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Trapezoidal Fuzzy Queue Model to an Interdependent Communication Network with Bulk Arrivals

Arti Tyagi[1] and T.P. Singh[2]

Research scholar M.M University Mullana. India.
Professor ,Yamuna institute of Engg .& Technology, Gadholi. Yamuna Nagar, India.
artityagi16@yahoo.co.in, tpsing78@yahoo.com

ABSTRACT

In this paper we apply an interdependence fuzzy queue model to a communication system with integrating data/voice packetized statistical multiplexing. In a state of overload the packet transmission is dropped in the multiplex queue to relive the congestion and transmitted through a co-transmitter. It is also approximated that the arrival processes are bulk arrival since in packetized data/voice transmission message is divided into various packets of random length. The messages, services and the transmission completion have been assumed trapezoidal fuzzy in nature. The model finds its application in reducing the average delay of transmission and variability of the buffer content and appears closer to real world situation.

Key words: Packet switching, fuzzy parameters, Trapezoidal fuzzy number, Message switching, statistical multiplexing etc.

1. INTRODUCTION:-

With the advent of faxes and the internet, the nature of traffic has changed dramatically. As a result packet switched networks have gained importance over circuit switching networks. The Circuit switching telephone is ill suited to interactive data traffic because it is designed for less frequent service requests with long holding times, whereas in packet switching the transmission links are shared on as needed basis. Each packet is transmitted as soon as the appropriate link is available but no transmission facilities are held by a source when it has nothing to send. Thus link utilization is improved at the expense of storage and control complexity in the model. Packet switching involves dividing data massages into small bundles of information and transmitting them through communication networks to their indented destination using computer control switches. In circuit switching, only few circuits are in use, much new transmission capacity is idle whereas in light load condition in packet switching the active users benefit by shorter than usual delay times. In all packet node communication system, network resources are managed by statistical multiplexing or dynamic memory allocation in which a communication channel is effectively divided into an arbitrary number of logical bit rate channels or data streams. The delay in packet switching can be reduced by utilizing the statistical multiplexing in communication system.

In recent years, researchers have been engaged in the study of communication system, data traffic process in the context of queuing analysis. A close resemblance has been observed between queue models and communication network. The massages can be regarded as customers, communication buffers as waiting line and all the concerned activity in transmission of messages as service pattern.

Srinivasa Rao etal (2000, 2003) developed an interdependent communication network with the assumption that arrival and transmission process at the node of the network are correlated and follows a bi-variate Poisson process having joint probability mass function of the form given by Milne (1974). Arrivals of packets are assumed to be single i.e. each packet arrives of its own. But in packet switching the messages is divided into small packets of random length. Therefore, message arrivals to the buffer free in bulk of packets. Srinivas Rao etal. (2006), Singh T.P.(2011) made an attempt to study an interdependent communication system assuming arrivals in bulk and showed the relevancy of the result with regard to expected number of packets in system as well as in buffer and variance of number of packets in system.

Recently, the work was further extended by Singh T.P., Kusum etal. (2012) on studying an interdependent fuzzy queue model to a communication system with voice packetized statistical multiplexing under triangular fuzzy environment. This study is further an extension of work done by Singh T.P. etal. (2012) as the fuzzy environment is considered trapezoidal in nature.

Communication has been studied due to unpredictable and uncertain nature of demand at transmission line, congestion occurs in communication system. Therefore, the messages as well as the concerned activity in transmission of massages, as service pattern etc. have been assumed trapezoidal fuzzy in nature. The co-variance between the composite arrival and transmission completion have also been assumed trapezoidal fuzzy in nature. Our model is more realistic than the work done of earlier authors. This interdependent network can reduce the mean buffer length and the variability of buffer contents when the environment is fuzzy. The delay in packet switching can be reduced by statistical multiplexing in communication network.

2. FUZZY SET:

Fuzzy logic extends Boolean logic to handle the expression of vague concepts. To express impression quantitatively a set membership function maps elements to real values between 0 & 1. The value indicates the degree to which an element belongs to a set. The degree is not describing probabilities that the item is in the set, but instead describes to what extent the item is in the set.

