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Some New Measures of Intuitionistic Fuzzy Entropy and Directed Divergences.

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Abstract

It is shown that to every measure of entropy for a probability distribution there corresponds a measure of entropy for a fuzzy set. In this paper some new measure of Intuitionitic fuzzy entropy and directed divergence have been obtained.

KeyWords- Intuitionistic Fuzzy Entropy / Intuitionistic Fuzzy Directed Divergences

INTRODUCTION:

The concept of entropy was developed to measure the uncertainty of a probability distribution. Zadeh [9] introduced the concept of fuzzy sets.

A fuzzy set A is represented as

A={ $x_i / \mu_A(x_i) : i = 1,2,..........., n.$ }

Where $\mu_A(x_i)$ is a membership function defined as follows:

If $\mu_A(x_i)$= 0, x_i dose not belong to A and there is no ambiguity.

If $\mu_A(x_i)$=1, x_i belong to A and there is no ambiguity.

If $\mu_A(x_i)$=0.5, there is maximum ambiguity whether x_i. belong to A or not.

If $x_1, x_2, ..., x_n$ are member of universe of discourse,then all $\mu_A(x_1), \mu_A(x_2), ..., \mu_A(x_n)$ lie between 0 and 1,but these are not probabilities because their sum is not unity.

Krassimir T.Atanassov [1],[2] introduced the concept of intuitionitic fuzzy set. Let a set E be fixed. An intuitionitic fuzzy set A in E is an object having the form

A={ $< x, \mu_A(x), \nu_A(x) > / x \in E$ }

Where the function $\mu_A(x)$ and $\nu_A(x)$ define the degree of membership and degree of nonmembership of element $x \in A$ to $A \subset E$ respectively.

The function $\mu_A(x)$ and $\nu_A(x)$ satisfy the following condition.

$(\forall x \in E)(0 \le \mu_A(x) + \nu_A(x) \le 1)$.

Obviously fuzzy set has the form $\{ < x,\ \mu_A(x), 1 - \mu_A(x) > / x \in E \}$

A measure of fuzziness $f(\mu_A(x_i), \nu_A(x_i))$ in an Intuitionistic fuzzy set should have atleast the following conditions.

(P-1): It should be defined for all $\mu_A(x_i)$ and $\nu_A(x_i)$ $(i = 1, 2, \ldots\ldots\ldots\ldots, n)$ in the range of $(0 \le \mu_A(x_i) + \nu_A(x_i) \le 1)$, $(i = 1, 2, \ldots\ldots\ldots\ldots, n)$.

(P-2): It should be continuous in this range.

(P-3): It should be zero when $\mu_A(x_i) = 0$ and $\nu_A(x_i) = 0$.

(P-4): It should be not changed when any $\mu_A(x_i)$ is changed into $\nu_A(x_i)$.

(P-5): It should be maximum when $\mu_A(x_i) = \frac{1}{2}$ and $\nu_A(x_i) = \frac{1}{2}$ $(i = 1, 2 \ldots\ldots\ldots, n)$.

(P-6): It should be increasing function of $\mu_A(x_i)$ when $0 \le \mu_A(x_i) \le \frac{1}{2}$ and decreasing function of $\mu_A(x_i)$ when $\frac{1}{2} \le \mu_A(x_i) \le 1$ and other variable are kept fixed. It should be decreasing function of $\nu_A(x_i)$ when $0 \le \nu_A(x_i) \le \frac{1}{2}$ and increasing function of $\nu_A(x_i)$ when $\frac{1}{2} \le \nu_A(x_i) \le 1$ and other variable are kept fixed. $(i = 1, 2, \ldots\ldots\ldots\ldots, n)$

(P-7): It should be concave function of $\mu_A(x_i)$, when $\nu_A(x_i)$ set as a constant.

2. SOME NEW MEASURES OF INTUITIONISTIC FUZZY ENTROPY.

(1) Shannon [7] suggested the following measure of entropy:

$$-\sum_{i=1}^{n} p_i \ln p_i$$

We propose the following Intuitionistic fuzzy entropy:

$$F_1(A) = -\sum_{i=1}^{n} \left[\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) \right]$$

This measure will be valid if and only if it satisfies the seven properties (P-1) to (P-7). So we shall verify these properties.

$F_1(A)$ is defined for all $\mu_A(x_i)$ and $\nu_A(x_i)$ in the range $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$

and it is continuous in this range.

