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Level Subsets of Intuitionistic Fuzzy Subsemiring of a Semiring

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Abstract

In this chapter, we introduce the concept of level subsets of intuitionistic fuzzy subsemirings of a semiring and establish some results on these. We also made an attempt to study the properties of level subsets of intuitionistic fuzzy subsemirings of semiring under homomorphism and anti-homomorphism.

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KEY WORDS: fuzzy set, fuzzy subsemiring, intuitionistic fuzzy set, intuitionistic fuzzy subsemiring, level subset.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R , + , .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R , +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a=a=a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0=0=0.a for all a in R. After the introduction of fuzzy sets by L.A. Zadeh [22], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T. Atanassov [3,4], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [20]. In this paper, we introduce the some theorems in level subsets of intuitionistic fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism.

1. PRELIMINARIES:

1.1 Definition: Let X be a non empty set. A fuzzy subset A of X is a function A: X $\rightarrow$ [0, 1].

1.2 Definition: Let R be a semiring. A fuzzy subset A of R is said to be a fuzzy subsemiring (FSSR) of R if it satisfies the following conditions:

$\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

1.3 Definition [3]: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A=\{\langle x, \mu_A(x),\nu_A(x)\rangle / x\in X\}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.1 Example : Let X={a,b,c} be a set. Then A={⟨a,0.52,0.34⟩,⟨ b,0.14,0.71⟩, ⟨ c, 0.25, 0.34 ⟩ } is an intuitionistic fuzzy subset of X.

1.4 Definition: Let A and B be any two intuitionistic fuzzy subsets of a set X. We define the following operations:

(i) $A\cap B=\{\langle x, \min\{ \mu_A(x) , \mu_B(x)\}, \max\{ \nu_A(x) , \nu_B(x)\}\rangle\}$, for all $x \in X$.

(ii) $A\cup B=\{\langle x, \max\{ \mu_A(x) , \mu_B(x)\}, \min\{ \nu_A(x) , \nu_B(x)\} \rangle \}$, for all $x \in X$.

1.5 Definition: Let R be a semiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subsemiring (IFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min \{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$,

(iii) $\nu_A(x + y) \leq \max \{ \nu_A(x), \nu_A(y) \}$,

(iv) $\nu_A(xy) \leq \max \{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

1.6 Definition: An intuitionistic fuzzy subsemiring A of a semiring R is called a intuitionistic fuzzy characteristic subsemiring of R if $\mu_A(x) = \mu_A(f(x))$ and $\nu_A(x) = \nu_A(f(x))$, for all x in R and f in Aut (R).

1.7 Definition: Let (R, +, ·) and (R^{l}, +, ·) be any two semirings. Let f: R $\rightarrow$ R^{l} be any function and A be an intuitionistic fuzzy subsemiring in R, V be an intuitionistic fuzzy subsemiring in f(R) = R^{l}, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.8 Definition: Let (R, +, •) and (R^{l}, +, •) be any two semirings. Then f: R$\rightarrow$ R^{l} is called a homomorphism if f(x+y) = f(x) + f(y), for all x and y in R and f (xy) = f(x)f(y), for all x and y in R.

1.9 Definition: Let (R, +, •) and (R^{I}, +, •) be any two semirings. Then f: R→ R^{I} is called an anti-homomorphism if f(x+y) = f(y) + f(x), for all x and y in R and f (xy) = f(y)f(x), for all x and y in R.

1.10 Definition: Let A be an intuitionistic fuzzy subset of X. For α, β in [0, 1], the level subset of A is the set $A_{(\alpha, \beta)}$ = { x∈X : $\mu_A(x) \geq \alpha$, $\mu_A(x) \leq \beta$ }.

2. Properties of level subsets of intuitionistic fuzzy subsemiring:

2.1 Theorem [18]: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). Then for α and β in [0,1], $A_{(\alpha, \beta)}$ is a subsemiring of R.

2.2 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). Then two level subsemirings $A_{(\alpha1, \beta1)}$, $A_{(\alpha2, \beta2)}$ and α_1 and α_2 in [0,1] and β_1, β_2 in [0,1] with $\alpha_2 < \alpha_1$ and $\beta_1 < \beta_2$ of A are equal if and only if there is no x in R such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$.