In the universe of discourse X, a fuzzy subset $\widetilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}$ (X) Which maps each element x into X to a real number in the interval [0,1]. $\mu_{\tilde{A}}$ (X) Denotes the grade or degree of membership and it is usually denoted as

$\mu_{\tilde{A}}$ (X) : X→[0,1]. If a fuzzy set A is defined on X, for any $\alpha \in [0,1]$, the α-cuts ${}^{\alpha}A$ is represented by the following crisp set,

Strong α-cuts: ${}^{\alpha+}A = \{ x \in X \;/\; \mu_A(x) > \alpha \}; \alpha \in [0,1]$

Weak α-cuts: ${}^{\alpha}A = \{ x \in X \;/\; \mu_A(x) \geq \alpha \}; \alpha \in [0,1]$

Hence, the fuzzy set A can be treated as crisp set ${}^{\alpha}A$ in which all the members have their membership values greater than or at least equal to α. It is one of the most important concepts in fuzzy set theory.

Notations:

$\tilde{\lambda}$: Fuzzy arrival rate

$\tilde{\mu}$: Fuzzy service rate

$\tilde{\rho}$: Fuzzy busy time of the server = $\frac{\tilde{\lambda}}{\tilde{\mu}}$

2.1 TRAPEZOIDAL FUZZY NUMBER:

A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. Taking <a1,a2,a3,a4,> trapezoidal fuzzy number in supporting interval [0,1] then,

$$\widetilde{A} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1 & a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise} \end{cases}$$

2.2 TRAPEZOIDAL FUZZY NUMBER OPERATION:

Let $\widetilde{A} = (a_1,a_2,a_3,a_4)$ and $\tilde{B} = (b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy number, then the arithmetic operation on $\widetilde{A}$ and $\tilde{B}$ are given as follows :

Addition $\widetilde{A} + \tilde{B} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4]$

Subtraction $\widetilde{A} - \tilde{B} = [a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1]$

Multiplication $\widetilde{A} * \tilde{B} = [a_1b_2, a_2b_2, a_3b_3, a_4b_4]$

Division $\widetilde{A} / \tilde{B} = [a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1]$

Provided $\widetilde{A}$ *and* $\tilde{B}$ are all non-zero positive numbers.

2.3 DEFUZZIFICATION OF TRAPEZOIDAL FUZZY NUMBER:

If $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then its associated crisp number is given by Yager's formula as follows:

$$A = \frac{a_1+2a_2+2a_3+a_4}{6}$$

MODEL DESCRIPTION & NOTATION:

Consider the arrival of packets and number of transmission are correlated. Both follows a bi-variate Poisson process having joint probability mass function based on the line of Milne (1974) & Srinivasa Rao, etal.(2000). The capacity of buffer is assumed to be infinite and the number of packets arrival in any module is taken as a random variable x in fuzzy environment.

$\widetilde{\lambda_x}$: Arrival rate of message of size having x packets in fuzzy.

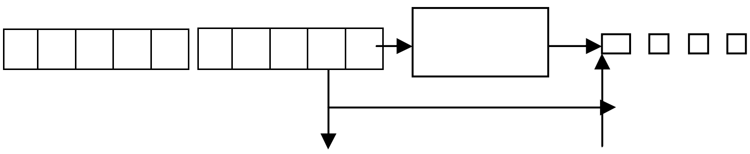
$\widetilde{\epsilon_x}$: Covariance between arrival of packets and number of transmission completion in fuzzy.

$\tilde{\mu}$: Average transmission rate in fuzzy.

$\widetilde{C_x}$: Probability that a batch of size x packets will arrive to buffer in fuzzy environment.

The composite arrival rate of packets $\tilde{\lambda} = \sum_x \widetilde{\lambda_x}$ and the covariance of the composite arrivals and transmission completions $\tilde{\epsilon} = \sum_x \widetilde{\epsilon_x}$

The covariance is generated through bit dropping of flow control mechanism inducing the dependence between arrival of messages and service transmissions. The network diagram is shown as:



Bit dropping or Flow control

4. MATHEMATICAL ANALYSIS:

Connecting the probability consideration, the differential difference equation of the fuzzy system is in transient form can be depicted as:

$$\widetilde{P_n'}(t) = -\left(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon}\right)\tilde{P}_n(t) + (\tilde{\mu} - \tilde{\varepsilon})\tilde{P}_{n+1}(t) + \left(\tilde{\lambda} - \tilde{\varepsilon}\right)\sum_{r=1}^{n} \tilde{P}_{n-r}(t) C_r, \qquad n \geq 1 \quad (1)$$

$$\widetilde{P_0'}(t) = -\left(\tilde{\lambda} - \tilde{\varepsilon}\right)\tilde{P}_0(t) + (\tilde{\mu} - \tilde{\varepsilon})\tilde{P}_1(t)$$

