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Nets in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we investigate the intuitionistic fuzzy net on the intuitionistic fuzzy topological space and we show that there is a relation between the convergence of intuitionistic fuzzy nets and prefilters.

Keywords: Intuitionistic fuzzy points, Fuzzy filters, Fuzzy Nets, Intuitionistic fuzzy topology.

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Introduction

Intuitionistic fuzzy sets constitute a generalization of the notion of a fuzzy set and were introduced by K.T.Atanassov in 1983 in [1] and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [4, 7, 10, 12].On the other hand, Lowen [11] introduced the concept of fuzzy filter and defined convergence in a fuzzy topological space which enables us to characterize fuzzy compactness. Many results on fuzzy filters and nets were obtained by M.A.De Prada and M.S.Aranguren [8, 9].Now, we investigate the intuitionistic fuzzy net on the intuitionistic fuzzy topological space (X,τ) and we show that there is a relation between the convergence of intuitionistic fuzzy nets and prefilters.

Definition1.1 [1] Let X be a nonempty set. An intuitionistic fuzzy set(IFS for short) A is an object having the form $A=\{\langle x,\mu(x),\gamma(x):x\in X\rangle\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership(namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A ,respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$ .For a given nonempty set X , let us denote the family of all IFS's in X by the symbol I^X .

Definition1.2 [2] Let X be a nonempty set and let A and B be two IFSs of X . Then

$A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$.

$A = B$ iff $A \subseteq B$ and $B \subseteq A$.

$$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$$

$$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$$

$$A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$$

Definition1.3 [3] Let $\{A_i : i \in J\}$ be an arbitrary family of IFSs in X .Then

(i) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}, \wedge\gamma_{A_i}\rangle : x \in X\}$

(ii) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}, \vee\gamma_{A_i}\rangle : x \in X\}$

Definition1.4 [6] $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$

Definition1.5 [6] Let X and Y two non-empty sets and $f : X \to Y$ be a map.

(i) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(ii) If $A = \{\langle x, \lambda_A(x), \vartheta_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f, denoted by $f(A)$, is the IFS in Y defined by $f(A) = \{\langle y, f(\lambda_A)(y), (1 - f(1-\vartheta_A))(y)\rangle : y \in Y\}$,

Where $$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1-\vartheta_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

Definition1.6 [6] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

$0_\sim, 1_\sim \in \tau$

$A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$

$\bigcup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\}$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

Definition1.7 [14] Let $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $x(\alpha,\beta)$ of X is an IFS of X defined by $x(\alpha,\beta)(y) = \begin{cases} (\alpha,\beta) & y = x \\ (0,1) & y \neq x \end{cases}$.

In this case x is called the support of $x(\alpha,\beta)$ and α and β are called the value and the non value of $x(\alpha,\beta)$ respectively. An IFP $x(\alpha,\beta)$ is said to belong to an IFS A of X denoted by $x(\alpha,\beta) \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy points as follows:

$x(\alpha,\beta) = \left(x_\alpha, 1 - x_{1\text{-}\beta}\right)$.We will denote the set of all IFPs in X as IFP(X)

Definition1.8 [14] Let $x(\alpha,\beta)$ be an IFP of an IFTS (X,τ).An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $x(\alpha,\beta)$ if there is an intuitionistic fuzzy open set B in X such that $x(\alpha,\beta) \in B \subseteq A$ .The family of all the IFN's of $x(\alpha,\beta)$ is called the system of IFNs of $x(\alpha,\beta)$ denoted by N ($x(\alpha,\beta)$).

Theorem 1.1 [14]
Let (X,τ) be an IFTS. Then an IFS A of X is an IFOS if and only if A is an IFN of $x(\alpha,\beta)$ for every IFP $x(\alpha,\beta) \in A$.