Proof: Assume that $A_{(\alpha1, \beta1)} = A_{(\alpha2, \beta2)}$. Suppose there exists x∈R such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$. Then $A_{(\alpha1, \beta1)} \subseteq A_{(\alpha2, \beta2)}$ which implies that x belongs to $A_{(\alpha2, \beta2)}$, but not in $A_{(\alpha1, \beta1)}$. This is contradiction to $A_{(\alpha1, \beta1)} = A_{(\alpha2, \beta2)}$. Therefore there is no x∈R such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$. Conversely, if there is no x∈R such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$. Then $A_{(\alpha1, \beta1)} = A_{(\alpha2, \beta2)}$.

2.3 Theorem: Let (R, +, .) be a semiring and A be an intuitionistic fuzzy subset of R such that $A_{(\alpha, \beta)}$ be a subsemiring of R. If α and β in [0,1], then A is an intuitionistic fuzzy subsemiring of R.

Proof: For x and y in R. Let $\mu_A(x)=\alpha_1$ and $\mu_A(y)=\alpha_2$, $\nu_A(x)= \beta_1$ and $\nu_A(y)= \beta_2$.

Case (i): If $\alpha_1<\alpha_2$ and $\beta_1>\beta_2$, then x and y in $A_{(\alpha1, \beta1)}$. As $A_{(\alpha1, \beta1)}$ is a subsemiring of R, x + y, xy in $A_{(\alpha1, \beta1)}$. Now, $\mu_A(x+y)\geq\alpha_1=\min\{\mu_A(x),\mu_A(y)\}$ which implies that $\mu_A(x+y)\geq\min\{\mu_A(x),\mu_A(y)\}$, for all x and y in R. Now, $\mu_A(xy) \geq \alpha_1= \min\{\mu_A(x),\mu_A(y)\}$ which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\nu_A(x+y) \leq \beta_1 =\max\{\nu_A(x),\nu_A(y)\}$ which implies that $\nu_A(x+y) \leq \max\{\nu_A(x),\nu_A(y)\}$, for all x and y in R. And , $\nu_A(xy) \leq \beta_1= \max\{ \nu_A(x), \nu_A(y) \}$ which implies that $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

Case (ii): If $\alpha_1<\alpha_2$ and $\beta_1<\beta_2$, then x and y in $A_{(\alpha1, \beta2)}$. As $A_{(\alpha1, \beta2)}$ is a subsemiring of R, x+y, xy in $A_{(\alpha1, \beta2)}$. Now, $\mu_A(x+y)\geq\alpha_1=\min\{\mu_A(x),\mu_A(y)\}$ which implies that $\mu_A(x+y)\geq\min\{\mu_A(x),\mu_A(y)\}$, for all x and y in R. Now, $\mu_A(xy) \geq \alpha_1 = \min\{ \mu_A(x), \mu_A(y) \}$ which implies that $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\nu_A(x+y)\leq \beta_2= \max\{\nu_A(x),\nu_A(y)\}$ which implies that $\nu_A(x+ y) \leq \max\{ \nu_A(x),\nu_A(y)\}$, for all x and y in R. And, $\nu_A(xy)\leq \beta_2 = \max\{\nu_A(x),\nu_A(y)\}$ which implies that $\nu_A(xy)$

$\leq$ max{ $\nu_A(x)$, $\nu_A(y)$ }, for all x and y in R.

Case (iii): If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$, then x and y in $A_{(\alpha 2, \beta 1)}$. As $A_{(\alpha 2, \beta 1)}$ is a subsemiring of R, x + y, xy in $A_{(\alpha 2, \beta 1)}$. Now, $\mu_A(x+y) \geq \alpha_2 = \min\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Now, $\mu_A(xy) \geq \alpha_2 = \min\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\nu_A(x+y) \leq \beta_1 = \max\{\nu_A(x), \nu_A(y)\}$ which implies that $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. And, $\nu_A(xy) \leq \beta_1 = \max\{\nu_A(x), \nu_A(y)\}$ which implies that $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R.

Case (iv): If $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$, then x and y in $A_{(\alpha 2, \beta 2)}$. As $A_{(\alpha 2, \beta 2)}$ is a subsemiring of R, x + y and xy in $A_{(\alpha 2, \beta 2)}$. Now, $\mu_A(x+y) \geq \alpha_2 = \min\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Now, $\mu_A(xy) \geq \alpha_2 = \min\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\nu_A(x+y) \leq \beta_2 = \max\{\nu_A(x), \nu_A(y)\}$ which implies that $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. And, $\nu_A(xy) \leq \beta_2 = \max\{\nu_A(x), \nu_A(y)\}$ which implies that $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. It is trivial.
In all the cases, A is an intuitionistic fuzzy subsemiring of a semiring R.