4.1 In steady state:

The steady state condition is reached when the behavior of the system becomes independent of the time ($t \rightarrow \infty$)

The steady state equations:

$$\left.\begin{aligned} 0 &= -(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon})\tilde{P}_n + (\tilde{\mu} - \tilde{\varepsilon})\tilde{P}_{n+1} + (\tilde{\lambda} - \tilde{\varepsilon})\sum_{r=1}^{n} \tilde{P}_{n-r}\, C_r, \quad n \geq 1 \\ 0 &= -(\tilde{\lambda} - \tilde{\varepsilon})\tilde{P}_0 + (\tilde{\mu} - \tilde{\varepsilon})\tilde{P}_1, \quad n \geq 1 \end{aligned}\right\} \quad (2)$$

Assuming P.g.f of the number of packets in the buffer and the number of packets that the message is divided:

$\tilde{P}(\tilde{z}) = \sum_{n=0}^{\infty} \tilde{P}_n \tilde{z}^n$, $|\tilde{z}| \leq 1$

$\tilde{C}(\tilde{z}) = \sum_{n=0}^{\infty} \tilde{C}_n \tilde{z}^n$, $|\tilde{z}| \leq 1$

Multiplying (2) with $\tilde{z}^n$ &summing over n =0 to ∞, after simplification, we get

$$0= -(\tilde{\lambda} - \tilde{\varepsilon})\tilde{P}(\tilde{z}) - (\tilde{\mu} - \tilde{\varepsilon})[\tilde{P}(\tilde{z}) - \tilde{P}(0)] + \frac{(\tilde{\mu} - \tilde{\varepsilon})}{\tilde{z}}[\tilde{P}(\tilde{z}) - \tilde{P}(0)] + (\tilde{\lambda} - \tilde{\varepsilon})\tilde{C}(\tilde{z})\tilde{P}(\tilde{z}) \quad (3)$$

From (3),

$$\tilde{P}(\tilde{z}) = \frac{(\tilde{\mu} - \tilde{\varepsilon})(1 - \tilde{z})}{(\tilde{\mu} - \tilde{\varepsilon})(1 - \tilde{z}) - (N - \tilde{\varepsilon})\tilde{z}\left(1 - \tilde{C}(\tilde{z})\right)} \tilde{P}_0 \quad (4)$$

Let the batch size $\tilde{C}_x$ is geometrically distributed:

i.e $\widetilde{C_x} = (1 - \alpha)\alpha^n$ $\quad 0 < \alpha < 1$ then,

$$\tilde{C}(\tilde{z}) = \frac{(1 - \alpha)\tilde{z}}{1 - \alpha\tilde{z}} \quad (5)$$

Use (5) in (4) we have

$$P(z) = \frac{(\tilde{\mu} - \tilde{\varepsilon})(1 - \alpha\tilde{z})\widetilde{P_0}}{(\tilde{\mu} - \tilde{\varepsilon})(1 - \alpha\tilde{z}) - (\tilde{\lambda} - \tilde{\varepsilon})\tilde{z}} \quad (6)$$

For $(\tilde{\lambda} - \tilde{\varepsilon}) < (\tilde{\mu} - \tilde{\varepsilon})(1 - \alpha)$

5. PERFORMANCE MEASURE:

Using condition $\tilde{P}(1)=1$

Probability that the system is empty i.e. $\tilde{P}(0) = 1 - \frac{(\tilde{\lambda} - \tilde{\varepsilon})}{(\tilde{\mu} - \tilde{\varepsilon})(1 - \alpha)}$

$= 1 - \tilde{\rho}_0$ (7)

Expanding $\tilde{P}(\tilde{z})$ and collecting the coefficient of $\tilde{z}^i$, the probability that the system size is ‘I ‘as,

$$\tilde{P}_x = (1 - \tilde{\rho}_0)[\alpha + (1 - \alpha)\tilde{\rho}_0]^{i-1}(1 - \alpha)\tilde{\rho}_0 \quad (8)$$

Note: In (3.2) if we take limit $\alpha \to 0$ we get

$\tilde{P}_i = (1 - \tilde{\rho}_0)(\tilde{\rho}_0) \qquad i > 0$

This gives the interdependent communication network without bulk arrivals in fuzzy environment.