Definition 1.9[10]
Let (X,τ)be an IFTS and let $x(\alpha,\beta) \in$IFP(X).Then B ($x(\alpha,\beta)$) $\subset \tau$ is called a local base at $x(\alpha,\beta)$ if it satisfies the following conditions:

If $B \in B(x(\alpha,\beta))$,then $x(\alpha,\beta) \in B$

If $U \in \tau$ and $x(\alpha,\beta) \in U$ then there exists a $B \in B(x(\alpha,\beta))$ such that $B \subset$ U .

Remark1.1: For simplicity we denote the intuitionistic fuzzy point $x(\alpha,\beta)$ by the alphabet p having support x and value α.

Definition 1.10[2] If $A = \{\langle x, \mu(x), \gamma(x) : x \in X\rangle\}$is an intuitionistic fuzzy set in (X,τ) then the support of A, denoted by Supp$A$ is defined as Supp$A$= $\{\langle x, \mu(x) \neq 0, \gamma(x) \neq 1 : x \in X\rangle\}$.Note that Supp$A = A_0$ is a subset of A.

Definition 1.11[6] Let (X, τ_1) and (Y, τ_2) be two IFTS's and let $f : X \to Y$ be a function .Then f is said to be fuzzy continuous iff the preimage of each IFOS in τ_2 is an IFOS in τ_1.

Definiton1.12 [5] Let $(X, \tau_1), (Y, \tau_2)$ be IFTS's and $f : X \to Y$ be a function. Let p be an IFP in X .Then the function f is said to be fuzzy continuous at p, if for each $M \in \mathcal{N}(f(p))$ there exists $N \in \mathcal{N}(p)$ such that $f(N) \subseteq M$.

Definiton1.13 [13] Let X be a nonempty set and $\mathcal{F}$ a non empty family of IFS's $\neq 0_\sim$ .We will call $\mathcal{F}$ an intuitionistic fuzzy prefilter in X if

for all $A_1, A_2 \in \mathcal{F}$ we have $A_1 \cap A_2 \in \mathcal{F}$

for all $A \in \mathcal{F}$ and each IFS B such that $A \subseteq B$ we have $B \in \mathcal{F}$

$C_0 \notin \mathcal{F}$ where C_t denotes the constant intuitionistic fuzzy set defined for $t \in I$.

Definiton1.14 [13]

A subset $\mathcal{B} \subset \tau$ is a base for an intuitionistic fuzzy prefilter on X iff $\mathcal{B}$ is nonempty and for all $B_1, B_2 \in \tau$ there is $B_3 \in \mathcal{B}$ such that $B_3 \subseteq B_1 \cap B_2$

$C_0 \notin \mathcal{B}$

The intuitionistic fuzzy prefilter $\mathcal{F}$ generated by $\mathcal{B}$ is defined as $\mathcal{F}$ = $\{A \in I^X : \text{there is } B \in \mathcal{B} \text{ such that } B \subseteq A\}$ and is denoted by $\langle \mathcal{B} \rangle$.

For two intuitionistic fuzzy prefilters $\mathcal{F}$ and $\mathcal{G}$ such that $\mathcal{F} \subset \mathcal{G}$ we say that $\mathcal{F}$ is coarser than $\mathcal{G}$ and that $\mathcal{G}$ *is* finer than $\mathcal{F}$.

Definition1.15 [13]

An intuitionistic fuzzy prefilter $\mathcal{F}$ is said to converge to the intuitionistic fuzzy point p $(\mathcal{F} \to p)$ if and only if $\mathcal{N}(p) \subset \mathcal{F}$ that is $\mathcal{F}$ is finer than the neighborhood prefilter at p. We say $\mathcal{F}$ has p as a cluster point if and only if for every $N \in \mathcal{N}(p)$ then $N \cap A \neq 0_\sim$ for every $A \in \mathcal{F}$. Also

A base for an intuitionistic fuzzy prefilter converges to an intuitionistic fuzzy point p $(\mathcal{B} \to p)$ if and only if for each $N \in \mathcal{N}(p)$ contains some $B \in \mathcal{B}$.

