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On Right Fuzzy Matrix Hemi Near Rings.

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Abstract

The concept of right fuzzy matrix hemi near rings is defined. Such hemi near rings have zero divisors. The property of fuzzy square matrices where sum of each row is zero or one is observed. Examples supporting not so obviousness of nilpotent ness of fuzzy matrix with respect to operation $\oplus$ is justified.

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Definition: Semi Near ring: (S ,+, x) is Semi Near ring if (S , +) and (S , x) are Semi groups and right distributive law holds.

Example: Z^+ be the set of positive integers. (Z^+ , +) and (Z^+. x) are semi groups.
Also right distributive law holds. $0 \notin Z^+$
So Z^+ is Semi Near Ring (SNR) without zero element.

Definition: Hemi Near ring (HNR) is Semi Near ring (SNR) with 0 element such that (H, +) is commutative.
If also (H , x) is commutative then (H , + , x) is commutative Hemi Near Ring.

Let P_{nxn} = { (aij)nxn / aij $\in$ [0 , 1] } denote collection of all nxn matrices with entries from [0 , 1] .
Define $\oplus$ on P as follows

Let A , B $\in$ P and if A = $\begin{pmatrix} a_{11} & -- & a_{1n} \\ -- & -- & -- \\ a_{n1} & -- & a_{nn} \end{pmatrix}$ B = $\begin{pmatrix} b_{11} & -- & b_{1n} \\ -- & -- & -- \\ b_{n1} & -- & b_{nn} \end{pmatrix}$

Then A $\oplus$ B = $\begin{pmatrix} a_{11}+b_{11} & -- & a_{1n}+b_{1n} \\ -- & -- & -- \\ a_{n1}+b_{n1} & -- & a_{nn}+b_{nn} \end{pmatrix}$

where aij + bij = aij + bij if aij + bij < 1
0 if aij + bij = 1
aij + bij – 1 if aij + bij > 1

The set P defined above satisfies following properties with respect to operation $\oplus$

1. Closure property.
2. Associativity

e.g.

$$\{ \begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1 & 1/4 \end{pmatrix} \} \oplus \begin{pmatrix} 1/2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1/2 \\ 0 & 1/2 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/2 \end{pmatrix} \ (1)$$

$$\begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/4 \end{pmatrix} \oplus \{ \begin{pmatrix} 1/2 & 0 \\ 1 & 1/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 1 \\ 1 & 1 \end{pmatrix} \}$$

$$\begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/4 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 \\ 1 & 1/4 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 0 & 1/2 \end{pmatrix} \qquad (2)$$

From (1) and (2) Associativity verified.

(3) Existence of identity:

Claim $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ acts as an identity element for P as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ Obviously.}$$

Consider A = $\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$ and K = $\begin{pmatrix} 1/2 & 0 \\ 0 & 0 \end{pmatrix}$ and K' = $\begin{pmatrix} 1/2 & 1 \\ 0 & 0 \end{pmatrix}$

Then it can be seen easily that A$\oplus$ K = A $\oplus$ K' = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

It is clear that both K and K' are elements of P and that A$\oplus$ K = A $\oplus$ K' = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

So inverse with respect to $\oplus$ is not unique.
Hence (P , $\oplus$) is not a group.
(P , $\oplus$) is a semigroup.
It has zero element viz. Zero matrix of order 2x2.
Also $\oplus$ is commutative over P.

Define $\otimes$ on P as follows
Let A and B be any two elements of P then A $\otimes$ B is defined as

$$A \otimes B = \begin{pmatrix} a_{11} & -- & a_{1n} \\ -- & -- & -- \\ a_{n1} & -- & a_{nn} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & -- & b_{1n} \\ -- & -- & -- \\ b_{n1} & -- & b_{nn} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}.b_{11} \oplus a_{12}.b_{21} b_{11} \oplus --- a_{1n}.b_{n1} & ------ & \\ -- & -- & -- \\ --- & -- & - \end{pmatrix}$$

where aij.bjk = aij and $\oplus$ as defined above.
It can be easily verified that (P , $\otimes$) is Semigroup.
Also (A $\oplus$ B) $\otimes$ C = (A $\otimes$ C) $\oplus$ (B $\otimes$ C)
It can be verified in case of P 2x2 as follows:

$$\{ \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1/2 \\ 1 & 0 \end{pmatrix} \} \otimes \begin{pmatrix} 1/2` & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1/2` & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (1)$$

Also $\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1/2` & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1/2 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1/2` & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 1/2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (2)$$

So the set (P_{2x2} , ⊕ , ⊗) is Hemi Near ring.