2.4 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring R. If any two level subsemiring of A belongs to R, then their intersection is also level subsemiring of A in R.

Proof: For α_1 and α_2 in [0,1], β_1 and β_2 in [0,1].

Case (i): If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha 2, \beta 2)} \subseteq A_{(\alpha 1, \beta 1)}$.
Therefore, $A_{(\alpha 1, \beta 1)} \cap A_{(\alpha 2, \beta 2)} = A_{(\alpha 2, \beta 2)}$, but $A_{(\alpha 2, \beta 2)}$ is a level subsemiring of A.

Case(ii): If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha 1, \beta 1)} \subseteq A_{(\alpha 2, \beta 2)}$.
Therefore, $A_{(\alpha 1, \beta 1)} \cap A_{(\alpha 2, \beta 2)} = A_{(\alpha 1, \beta 1)}$, but $A_{(\alpha 1, \beta 1)}$ is a level subsemiring of A.

Case (iii): If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha 2, \beta 1)} \subseteq A_{(\alpha 1, \beta 2)}$.
Therefore, $A_{(\alpha 2, \beta 1)} \cap A_{(\alpha 1, \beta 2)} = A_{(\alpha 2, \beta 1)}$, but $A_{(\alpha 2, \beta 1)}$ is a level subsemiring of A.

Case (iv): If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha 1, \beta 2)} \subseteq A_{(\alpha 2, \beta 1)}$.
Therefore, $A_{(\alpha 1, \beta 2)} \cap A_{(\alpha 2, \beta 1)} = A_{(\alpha 1, \beta 2)}$, but $A_{(\alpha 1, \beta 2)}$ is a level subsemiring of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha 1, \beta 1)} = A_{(\alpha 2, \beta 2)}$.
In all cases, Intersection of any two level subsemiring is a level subsemiring of A.

2.5 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). If

α_i and β_j in [0,1] and $A_{(\alpha i, \beta j)}$, i and j in I, is a collection of level subsemirings of A, then their intersection is also a level subsemiring of A.

Proof: It is trivial.

2.6 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring R. If any two level subsemirings of A belongs to R, then their union is also a level subsemiring of A in R.

Proof: Let α_1, α_2, β_1 and β_2 in [0,1].

Case (i): If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha 2, \beta 2)} \subseteq A_{(\alpha 1, \beta 1)}$.
Therefore, $A_{(\alpha 1, \beta 1)} \cup A_{(\alpha 2, \beta 2)} = A_{(\alpha 1, \beta 1)}$, but $A_{(\alpha 1, \beta 1)}$ is a level subsemiring of A.

Case (ii): If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha 1, \beta 1)} \subseteq A_{(\alpha 2, \beta 2)}$.
Therefore, $A_{(\alpha 1, \beta 1)} \cup A_{(\alpha 2, \beta 2)} = A_{(\alpha 2, \beta 2)}$, but $A_{(\alpha 2, \beta 2)}$ is a level subsemiring of A.

Case (iii): If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < \nu_A(x) < \beta_2$, then $A_{(\alpha 2, \beta 1)} \subseteq A_{(\alpha 1, \beta 2)}$.
Therefore, $A_{(\alpha 2, \beta 1)} \cup A_{(\alpha 1, \beta 2)} = A_{(\alpha 1, \beta 2)}$, but $A_{(\alpha 1, \beta 2)}$ is a level subsemiring of A.

Case (iv): If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 > \nu_A(x) > \beta_2$, then $A_{(\alpha 1, \beta 2)} \subseteq A_{(\alpha 2, \beta 1)}$.
Therefore, $A_{(\alpha 1, \beta 2)} \cup A_{(\alpha 2, \beta 1)} = A_{(\alpha 2, \beta 1)}$, but $A_{(\alpha 2, \beta 1)}$ is a level subsemiring of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha 1, \beta 1)} = A_{(\alpha 2, \beta 2)}$.
In all cases, union of any two level subsemiring is also a level subsemiring of A.