A base for an intuitionistic fuzzy prefilter has p as a cluster point iff for each $N \in \mathcal{N}(p)$, $N \cap B \neq 0_\sim$ for each $B \in \mathcal{B}$.

The above are valid if we use the neighborhood bases at p, $\mathcal{B}(p)$.Clearly if $\mathcal{F} \to p$, then $\mathcal{F}$ has p as an intuitionistic fuzzy cluster point.

Intuitionistic fuzzy nets

Definition 2.1 Let $(\mathrm{D},\leq)$ be a directed set, X a non empty set and IFP(X) the collection of all intuitionistic fuzzy points in X. A mapping $\psi : \mathrm{D} \to \mathrm{IFP}(X)$ is called an intuitionistic fuzzy net in X and is denoted by $\{\psi(\mathrm{d}); \mathrm{d} \in \mathrm{D}\}$. If $\psi = x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})}$ for each $\mathrm{d} \in \mathrm{D}$ where $x \in X$, $\mathrm{d} \in \mathrm{D}$, $\alpha_{\mathrm{d}} \in (0,1]$ and $\beta_{\mathrm{d}} \in [0,1)$, then the intuitionistic fuzzy net ψ is denoted as $\left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} : \mathrm{d} \in \mathrm{D}\right\}$ or simply $\left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})}\right\}$.

Definition 2.2 Let (X,τ) be intuitionistic fuzzy topological space and let $\psi = \left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} : \mathrm{d} \in \mathrm{D}\right\}$ be an intuitionistic fuzzy net in X and $A \in \mathrm{I}^X$ then we say that

ψ is in an intuitionistic fuzzy subset A if $\psi(d) \in A, \forall d \in D$.

ψ is residually in an intuitionistic fuzzy subset A if there is some $d_0 \in D$ such that $\psi(d) \in A, \forall d \geq d_0$.

ψ is cofinally in an intuitionistic fuzzy subset A if for each $d \in D$ there is some $d_0 \in D, d \geq d_0$ such that $\psi(d_0) \in A$.

Definition2.3 An intuitionistic fuzzy net $\theta = \left\{y^{e}_{(l_e, m_e)} : e \in E\right\}$ in X is called an intuitionistic fuzzy subnet of an intuitionistic fuzzy net $\psi = \left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} : \mathrm{d} \in \mathrm{D}\right\}$ if and only if there is mapping λ from $(\mathrm{E},\leq)$ into $(\mathrm{D},\leq)$ such that

$\theta = \psi \circ \lambda$ i.e.) $\theta(e) = \psi_{\lambda(e)}$ for each $e \in E$

For each $d \in D$, there exists $e \in E$ such that if $e^* \in E$ with $e^* \geq e$, then $\lambda(e^*) \geq d$.

We denote an intuitionistic fuzzy subnet of an intuitionistic fuzzy net $\left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} : \mathrm{d} \in \mathrm{D}\right\}$ by $\left\{x^{f(e)}_{(\alpha_{f(e)},\beta_{f(e)})} : \mathrm{e} \in \mathrm{E}\right\}$

Definition 2.4 Let (X,τ) be intuitionist fuzzy topological space and let $\psi = \left\{x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} : \mathrm{d} \in \mathrm{D}\right\}$ be an intuitionistic fuzzy net in X and $x(\alpha,\beta) \in$ IFP(X).Then ψ is said to be convergent to $x(\alpha,\beta)$, denoted by $\psi \to x(\alpha,\beta)$, if for every $N \in \mathcal{N}(x(\alpha,\beta))$ there is some $d_0 \in D$ such that $x^{\mathrm{d}}_{(\alpha_{\mathrm{d}},\beta_{\mathrm{d}})} \in N, \forall \mathrm{d} \geq \mathrm{d}_0$ i.e.) if ψ is residually in each neighborhood N of $x(\alpha,\beta)$.

