Cl_\mu(f^{-1}(B))$. But $f(1_\sim) = A$ is not an IFγ-closed set in (X,σ). Hence f is not $\overline{M}(\mu,\gamma)$-map.

The above situation will not arise if f is a bijection as shown in the following theorem.

Theorem 4.12. Let $f : (X,\tau) \to (Y,\sigma)$ be a bijection. Then the following statements are equivalent:

(a) f is $\overline{M}(\mu,\gamma)$-map.

(b) $Cl_\gamma(f(A)) \subseteq f(Cl_\mu(A))$ for each IFS A of X.

(c) $f^{-1}(Cl_\gamma(B)) \subseteq Cl_\mu(f^{-1}(B))$ for each IFS B of Y.

Proof. By theorem 4.9 it suffices to show that (b) is equivalent to (c)
Let B be any IFS of Y. Then $f^{-1}(B)$ is an IFS of X. Replacing A by $f^{-1}(B)$in (b), $Cl_\gamma(B) \subseteq Cl_\gamma(ff^{-1}(B)) = f(Cl_\mu(f^{-1}(B)))$.

So, $f^{-1}(Cl_\gamma(B)) \subseteq f^{-1}f(Cl_\mu(f^{-1}(B))) = Cl_\mu(f^{-1}(B))$, since f is one to one. Conversely, Let A be any IFS of X. Then $f(A)$ is an IFS of Y and
$f^{-1}(Cl_\gamma(f(A))) \subseteq Cl_\mu(f^{-1}f(A)) = Cl_\mu(A)$.
Hence, $ff^{-1}(Cl_\gamma(f(A))) \subseteq f(Cl_\mu(A))$, and f is on to. Thus, $Cl_\gamma(f(A))) \subseteq f(Cl_\mu(A))$and the theorem follows. ■

The following theorem is a consequence of the Theorem 4.7, Theorem 4.8 and Theorem 4.12.

Theorem 4.13. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijection. Then the following statements are equivalent:

(a) f is a (β, δ)-homeomorphism.

(b) f is $I(\beta, \delta)$-map and $M(\beta, \delta)$-map.

(c) $f(Cl_\beta(A)) = Cl_\delta(f(A))$ for each IFS A of X.

(d) $Cl_\beta(f^{-1}(B)) = f^{-1}(Cl_\delta(B))$ for each IFS B of Y.

(e) $f^{-1}(Int_\delta(C)) = Int_\beta(f^{-1}(C))$ for each IFS C of Y.

(f) $Int_\delta(f(A)) = f(Int_\beta(A))$ for each IFS A of X.

5. Conclusion

The study of $I(\beta, \delta)$-mappings unified the theory of continuous and irresolute, open maps, closed maps and homeomorphism. It can be viewed as an aid to get the result of image processing.

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