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Intuitionistic Fuzzy Almost Weakly Generalized Closed Mappings

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy almost weakly generalized closed mappings and intuitionistic fuzzy almost weakly generalized open mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy weakly generalized closed set, intuitionistic fuzzy almost weakly generalized closed mappings and intuitionistic fuzzy almost weakly generalized open mappings.

AMS Subject Classification (2000): 54A40, 03F55

Introduction

Fuzzy set (FS) as proposed by Zadeh [15] in 1965 is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

In this paper, we introduce the notion of intuitionistic fuzzy almost weakly generalized closed mappings and intuitionistic fuzzy almost weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy almost weakly generalized closed mappings and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.

Then

a. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
b. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
c. $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,
d. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$,
e. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A \rangle$ shall be used instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A= \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

a. $0_\sim, 1_\sim \in \tau$,
b. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
c. $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i\in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in X and } G \subseteq A \}$,

$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in X and } A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ in an IFTS (X, τ) is said to be

a. *intuitionistic fuzzy semi closed set* [4] (IFSCS in short) if $int(cl(A)) \subseteq A$,
b. *intuitionistic fuzzy α-closed set* [4] (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,
c. *intuitionistic fuzzy pre-closed set* [4] (IFPCS in short) if $cl(int(A)) \subseteq A$,
d. *intuitionistic fuzzy regular closed set* [4] (IFRCS in short) if $cl(int(A)) = A$,
e. *intuitionistic fuzzy generalized closed set* [14] (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
f. *intuitionistic fuzzy generalized semi closed set* [12] (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
g. *intuitionistic fuzzy α generalized closed set* [10] (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IFαGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6: [6] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

The family of all IFWGCSs in an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7: [6] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs in an IFTS (X, τ) is denoted by IFWGO(X).

Result 2.8: [6] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9: [6] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

$wgint(A) = \cup \{ G / G \text{ is an IFWGOS in X and } G \subseteq A \}$,

$wgcl (A) = \cap \{ K / K \text{ is an IFWGCS in X and } A \subseteq K \}$.

Definition 2.10: [3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle / x \in X\}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If A = {⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x ∈ X} is an IFS in X, then the *image* of A under f denoted by f(A) is the IFS in Y defined by f (A) = {⟨ y, f ($\mu_A(y)$), f_($\nu_A(y)$) ⟩ / y ∈ Y} where f_(ν_A) = 1-f(1- ν_A).

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be

a. [8] *intuitionistic fuzzy weakly generalized closed mapping* (IFWGCM in short) if f (A) is an IFWGCS in Y for every IFCS A in X,
b. [8] *intuitionistic fuzzy weakly generalized * -closed mapping* (IFWG*CM in short) if f(A) is an IFWGCS in Y for every IFWGCS A in X,
c. [13] *intuitionistic fuzzy closed mapping* (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X,
d. [4] *intuitionistic fuzzy semi closed mapping* (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X,
e. [4] *intuitionistic fuzzy pre-closed mapping* (IFPCM for short) if f(A) is an IFPCS in Y for every IFCS A in X,
f. [4] *intuitionistic fuzzy α-closed mapping* (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X,
g. [11] *intuitionistic fuzzy α-generalized closed mapping* (IFαGCM for short) if f(A) is an IFαGCS in Y for every IFCS A in X,.
h. [5] *intuitionistic fuzzy almost closed mapping* (IFACM in short) if f(A) is an IFCS in Y for every IFRCS A in X,
i. [9] *intuitionistic fuzzy almost weakly generalized continuous* (IFAWG continuous in short) if $f^{-1}(A)$ is an IFWGCS in X for every IFRCS A in Y.

Definition 2.12: [6] An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_wT_{1/2}$ *space* ($IF_wT_{1/2}$ space in short) if every IFWGCS in X is an IFCS in X.

Definition 2.13: [6] An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_{wg}T_q$ *space* ($IF_{wg}T_q$ space in short) if every IFWGCS in X is an IFPCS in X.