Definition 2.5
Let (X,τ) be an IFTS. We say ψ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point if and only if for each $N \in \mathcal{N}(x(\alpha,\beta))$, for each $d_0 \in D$, there is some $d \in D$ such that

$x^{d}_{(\alpha_d,\beta_d)} \in N$, with $d \ge d_0$ i.e.) $\psi(d)$ is cofinally in each neighborhood N of $x(\alpha,\beta)$.

Theorem2.1
Let (X,τ) be an IFTS, $x(\alpha,\beta) \in$ IFP(X) and $\psi = \left\{x^{d}_{(\alpha_d,\beta_d)} : d \in D\right\}$.If ψ converges to $x(\alpha,\beta)$, then every intuitionistic fuzzy subnet of ψ is also convergent to $x(\alpha,\beta)$.

Proof:
Let $\psi = \left\{x^{d}_{(\alpha_d,\beta_d)} : d \in D\right\}$ be an intuitionistic fuzzy net in X such that ψ converges to $x(\alpha,\beta)$ and let $\theta = \left\{y^{e}_{(l_e,m_e)} : e \in E\right\}$ be an intuitionistic fuzzy subnet of ψ .

Let $N \in \mathcal{N}(x(\alpha,\beta))$.Then there exists $d_0 \in D$ such that for every $d \in D$, $d \ge d_0$ we have $x^{d}_{(\alpha_d,\beta_d)} \in N$

By definition of θ , for this d_0, there exists $e \in E$ such that for each $e^* \in E$, $e^* \ge e$ we have $\lambda(e^{*}) \ge d$ where $\lambda : E \to D$.

Now $y^{e}_{(l_e,m_e)} = x^{f(e)}_{(\alpha_{f(e)},\beta_{f(e)})}$. Then $y^{e}_{(l_e,m_e)} \in N$ for all $e^* \ge e$.

So θ converges to $x(\alpha,\beta)$.

Theorem2.2
Let (X,τ) be an IFTS. An intuitionistic fuzzy net ψ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point if and only if ψ has an intuitionistic fuzzy subnet θ which converges to $x(\alpha,\beta)$.

Proof
Let $x(\alpha,\beta)$ be an intuitionistic fuzzy cluster point of ψ .

Then for any $N \in \mathcal{N}(x(\alpha,\beta))$, there exists an element $x^{d}_{(\alpha_d,\beta_d)}$ of the intuitionistic fuzzy net ψ such that $x^{d}_{(\alpha_d,\beta_d)} \in N$

Let $E = \left\{(d, N) : d \in D, N \in \mathcal{N}\left(x(\alpha,\beta)\right), x^{d}_{(\alpha_d,\beta_d)} \in N\right\}$.

We define an ordering in E as follows: $(d_1, N_1) \ge (d_2, N_2)$ if and only if $d_1 \ge d_2$ and $N_1 \subseteq N_2$.Clearly E is a directed set.

Let $\lambda : E \to D$ and let $\theta : E \to$ IFP(X) given by $\theta(d_1, N_1) = x^{d_1}_{(\alpha_{d_1},\beta_{d_1})}$.θ is an intuitionistic fuzzy subnet of ψ .

To show that θ converges to $x(\alpha,\beta)$, let $M \in \mathcal{N}(x(\alpha,\beta))$.Then there exists $d \in D$ such that $(d, M) \in E$ and $x^{d}_{(\alpha_d,\beta_d)} \in M$ .Thus for any $(e, N) \in E$ with $(e, N) \ge (d, M)$, we have

$\theta(e,N) = x^{e}_{(\alpha_e,\beta_e)} \in N \subseteq M$. Hence θ converges to $x(\alpha,\beta)$.

Converse Part:

Let $M \in N(x(\alpha,\beta))$.Then there exists $d \in D$ such that $(d, M) \in E$ and then $x^{d}_{(\alpha_d,\beta_d)} \in M$.

Thus for any $(d_0, N) \in E$ such that $(d, M) \geq (d_0, N) \in E$, we have $\psi(d, M) = x^{d}_{(\alpha_d,\beta_d)} \in M \subseteq N$.