It is zero for $\mu_A(x_i)=0$ and $\nu_A(x_i)=0$.

It dose not change when $\mu_A(x_i)$ is changed into $\nu_A(x_i)$.

It is maximum when $\mu_A(x_i)=\frac{1}{2}$ and $\nu_A(x_i)=\frac{1}{2}$, $(i=1,2,..................,n)$.

It is increasing function of $\mu_A(x_i)$ when $0\le\mu_A(x_i)\le\frac{1}{2}$ and decreasing function of $\nu_A(x_i)$ when $0\le\nu_A(x_i)\le\frac{1}{2},(i=1,2,.............,n)$.

To examine its concavity, we consider the following function.

$\psi(x) = -x\ln x + c$

$\psi''(x) = -\frac{1}{x(1-x)} \le 0$, as $0<x<1$

Thus $\psi(x)$ is a concave function.

Hence $F_1(A)$ is a valid measure of Intuitionistic fuzzy entropy.

(2) Renyi [6] suggested the following measure of entropy:

$$\frac{1}{1-\alpha}\ln\sum_{i=1}^{n} p_i^{\alpha} , \alpha > 0, \alpha \neq 1.$$

We propose the following Intuitionistic fuzzy entropy:

$$F_2(A)=\frac{1}{1-\alpha}\ln\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)\right],\ \alpha>0,\alpha\neq 1$$

(3) Havrda and Charvat [3] suggested the following measure of entropy:

$$\frac{1}{1-\alpha}\sum_{i=1}^{n} p_i^{\alpha} -1 , \alpha > 0, \alpha \neq 1.$$

We propose the following Intuitionistic fuzzy entropy:

$$F_3(A)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)-1\right],\alpha>0,\alpha\neq 1$$

(4) Sharma and Taneja [8] suggested the following measure of entropy:

$$\frac{1}{\beta-\alpha}\left[\sum_{i=1}^{n} p_i^{\alpha} - \sum_{i=1}^{n} p_i^{\beta}\right], \alpha \neq \beta .$$

We propose the following Intuitionistic fuzzy entropy:

$$F_4(A)=\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)-\mu_A^{\beta}(x_i)-\nu_A^{\beta}(x_i)\right],$$

Where either $\alpha \geq 1, \beta \leq 1$ or $\alpha \leq 1, \beta \geq 1$ and $\alpha \neq \beta$.

(5) Kapur [4] suggested the following measure of entropy of degree α , β.

$$\frac{1}{\alpha+\beta-2}\left[\sum_{i=1}^{n} p_i^{\alpha}+\sum_{i=1}^{n} p_i^{\beta}-2\right].$$

We propose the following Intuitionistic fuzzy entropy:

$$F_5(A)=\frac{1}{\alpha+\beta-2}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)+\mu_A^{\beta}(x_i)+\nu_A^{\beta}(x_i)-2\right].$$

(6) Kapur [4] suggested the following measure of entropy of order α and type β :

$$\frac{1}{\beta-\alpha}\ln\frac{\sum_{i-1}^{n} p_i^{\alpha}}{\sum_{i-1}^{n} p_i^{\beta}}, \alpha \neq \beta$$

We propose the following Intuitionistic fuzzy entropy:

$$F_6(A)=\frac{1}{\beta-\alpha}\ln\frac{\sum_{i-1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)\right]}{\sum_{i-1}^{n}\left[\mu_A^{\alpha}(x_i)+\nu_A^{\alpha}(x_i)\right]}$$

where either $\alpha \geq 1, \beta \leq 1$ or $\alpha \leq 1, \beta \geq 1$ and $\alpha \neq \beta$.

(7) Kapur [5] suggested the following measure of entropy:

$$-\sum_{i=1}^{n} p_i \ln p_i+\frac{1}{a}(1+ap_i)\ln(1+ap_i)-\frac{1}{a}(1+a)\ln(1+a),\ a\geq -1$$

We propose the following Intuitionistic fuzzy entropy:

$$F_7(a)=-\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)+\nu_A(x_i)\ln\nu_A(x_i)\right]+\frac{1}{a}\left[(1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))\right]+$$
$$\frac{1}{a}\left[(1+a\nu_A(x_i))\ln(1+a\nu_A(x_i))\right]-\frac{1}{a}(1+a)\ln(1+a),\ a\geq -1.$$

The expression $F_2(A)$ to $F_7(A)$ satisfy all the essential properties of a Intuitionistic fuzzy entropy , so they are valid measure of Intuitionistic fuzzy entropy.