Intuitionistic Fuzzy Almost Weakly Generalized Closed Mappings

In this section, we introduce intuitionistic fuzzy almost weakly generalized closed mappings and study some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy almost weakly generalized closed mapping* (IFAWGCM in short) if f(A) is an IFWGCS in Y for every IFRCS A in X.

Example 3.2: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.3), (0.5, 0.6)⟩, T_2 = ⟨ y, (0.3, 0.3), (0.6, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Now int(T_1^C) = T_1 and cl(int(T_1^C)) = T_1^C. Therefore T_1^C is an IFRCS in X. Consider a mapping f: (X, τ) → (Y,σ) defined as f (a) = u and f (b) = v. Then

$f(T_1^C) = \langle y, (0.5, 0.6), (0.4, 0.3)\rangle$ is an IFWGCS in Y, since $cl(int(T_1^C)) = T_2^C \subseteq 1_\sim$. Hence f is an IFAWGCM.

Theorem 3.3: Every IFWGCM is an IFAWGCM but not conversely.

Proof: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IFWGCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.4: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.2), (0.4, 0.4)\rangle$, $T_2 = \langle x, (0.2, 0), (0.5, 0.4)\rangle$, $T_3 = \langle y, (0.5, 0.6), (0.2, 0)\rangle$, $T_4 = \langle y, (0.5, 0.4), (0.5, 0.1)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, T_4, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: $(X, \tau) \rightarrow (Y,\sigma)$ defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFWGCM, since the IFS $T_2^C = \langle x, (0.5, 0.4), (0.2, 0) \rangle$ is an IFCS in X but $f(T_2^C) = \langle y, (0.5, 0.4), (0.2, 0) \rangle$ is not an IFWGCS in Y, since $cl(int(f(T_2^C))) = 1_\sim \not\subseteq T_3$.

Theorem 3.5: Every IFCM is an IFAWGCM but not conversely.

Proof: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IFCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFCS in Y. Since every IFCS is an IFWGCS, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.6: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$, $T_2 = \langle y, (0.3, 0.2), (0.6, 0.8)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: $(X, \tau) \rightarrow (Y,\sigma)$ defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFCM, since the IFS $T_1^C = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ is an IFCS in X but $f(T_1^C) = \langle y, (0.6, 0.7), (0.4, 0.2) \rangle$ is not an IFCS in Y, since $cl(f(T_1^C)) = T_2^C \neq f(T_1^C)$.

Theorem 3.7: Every IFαCM is an IFAWGCM but not conversely.

Proof: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IFαCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFαCS in Y. Since every IFαCS is an IFWGCS, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.8: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$, $T_2 = \langle y, (0.3, 0.2), (0.6, 0.6)\rangle$, $T_3 = \langle y, (0.7, 0.7), (0.2, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: $(X, \tau) \rightarrow (Y,\sigma)$ defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFαCM, since the IFS $T_1^C = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle$ is an IFCS in X but $f(T_1^C) = \langle y, (0.6, 0.5), (0.3, 0.4) \rangle$ is not an IFαCS in Y, since $cl(int(cl(f(T_1^C)))) = T_2^C \not\subseteq f(T_1^C)$.

Theorem 3.9: Every IFPCM is an IFAWGCM but not conversely.

Proof: Let f: (X, τ) $\rightarrow$ (Y,σ) be an IFPCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFPCS in Y. Since every IFPCS is an IFWGCS, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.10: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$, $T_2 = \langle y, (0.2, 0.4), (0.8, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: (X, τ) $\rightarrow$ (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFPCM, since the IFS $T_1^C = \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$ is an IFCS in X but $f(T_1^C) = \langle y, (0.7, 0.6), (0.3, 0.4) \rangle$ is not an IFPCS in Y, since $cl(int(f(T_1^C))) = T_2^C \not\subset f(T_1^C)$.

Theorem 3.11: Every IFαGCM is an IFAWGCM but not conversely.