2.7 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). If α_i and β_j in [0,1] and $A_{(\alpha i, \beta j)}$, i and j in I, is a collection of level subsemirings of A, then their union is also a level subsemiring of A.

Proof: It is trivial.

2.8 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). If A is an intuitionistic fuzzy characteristic subsemiring of R, then each level subsemiring of A is a characteristic subsemiring of R.

Proof: Let A be an intuitionistic fuzzy characteristic subsemiring of a semiring (R, +, .). Let x and y in R, α in Im μ_A, β in Im ν_A; f in Aut(R) and x in $A_{(\alpha, \beta)}$. Clearly $\mu_A(f(x)) \geq \alpha$ and $\nu_A(f(x)) \leq \beta$. Therefore, $f(x) \in A_{(\alpha, \beta)}$. Hence, $f(A_{(\alpha, \beta)}) \subseteq A_{(\alpha, \beta)}$ -----(1). For the reverse inclusion, let x in $f(A_{(\alpha,\beta)})$ and y in R such that f(y)=x. Then, $\mu_A(y) = \mu_A(f(y)) = \mu_A(x) \geq \alpha$. And, $\nu_A(y) = \nu_A(f(y)) = \nu_A(x) \leq \beta$. Therefore, $\mu_A(y) \geq \alpha$ and $\nu_A(y) \leq \beta$. Hence, $y \in A_{(\alpha, \beta)}$, when $x \in f(A_{(\alpha, \beta)})$. Hence, $f(A_{(\alpha, \beta)}) \subseteq A_{(\alpha, \beta)}$ ------ (2). From (1) and (2), we get $A_{(\alpha, \beta)}$ is a characteristic subsemiring of a semiring R.

2.9 Theorem: Any subsemiring H of a semiring (R, +, .) can be realized as a level subsemiring of some intuitionistic fuzzy subsemiring of R.

Proof: Let A be the intuitionistic fuzzy subset of a semiring (R, +, .) defined by

$$\mu_A(x) = \begin{cases} \alpha & \text{if } x \in H,\ 0 < \alpha < 1 \\ 0 & \text{if } x \notin H \end{cases} \quad \text{and}$$

$$\nu_A(x) = \begin{cases} \beta & \text{if } x \in H,\ 0 < \beta < 1 \\ 0 & \text{if } x \notin H \end{cases}$$

and $\alpha + \beta \le 1$, where H is a subsemiring of R. Let x and y in R. If x and y in H, then x+y, xy in H, since H is a subsemiring of R. So, $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. So, $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. If x in H, y not in H, then x+y, xy not in H. Therefore, $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Therefore, $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R, $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. If x and y not in H, then x+y, xy not belong to H. Clearly $\mu_A(x+y) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R, $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Also, $\nu_A(x+y) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R, $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. Hence A is an intuitionistic fuzzy subsemiring of R.

2.10 Theorem: Let f be any mapping from a semiring R_1 to R_2 and let A be an intuitionistic fuzzy subsemiring of R_1. Then for α and β in [0,1], we have

$$f(A_{(\alpha,\beta)}) = \bigcap_{\substack{\alpha > \varepsilon_1 > 0, \\ \beta > \varepsilon_2 > 0}} f(A_{(\alpha-\varepsilon_1, \beta+\varepsilon_2)}).$$

Proof: Suppose that α and β in [0,1] and y = f(x) in R_2. If y in $f(A_{(\alpha,\beta)})$, then $f(\mu_A)(y) = \sup_{x \in f^{-1}(y)} \{\mu_A(x)\} \ge \alpha$ and $f(\nu_A)(y) = \inf_{x \in f^{-1}(y)} \{\nu_A(x)\} \le \beta$. Therefore, for every real number $\varepsilon_1, \varepsilon_2 > 0$, there exist $x_0 \in f^{-1}(y)$ such that $\mu_A(x_0) > \alpha - \varepsilon_1$ and $\nu_A(x_0) < \beta + \varepsilon_2$. So, for every $\varepsilon_1, \varepsilon_2 > 0$, $y = f(x_0) \in f(A_{(\alpha-\varepsilon_1, \beta+\varepsilon_2)})$ and hence

$$y \in \bigcap_{\substack{\alpha > \varepsilon_1 > 0, \\ \beta > \varepsilon_2 > 0}} f(A_{(\alpha-\varepsilon_1, \beta+\varepsilon_2)}).$$