Definition: The Hemi Near ring (P_{nxn} , ⊕ , ⊗) is right fuzzy matrix Hemi Near Ring.

Proposition: Right fuzzy matrix Hemi Near Ring need not be commutative.

Proof: As $A \otimes B \neq B \otimes A$ for all A , B $\in P_{nxn}$ Hence proved.

Proposition 2: Right fuzzy matrix Hemi Near Ring (P_{nxn} , ⊕ , ⊗) has zero divisors.

Proof: Obvious. Supported by example from (P_{2x2} , ⊕ , ⊗)

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Example: Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 2/3 & 0 & 1/3 \\ 1/5 & 2/5 & 1/5 \end{pmatrix} \in P_{3x3}$

And B be any matrix of P 3x3 i.e. $B = \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix}$

$$A \otimes B = \begin{pmatrix} 1 & 0 & 1 \\ 2/3 & 0 & 1/3 \\ 1/5 & 2/5 & 1/5 \end{pmatrix} \otimes \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

But if sum in particular row exceeds 2 or it becomes 2 in any matrix of P_{2x2} then in that case $A \otimes B \neq 0$ matrix.

Proposition 3: Let (P_{nxn} , ⊕ , ⊗) be right fuzzy Hemi near Ring. All matrices $A \in P_{nxn}$ with sum of each row in A is one or 0 is such that $A \otimes B = 0_{nxn}$ for all $B \in P_{nxn}$

Proof: Let $A = \begin{pmatrix} a_{11} & -- & a_{1n} \\ -- & -- & -- \\ a_{n1} & -- & a_{nn} \end{pmatrix}$ where $A \in P_{nxn}$

where sum of each row is zero or one.

Then if B is any matrix of P_{nxn} then

A ⊗ B = (Cik) where C ik = ∑ aij ⊗ bjk where summation is taken

Over j where j ranges from 1 to n and summation stands for ⊕

So each Cik = ∑ aij where j ranges from 1 to n

It follows that each Cik = 0

Remark: Clearly I nx n ⊗ A ≠ I nxn and A ⊗ I nxn ≠ A

$$\begin{pmatrix} 1/2 & 1/2 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The remark is verified in case of (P2x2 , ⊕ , ⊗)

Proposition 4: (P nxn , ⊕ , ⊗) has no fuzzy idempotents other than O nxn.

Proof: Obvious.

Proposition 5: Let A nxn be the set of matrices with the property defined in proposition 3

Then A nxn is right fuzzy sub matrix Hemi Near ring of P nxn

Proof: Obvious from definition of A nxn and definitions of operations ⊕ , ⊗

Proposition 6: Let P nxn be a right fuzzy matrix Hemi Near Ring. For A ∈ P nxn we Have A + A +A + - - - - - - + A = 0 nxn

Where the sum is made a suitable number of times.

Not So obvious .

E.g $A = \begin{pmatrix} 1/2 & 1 \\ 1/2 & 1/2 \end{pmatrix} \in P_{2x2}$

$$A \oplus A = \begin{pmatrix} 1/2 & 1 \\ 1/2 & 1/2 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 1 \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A \oplus A \oplus A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 1 \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1 \\ 1/2 & 1/2 \end{pmatrix}$$

This matrix A will never be 0_{2x2} by this process.

Consider $A = \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} \in P_{2x2}$

Then $A \oplus A = \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2/3 & 2/4 \end{pmatrix}$

$$A \oplus A \oplus A = \begin{pmatrix} 0 & 0 \\ 2/3 & 2/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 3/4 \end{pmatrix}$$

$$A \oplus A \oplus A \oplus A = \begin{pmatrix} 1/2 & 0 \\ 0 & 3/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1/3 & 0 \end{pmatrix}$$

A added 5 times $= \begin{pmatrix} 0 & 0 \\ 1/3 & 0 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 2/3 & 1/4 \end{pmatrix}$

A added 6 times $= \begin{pmatrix} 1/2 & 0 \\ 2/3 & 1/4 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0 \\ 1/3 & 1/4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1/2 \end{pmatrix}$

Continuing like this after 12 times when we add matrix A to itself we get 0 matrix.

Remark:
In above proposition $A \in P$ nxn is such that $A = (aij)_{nxn}$ where $a_{ij} \in [0, 1)$

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