Proof: Let f: (X, τ) $\rightarrow$ (Y,σ) be an IFαGCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFαGCS in Y. Since every IFαGCS is an IFWGCS, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.12: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$, $T_2 = \langle y, (0.6, 0.6), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: (X, τ) $\rightarrow$ (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFαGCM, since the IFS $T_1^C = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle$ is an IFCS in X but $f(T_1^C) = \langle y, (0.6, 0.5), (0.3, 0.4) \rangle$ is not an IFαGCS in Y, since $\alpha cl(f(T_1^C)) = 1_\sim \not\subset T_2$.

Remark 3.13: An IFSCM and an IFAWGCM are independent to each other as seen from the following examples.

Example 3.14: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.4, 0.2), (0.5, 0.2)\rangle$, $T_2 = \langle y, (0.5, 0.4), (0.2, 0.1)\rangle$, $T_3 = \langle y, (0.3, 0.1), (0.7, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: (X, τ) $\rightarrow$ (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFSCM but not an IFAWGCM, since the IFS $T_1^C = \langle x, (0.5, 0.2), (0.4, 0.2) \rangle$ is an IFRCS in X but $f(T_1^C) = \langle y, (0.5, 0.2), (0.4, 0.2) \rangle$ is not an IFWGCS in Y, since $cl(int(f(T_1^C))) = T_3^C \not\subset T_2$.

Example 3.15: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.4, 0.1), (0.6, 0.9)\rangle$, $T_2 = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping f: (X, τ) $\rightarrow$ (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFSCM, since the IFS $T_1^C = \langle x, (0.6, 0.9), (0.4, 0.1) \rangle$ is an IFCS in X but $f(T_1^C) = \langle y, (0.6, 0.9), (0.4, 0.1) \rangle$ is not an IFSCS in Y, since $int(cl(f(T_1^C))) = 1_\sim \not\subset T_1^C$.

Theorem 3.16: Every IFACM is an IFAWGCM but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IFACM. Let A be an IFRCS in X. By hypothesis, f(A) is an IFCS in Y. Since every IFCS is an IFWGCS, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.17: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.2), (0.6, 0.5)⟩, T_2 = ⟨ y, (0.2, 0.3), (0.6, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f: (X, τ) → (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFACM, since the IFS T_1^C = ⟨ x, (0.6, 0.5), (0.4, 0.2) ⟩ is an IFRCS in X but f (T_1^C) = ⟨ y, (0.6, 0.5), (0.4, 0.2) ⟩ is not an IFCS in Y, since cl(f (T_1^C)) = $T_2^C \not\subset T_1^C$.

Theorem 3.18: Let f: (X, τ) → (Y,σ) be an IFAWGCM and (Y,σ) an $IF_wT_{1/2}$ space. Then f is an IFACM.

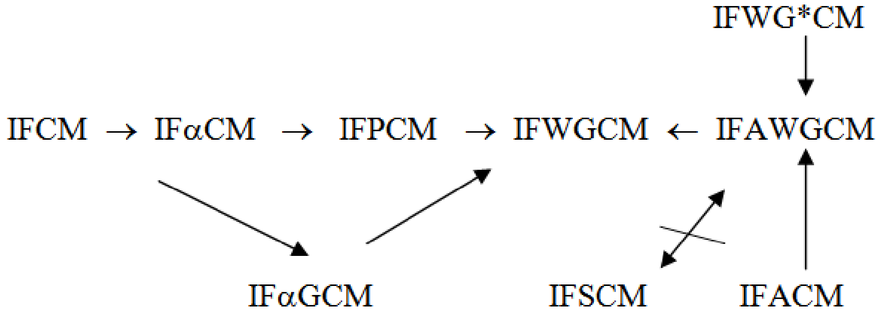
Proof: Let A be an IFRCS in X. By hypothesis, f(A) is an IFWGCS in Y. Since (Y,σ) is an $IF_wT_{1/2}$ space, f (A) is an IFCS in Y. Hence f is an IFACM.