Hence ψ has $x(\alpha,\beta)$ be an intuitionistic fuzzy cluster point.

Theorem2.3

Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a mapping. Then f is fuzzy continuous if and only if $f(\psi) \to f(x(\alpha,\beta))$ whenever $\psi \to x(\alpha,\beta)$ for each $x(\alpha,\beta) \in \text{IFP}(X)$.

Proof

Let $\psi = \{x^{d}_{(\alpha_d,\beta_d)} : d \in D\}$ is an intuitionistic fuzzy net in X such that $\psi \to x(\alpha,\beta)$.

If $B \in N(f(x(\alpha,\beta)))$ then $f^{-1}(B) \in N(x(\alpha,\beta))$.

Since ψ converges to $x(\alpha,\beta)$, there exists some $d_0 \in D$, $x^{d}_{(\alpha_d,\beta_d)} \in f^{-1}(B)\ \forall d \geq d_0$.

This implies $f(x^{d}_{(\alpha_d,\beta_d)}) \in B$ .Thus $f(\psi)$ converges to $f(x(\alpha,\beta))$.

Converse Part:

Let $B \in I^Y$ be open and $x(\alpha,\beta) \in \text{IFP}(X)$ be such that $x(\alpha,\beta) \in f^{-1}(B)$

Then $f(x(\alpha,\beta)) \in B$ and $B \in N(f(x(\alpha,\beta)))$.

Suppose that for each $A \in N(x(\alpha,\beta))$, we have $A \not\subset f^{-1}(B)$.

Let $x^a \in X$ .Then $\mu_A(x^a) > \mu_{f^{-1}(B)}(x^a)$ and $\gamma_A(x^a) < \gamma_{f^{-1}(B)} x^a)$.

Take $x^{a}_{(\alpha_a,\beta_a)} = q$ to be an intuitionistic fuzzy point with $q(x^a) = \mu_A(x^a)$. Then $x^{a}_{(\alpha_a,\beta_a)} \in A$.

Let $D = (N(p), \leq)$.Then $\psi = \{x^{a}_{(\alpha_a,\beta_a)} : A \in N(x(\alpha,\beta))\}$ is an intuitionistic fuzzy net in X converges to $x(\alpha,\beta)$. Since $B \in N(f(x(\alpha,\beta)))$, and $f(x(\alpha,\beta)) \notin B$, $f(\psi)$ does not converge to $f(x(\alpha,\beta))$.This contradiction shows that there is some $A \in N(x(\alpha,\beta))$ with $A \subseteq f^{-1}(B)$ .Thus f is fuzzy continuous.

Definition2.6
An intuitionistic fuzzy net ψ in X is residually (cofinally) in a subset Y of X if it is residually (cofinally) in any intuitionistic fuzzy subset with support Y.

Definition2.7
An intuitionistic fuzzy net ψ in X is an intuitionistic fuzzy ultranet if for each $Y \subset X$, ψ is residually in Y or in Y^c.

Theorem 2.4
Let X and Y be two sets and f a map from X into Y. If $\bar{f}$ is an induced map from IFP(X) into IFP(Y) and ψ is an intuitionistic fuzzy ultranet in X then $\bar{f} \circ \psi$ is an intuitionistic fuzzy ultranet in Y.

Proof
Let $Y_0 \subset Y$ and $f^{-1}(Y_0) = X_0$. If X_0 is empty, ψ is residually in $X_0{}^c$.Then ψ is residually in any intuitionistic fuzzy subset A with support $X_0{}^c$. For each intuitionistic fuzzy set B in Y with support Y_0,the intuitionistic fuzzy ultra net ψ is residually in $f^{-1}(B)$.Since Supp $f^{-1}(B) = f^{-1}(\text{Supp}\, B) = X_0$.i.e.) there is some $d_0 \in D$ such that $\psi(d) \in f^{-1}(B)$ if $d \geq d_0$.