Therefore, $$f(A_{(\alpha,\beta)}) \subseteq \bigcap_{\substack{\alpha > \varepsilon_1 > 0, \\ \beta > \varepsilon_2 > 0}} f(A_{(\alpha-\varepsilon_1, \beta+\varepsilon_2)}). \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Conversely, $y \in \bigcap_{\substack{\alpha>\varepsilon_1>0,\\ \beta>\varepsilon_2>0}} f(A_{(\alpha-\varepsilon_1,\beta+\varepsilon_2)})$, then for each ε_1, $\varepsilon_2 > 0$, we have $y \in f(A_{(\alpha-\varepsilon_1,\beta+\varepsilon_2)})$ and there exist $x_0 \in A_{(\alpha-\varepsilon_1,\beta+\varepsilon_2)}$ such that $y = f(x_0)$. Therefore for each $\varepsilon_1, \varepsilon_2 > 0$, there exist $x_0 \in f^{-1}(y)$ and $\mu_A(x_0) \geq \alpha - \varepsilon_1$ and $\nu_A(x_0) \leq \beta + \varepsilon_2$.

Hence, $f(\mu_A)(y) = \underset{x_i \in f^{-1}(y)}{Sup}\{\mu_A(x_i)\} \geq \underset{\alpha>\varepsilon_1>0}{Sup}\{\alpha - \varepsilon_1\} = \alpha$ and

$f(\nu_A)(y) = \underset{x_i \in f^{-1}(y)}{Inf}\{\nu_A(x_i)\} \leq \underset{\beta>\varepsilon_2>0}{Inf}\{\beta + \varepsilon_2\} = \beta$. So, $y \in f(A_{(\alpha,\beta)})$.

Therefore, $\bigcap_{\substack{\alpha>\varepsilon_1>0,\\ \beta>\varepsilon_2>0}} f(A_{(\alpha-\varepsilon_1,\beta+\varepsilon_2)}) \subseteq f(A_{(\alpha,\beta)})$ …………….. (2).

From (1) and (2) we get, $f(A_{(\alpha,\beta)}) = \bigcap_{\substack{\alpha>\varepsilon_1>0,\\ \beta>\varepsilon_2>0}} f(A_{(\alpha-\varepsilon_1,\beta+\varepsilon_2)})$.

2.11 Theorem: Let A be an intuitionistic fuzzy subset of a set X. Then $\mu_A(x) = \max\{\alpha / x \in A_{(\alpha,\beta)}\}$ and $\nu_A(x) = \min\{\beta / x \in A_{(\alpha,\beta)}\}$, where $x \in X$.

Proof: It is trivial.

2.12 Theorem: Any two different intuitionistic fuzzy subsemirings of a semiring may have identical family of level subsemirings.

Proof: We consider the following example:

Consider the semiring $R = Z_5 = \{0, 1, 2, 3, 4\}$ with addition modulo 5 and multiplication modulo 5 operations. Define intuitionistic fuzzy subsets A and B of R by $A=\{\langle 0, 0.7, 0.1\rangle,\langle 1, 0.5, 0.4\rangle,\langle 2, 0.5, 0.4\rangle,\langle 3, 0.5, 0.4\rangle,\langle 4, 0.5, 0.4\rangle\}$ and $B=\{\langle 0, 0.8, 0.2\rangle,\langle 1, 0.6, 0.3\rangle,\langle 2, 0.6, 0.3\rangle,\langle 3, 0.6, 0.3\rangle,\langle 4, 0.6, 0.3\rangle\}$.Clearly A and B are two different intuitionistic fuzzy subsemirings of R. And, Im $\mu_A=\{0.7,0.5\}$, Im$\nu_A=\{0.1,0.4\}$. The level subsemirings of A are $A_{(0.7,0.1)}=A_{(0.7,0.4)}=A_{(0.5,0.1)}=\{0\}$, $A_{(0.5,0.4)}=\{0,1,2,3,4\}=R$. And, Im $\mu_B=\{0.8,0.6\}$, Im $\nu_B=\{0.2,0.3\}$.The level subsemirings of B are $B_{(0.8,0.2)}=B_{(0.8,0.3)}=B_{(0.6,0.2)}=\{0\}$, $B_{(0.6,0.3)}=\{0, 1, 2, 3, 4\} = R$. Thus the two intuitionistic fuzzy subsemirings A and B have the same family of level subsemirings.