The average fuzzy number of packets in the system is

$\tilde{L} = \frac{\tilde{\rho}_0}{(1-\alpha)(1-\tilde{\rho}_0)}$ where $\tilde{\rho}_0 = \frac{(\tilde{\lambda}-\tilde{\varepsilon})}{(1-\alpha)(\tilde{\mu}-\tilde{\varepsilon})}$

(ii) The variance of the fuzzy number of packets in the system

Variance = $\frac{\alpha\tilde{\rho}_0(1-\tilde{\rho}_0)+\tilde{\rho}_0}{(1-\alpha)^2(1-\tilde{\rho}_0)^2}$

Value of Average number of packets i.e. $\tilde{L}$

Table-1

	$\tilde{\lambda}$=(3,4,5,6)	$\tilde{\mu}$= (9,8,7,6)		
Average fuzzy number of packets in the system				
α	(.1,.2,.25,.3)	(.26,.30,.34,.36)	(.35,.40,.42,.45)	(.5,.6,.7,.72)
0.1	(2.2, 2.14, 1.97, 1.73)	(2.08, 2.01, 1.9 ,1.64)	(2.05,1.95,1.83,1.62)	(1.89,1.85,1.7,1.5)
0.15	(3.23,2.87,2.51,1.77)	(2.75,2.58,2.31,1.96)	(2.65,2.47,2.22,1.88)	(2.50,2.35,2.05,1.72)
0.2	(4.82,3.8,3.2,2.52)	(3.95,3.57,2.96,2.37)	(3.87,3.35,2.83,2.32)	(3.67,2.63,2.52,2.07)
0.25	(8.26,5.48,4.2,3.14)	(6.66,5.12,4,3)	(6.6,4.8,3.8,2.9)	(6.13,4.53,3.3,2.50)

Table-II

On Defuzzification, the average number of packets we get:

α	(.1,.2,.25,.3)	(.26,.30,.34,.36)	(.35,.40,.42,.45)	(.5,.6,.7,.72)
0.1	2.58	1.92	1.87	1.74
0.15	2.62	2.39	2.31	2.12
0.2	3.46	3.23	3.09	2.67
0.25	5.12	4.66	4.45	3.01

Table-III

Variance of number of packets in the system					
α	έ	(.1,.2,.25,.3)	(.26,.30,.34,.36)	(.35,.40,.42,.45)	(.5,.6,.7,.72)
0.1		(11.2,7.9,6.1,4.07)	(9.4,7.6,5.6,3.85)	(9.1,6.9,5.2,3.65)	(7.8,6.5,4.5,3.1)
0.15		(20.1,13.6,9.4,6.6)	(15.8,11.5,8.5,5.1)	(15.2,10.1,7.7,4.7)	(14.6,9.7,6.2,3.9)
0.2		(48.4,23.1,13.7,7.9)	(30.1,22.5,12.3,7.3)	(29.2,17.8,11.2,6.9)	(27.8,17.3,9.4,5.3)
0.25		(120.8,43.7,22.6,11.7)	(99.6,38.5,20.5,10.7)	(84.17,34.2,14.2,10)	(82.5,32.5,15,8)

Table-IV

On Defuzzification, the Variance of number of packets in the system					
α	έ	(.1,.2,.25,.3)	(.26,.30,.34,.36)	(.35,.40,.42,.45)	(.5,.6,.7,.72)
0.1		7.21	6.60	6.15	5.4
0.15		12.1	10.1	9.2	8.3
0.2		21.6	17.8	15.6	14.4
0.25		44.1	38.05	33.4	30.9

6. CONCLUSION:

From the table, it is clear that the average numbers of packets in network as well as in the buffer are decreasing as the independent parameters are increasing provided these are positive and other parameters are fixed. Also we find that the average number of packets in fuzzy network and in buffer are increasing as α increases for given value of λ, μ, ϵ. The mean buffer length of this network is less than that of classical network with bulk arrival without interdependence. But when the covariance of the composite arrivals and transmission completion are fuzzy in nature then result horizontally are not so restrict. The whole system parameters become fuzzy in nature and the system behavior shows monotonicity not necessarily to be decreasing strictly. From table (III) & (IV) we again observe the monotonicity in the behavior of the variability of number of packets in buffer. In general, as α increases variance also increases provided the other parameters fixed. On defuzzification, the result also show the similar behavior.

i. if in the model the arrivals, services & co-variances are taken in crisp set then the result of the model tally with the work made by the Srinivasa Rao (2006) and Singh T.P (2011).
ii. If instead of trapezoidal fuzzy number, the data are considered in triangular fuzzy nature then the result are in agreement with the work done by Singh T. P., Kusam etal(2012).

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