Thus $f(\psi(d)) \in f(f^{-1}(B)) \subset B$, $\forall\, d \geq d_0$. Hence $\bar{f} \circ \psi$ is residually in Y_0.Thus $\bar{f} \circ \psi$ is an intuitionistic fuzzy ultra net.

Relation between Nets and Prefilters

Definition 3.1 Let $\mathcal{F}$ be an intuitionistic fuzzy prefilter on X, IFP(X) the collection of all intuitionistic fuzzy points in X and $D_{\mathcal{F}} = \{(p, F) : p \in F, p \in \text{IFP}(X), F \in \mathcal{F}\}$ $\subset \text{IFP}(X) \times \mathcal{F}$ directed by the relation $(p_1, F_1) \leq (p_2, F_2)$ if and only if $F_2 \subseteq F_1$. The map $\psi_{\mathcal{F}} : D_{\mathcal{F}} \to \text{IFP}(X)$ defined by $\psi_{\mathcal{F}}(p, F) = p$ is an intuitionistic fuzzy net in X, called the intuitionistic fuzzy net based on the intuitionistic fuzzy prefilter $\mathcal{F}$.

Definition 3.2 Let $\psi = \{x^{a}_{(\alpha_a, \beta_a)} : d \in D\}$ be an intuitionistic fuzzy net in X. For each $d \in D$, let $B_d = \cup \{x^{d_0}_{(\alpha_{d_0}, \beta_{d_0})} : d_0 \in D, d_0 \geq d\}$.Then the family $\mathcal{F}_\psi = \{B_d : d \in D\}$ forms an intuitionistic fuzzy filter base on X, called the intuitionistic fuzzy filter generated by the intuitionistic fuzzy net ψ.

Theorem 3.1 Let (X,τ) be an intuitionistic fuzzy topological space, $x(\alpha,\beta) \in \text{IFP}(X)$ and let ψ be an intuitionistic fuzzy net in X then
ψ converges to $x(\alpha,\beta)$ if and only if $\mathcal{F}_\psi$ converges to $x(\alpha,\beta)$
ψ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point if and only if
$\mathcal{F}_\psi$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point.

Proof

(i) Since $\mathcal{F}_\psi = \{B_d : d \in D\}$, $B_d = \cup \left\{ x^{d_0}_{(\alpha_{d_0},\beta_{d_0})} : d_0 \in D \right\}$ we have $\mathcal{F}_\psi$ converges to $x(\alpha,\beta)$ if and only if for each $N \in \mathcal{N}(x(\alpha,\beta))$,there exists $B \in \mathcal{F}_\psi$, $B \subseteq N$ if and only if for each $N \in \mathcal{N}(x(\alpha,\beta))$, there exists $d \in D$, $B = \bigcup \left\{ x^{d_0}_{(\alpha_{d_0},\beta_{d_0})} \right\} \subseteq N$ if and only if for each $N \in \mathcal{N}(x(\alpha,\beta))$,there exists $d \in D$, for every $d_0 \in D$, $d_0 \geq d$, $x^{d_0}_{(\alpha_{d_0},\beta_{d_0})} \in N$

Hence ψ converges to $x(\alpha,\beta)$.

(ii)Since $\mathcal{F}_\psi$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point if and only if for every $N \in \mathcal{N}(x(\alpha,\beta))$ and for every $B \in \mathcal{F}_\psi$, $B \cap N \neq 0_\sim$ if and only if for every $N \in N(x(\alpha,\beta))$,

$\forall d \in D$, $d_0 \geq d$, $\left\{ \cup \{ x^{d_0}_{(\alpha_{d_0},\beta_{d_0})} \} \right\} \cap N \neq 0_\sim$ if and only if $N \in N(x(\alpha,\beta))$,

$\forall d \in D$ there exists $d_0 \in D, d_0 \geq d$, $x^{d_0}_{(\alpha_{d_0},\beta_{d_0})} \in N$.