2.13 Theorem: Let A and B be intuitionistic fuzzy subsets of the sets G and H respectively and α and β in [0,1]. Then $(A \times B)_{(\alpha,\beta)} = A_{(\alpha,\beta)} \times B_{(\alpha,\beta)}$.

Proof: It is trivial.

2.14 Theorem: Let (R, +, .) be a finite semiring and A be an intuitionistic fuzzy subsemiring of R. If α, γ are elements of the image set of μ_A of A and β, δ are elements of the image set of ν_A of A such that $A_{(\alpha,\beta)} = A_{(\gamma,\delta)}$, then need not be $\alpha = \gamma$ and $\beta = \delta$.

Proof: We consider the following example: Consider the semiring R = Z_5 ={0,1,2, 3, 4 } with addition modulo 5 and multiplication modulo 5 operations. Define intuitionistic fuzzy subsemiring A by A= {⟨ 0, 0.7, 0.1 ⟩,⟨1 , 0.5, 0.4 ⟩, ⟨ 2, 0.5, 0.4 ⟩,⟨ 3, 0.5, 0.4 ⟩, ⟨4, 0.5, 0.4 ⟩ }. If α = 0.7, γ = 0.5, are in μ_A of A and β = 0.1, δ = 0.1 are in ν_A of A, then $A_{(0.7,0.1)}$ = { 0 }, $A_{(0.5,0.1)}$ = { 0 } are the level subsemirings of R. Clearly $\alpha \neq \gamma$.

2.15 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). Then for α and β in [0,1], μ-level α-cut $U(\mu_A,\alpha)$ is a subsemiring of R.

Proof: For all x and y in $A_{(\alpha,\beta)}$. Now, $\mu_A(x+y) \geq \min\{\mu_A(x),\mu_A(y)\} \geq \alpha$ which implies that $\mu_A(x+y) \geq \alpha$. Now, $\mu_A(xy) \geq \min\{ \mu_A(x),\mu_A(y)\} \geq \alpha$ which implies that $\mu_A(xy) \geq \alpha$. Therefore, $\mu_A(x+y) \geq \alpha$ and $\mu_A(xy) \geq \alpha$, we get x+y and xy in $U(\mu_A,\alpha)$. Hence $U(\mu_A,\alpha)$ is a subsemiring of R.

2.16 Theorem: Let A be an intuitionistic fuzzy subsemiring of a semiring (R, +, .). Then for α and β in [0,1], ν-level β-cut $L(\nu_A, \beta)$ is a subsemiring of R.

Proof: For all x and y in $A_{(\alpha, \beta)}$. Now, $\nu_A(x + y) \leq \max \{\nu_A(x), \nu_A(y) \} \leq \beta$ which implies that $\nu_A(x+y) \leq \beta$. And also, $\nu_A(xy) \leq \max \{\nu_A(x), \nu_A(y) \} \leq \beta$ which implies that $\nu_A(xy) \leq \beta$. We get x+y and xy in $L(\nu_A, \beta)$. Hence $L(\nu_A, \beta)$ is a subsemiring of R.

2.17 Theorem [18]: Let (R, +, •) and (R^I, +, •) be any two semirings. If f: R→ R^I is a homomorphism, then the homomorphic image of a level subsemiring of an intuitionistic fuzzy subsemiring of R is a level subsemiring of an intuitionistic fuzzy subsemiring of R^I.

2.18 Theorem [18]: Let (R, +, •) and (R^I, +, •) be any two semirings. If f: R→ R^I is a homomorphism, then the homomorphic pre-image of a level subsemiring of an intuitionistic fuzzy subsemiring of R^I is a level subsemiring of an intuitionistic fuzzy subsemiring of R.

2.19 Theorem [18]: Let (R, +, •) and (R^I, +, •) be any two semirings. If f: R→R^I is an anti-homomorphism, then the anti-homomorphic image of a level subsemiring of an intuitionistic fuzzy subsemiring of R is a level subsemiring of an intuitionistic fuzzy subsemiring of R^I.

2.20 Theorem [18]: Let (R, +, •) and (R^{l}, +, •) be any two semirings. If f: R →R^{l} is an anti-homomorphism, then the anti-homomorphic pre-image of a level subsemiring of an intuitionistic fuzzy subsemiring of R^{l} is a level subsemiring of an intuitionistic fuzzy subsemiring of R.

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