3. SPECIAL CASES.

1. When $\alpha \to 1$, $F_2(A) \to F_1(A)$
2. When $\alpha \to 1$, $F_3(A) \to F_1(A)$
3. When $\beta \to 1$, $F_4(A) \to F_3(A)$

   When $\beta \to 1, \alpha \to 1, F_4(A) \to F_1(A)$

4. When $\beta \to 1$, $F_5(A) \to F_3(A)$

   When $\beta \to 1, \alpha \to 1, F_5(A) \to F_1(A)$

5. When $\beta \to 1$, $F_6(A) \to F_2(A)$

   When $\beta \to 1, \alpha \to 1, F_6(A) \to F_1(A)$

6. When $a \to 1$, $F_7(A) \to F_1(A)$

4. SOME NEW MEASURES OF INTUITIONISTIC FUZZY DIRECTED DIVERGENCES

If $\phi(.)$ is a twice differentiable function with $\phi(1) = 0$.

The measure of directed divergence between two Intuitionistic fuzzy sets A and B is defined as

$$D(A:B) = \sum_{i=1}^{n} \left[\mu_B(x_i)\phi\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right) + \nu_B(x_i)\phi\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right) \right]$$

Here $D(A:B)$ is a convex function, which has minimum value zero when $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$, for all i $(i = 1,2,..............,n)$ and $D(A:B) \geq 0$.

I. Corresponding to $F_1(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_1(A:B) = \sum_{i=1}^{n} \left[\mu_A(x_i) \ln \frac{\mu_A(x_i)}{\mu_B(x_i)} + \nu_A(x_i) \ln \frac{\nu_A(x_i)}{\nu_B(x_i)} \right]$$

II. Corresponding to $F_2(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_2(A:B)=\frac{1}{\alpha-1}\ln\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)\,\mu_B^{1-\alpha}(x_i)+\nu_A^{\alpha}(x_i)\,\nu_B^{1-\alpha}(x_i)\right]$$

III. Corresponding to $F_3(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_3(A:B)=\frac{1}{\alpha-1}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)\,\mu_B^{1-\alpha}(x_i)+\nu_A^{\alpha}(x_i)\,\nu_B^{1-\alpha}(x_i)\right)-1\right]$$

IV. Corresponding to $F_4(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_4(A:B)=\frac{1}{\alpha-\beta}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)\,\mu_B^{1-\alpha}(x_i)+\nu_A^{\alpha}(x_i)\,\nu_B^{1-\alpha}(x_i)\right)-\left(\mu_A^{\beta}(x_i)\,\mu_B^{1-\beta}(x_i)+\nu_A^{\beta}(x_i)\,\nu_B^{1-\beta}(x_i)\right)\right]$$

V. Corresponding to $F_5(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_5(A:B)=\frac{1}{2-\alpha-\beta}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)\,\mu_B^{1-\alpha}(x_i)+\nu_A^{\alpha}(x_i)\,\nu_B^{1-\alpha}(x_i)\right)+\left(\mu_A^{\beta}(x_i)\,\mu_B^{1-\beta}(x_i)+\nu_A^{\beta}(x_i)\,\nu_B^{1-\beta}(x_i)\right)-2\right]$$

VI. Corresponding to $F_6(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_6(A:B)=\frac{1}{\alpha-\beta}\ln\frac{\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i)+\nu_A^{\alpha}(x_i)\nu_B^{1-\alpha}(x_i)\right)}{\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)\mu_B^{1-\beta}(x_i)+\nu_A^{\beta}(x_i)\nu_B^{1-\beta}(x_i)\right)}$$

VII. Corresponding to $F_7(A)$ we propose the following measure of Intuitionistic fuzzy directed divergence.

$$D_7(A:B)=\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\frac{\mu_A(x_i)}{\mu_B(x_i)}+\nu_A(x_i)\ln\frac{\nu_A(x_i)}{\nu_B(x_i)}\right]-$$

$$\frac{1}{a}\sum_{i=1}^{n}\left[(1+a\mu_A(x_i))\ln\frac{(1+a\mu_A(x_i))}{(1+a\mu_B(x_i))}+(1+a\nu_A(x_i))\ln\frac{(1+a\nu_A(x_i))}{(1+a\nu_B(x_i))}\right]$$

We can easily verify that they are valid measures of Intuitionistic fuzzy directed divergence.

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