Hence ψ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point.

Theorem 3.2 Let (X,τ) be an intuitionistic fuzzy topological space, $x(\alpha,\beta) \in \text{IFP}(X)$ and let $\mathcal{F}$ be an intuitionistic fuzzy prefilter in X then
$\mathcal{F}$ converges to $x(\alpha,\beta)$ if and only if $\psi_\mathcal{F}$ converges to $x(\alpha,\beta)$
$\mathcal{F}$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point if and only if
$\psi_\mathcal{F}$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point.

Proof

(i) Let $\mathcal{F}$ converges to $x(\alpha,\beta)$ and let $\psi_\mathcal{F} : \text{D}_\mathcal{F} \to \text{IFP}(X)$ be the intuitionistic fuzzy net based on $\mathcal{F}$ where $\text{D}_\mathcal{F} = \{(p,F) : p \in F, p \in \text{IFP}(X), F \in \mathcal{F}\}$ and $\psi_\mathcal{F}(p,F) = p$. If $N \in \mathcal{N}(\text{x}(\alpha,\beta))$, then there exists $A \in F$ such that $A \subseteq N$. Choose $q = y(\text{h},\text{k}) \in A$ such that $(q,A) \in \text{D}_\mathcal{F}$. If $r = z(\text{l},\text{m})$, $(r,A_1) \in \text{D}_\mathcal{F}$ is such that $(q, A) \leq (r, A_1)$ then $\psi_\mathcal{F}(r,A_1) = r \in A_1$

Since $A_1 \subseteq A \subset N$, we have $r \in N$. Hence $\psi_F\ (r, A_1) \in N$.

Thus ψ_F converges to $x(\alpha,\beta)$.

Conversely, let ψ_F converges to $x(\alpha,\beta)$. For every $N \in \mathcal{N}\ (x(\alpha,\beta))$ there exists $(r, A_1) \in \mathrm{D}_F$ such that for each $(q,A) \in \mathrm{D}_F$ with $(r, A_1) \le (q, A)$ we have $\psi_F\ (q, A) = q \in N$.

For each $s \in A_1$, we have $(r, A_1) \le (s, A_1)$ and hence $\psi_F\ (s, A_1) = s \in N$.

Hence $A_1 \subseteq N$ .Thus $\mathcal{F}$ converges to $x(\alpha,\beta)$.

(ii) Let $\mathcal{F}$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point .Let $\psi_F : \mathrm{D}_F \to \mathrm{IFP}(X)$ be the intuitionistic fuzzy net based on $\mathcal{F}$, where $\mathrm{D}_F = \{(p,F) : p \in F, p \in \mathrm{IFP}(X), F \in \mathcal{F}\}$ and $\psi_F\ (p,F) = p$. Let $N \in \mathcal{N}\ (\mathrm{x}(\alpha,\beta))$ and $(q,A) \in \mathrm{D}_F$ then $A \cap N \neq 0_\sim$.

If $r = z(\mathrm{l,m}) \in A \cap N$ then $(r,A) \in \mathrm{D}_F$ is such that $(q,A) \le (r,A)$. Hence $\psi_F\ (r,A) = r \in N$. ψ_F has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point.

Conversely, let ψ_F has $x(\alpha,\beta)$ as intuitionistic fuzzy cluster point and let $N \in \mathcal{N}\ (x(\alpha,\beta))$ and $A \in \mathcal{F}$. If $q = y(\mathrm{h,k}) \in A$ then $(q,A) \in \mathrm{D}_F$ and hence there is $(r,A_1) \in \mathrm{D}_F$ with $(q, A) \le (r, A_1)$ and $\psi_F\ (r,A_1) = r \in N$. Since $A_1 \subseteq A$ and $r \in A_1 \cap N$, we have $r \in A \cap N$ and so $A \cap N \neq 0_\sim$. Thus $\mathcal{F}$ has $x(\alpha,\beta)$ as an intuitionistic fuzzy cluster point.

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