Theorem 3.19: Every IFWG*CM is an IFAWGCM but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IFWGCM. Let A be an IFRCS in X. Since every IFRCS is an IFWGCS, A is an IFWGCS in X. By hypothesis, f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Example 3.20: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.2, 0.2), (0.4, 0.4)⟩, T_2 = ⟨ x, (0.2, 0), (0.5, 0.4)⟩, T_3 = ⟨ y, (0.5, 0.6), (0.2, 0)⟩, T_4 = ⟨ y, (0.5, 0.4), (0.5, 0.1)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, T_4, $1_\sim$} are IFTs on X and Y respectively. Consider a mapping f: (X, τ) → (Y,σ) defined as f (a) = u and f (b) = v. This f is an IFAWGCM but not an IFW*GCM, since the IFS T_2^C = ⟨ x, (0.5, 0.4), (0.2, 0) ⟩ is an IFWGCS in X but f(T_2^C) = ⟨ y, (0.5, 0.4), (0.2, 0) ⟩ is not an IFWGCS in Y, since cl(int(f^{-1}(T_2^C))) = $1_\sim \not\subset T_3$.

The relations among various types of intuitionistic fuzzy closedness are given in the following diagram.



The reverse implications are not true in general in the above diagram.

In this diagram "A $\rightarrow$ B" we mean A implies B but not conversely and "A $\nleftrightarrow$ B" means A and B are independent to each other.

Definition 3.21: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy almost weakly generalized open mapping* (IFAWGOM in short) if f(A) is an IFWGOS in Y for every IFROS A in X.

Theorem 3.22: Let f: (X, τ) → (Y,σ) be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then the following statements are equivalent.

a. f is an IFAWGCM,
b. f is an IFAWGOM.

Proof:

(a) ⇒(b): Let A be an IFROS in X. This implies A^c is an IFRCS in X. Then f(A^c) is an IFWGCS in Y, by hypothesis. Since f (A^c) = $(f(A))^c$, f (A) is an IFWGOS in Y. Hence f is an IFAWGOM.

(b) ⇒(a): Let A be an IFRCS in X. Then A^c is an IFROS in X. By hypothesis, f (A^c) = $(f(A))^c$ is an IFWGOS in Y. Therefore f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Theorem 3.23: Let f: (X, τ) → (Y,σ) be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then the following statements are equivalent.

a. f (wgint(B)) ⊆ wgint(f (B)) for every IFROS B in X,
b. f is an intuitionistic fuzzy almost weakly generalized closed mapping.

Proof:

(a) ⇒(b): Let B be an intuitionistic fuzzy regular open set in X. By hypothesis, f(wgint(B)) ⊆ wgint(f(B)). Since every IFROS is an IFWGOS, B is an IFWGOS in X. Therefore wgint(B) = B. Hence f(B) = f(wgint(B)) ⊆ wgint(f(B)) ⊆ f(B). This implies f(B) is an IFWGOS in Y. Thus f is an intuitionistic fuzzy almost weakly generalized closed mapping.

(b) ⇒(a): Let f be an intuitionistic fuzzy almost weakly generalized closed mapping. Let B be an IFROS in X. Then f(B) is an IFWGOS in Y. Therefore f(B) = f(wgint(B)) ⊆ wgint(f(B)).

Theorem 3.24: Let f: (X, τ) → (Y,σ) be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then the following statements are equivalent.

a. wgcl(f(B)) ⊆ f (wgcl(B)) for every IFRCS B in X,
b. f is an intuitionistic fuzzy almost weakly generalized closed mapping.

Proof:

(a) ⇒(b): Let B be an intuitionistic fuzzy regular closed set in X. By hypothesis, wgcl(f(B)) ⊆ f(wgcl(B)). Since every IFRCS is an IFWGCS, B is an IFWGCS in X.

Therefore wgcl(B) = B. Hence f(B) = f(wgcl(B)) $\supseteq$ wgcl(f(B)) $\supseteq$ f(B). This implies f(B) is an IFWGCS in Y. Hence f is an intuitionistic fuzzy almost weakly generalized closed mapping.
(b) $\Rightarrow$(a): Let f be an intuitionistic fuzzy almost weakly generalized closed mapping. Let B be an IFRCS in X. Then f(B) is an IFWGCS in Y. Therefore wgcl(f(B)) = f(B) $\subseteq$ f(wgcl(B)).

Theorem 3.25: Let f: (X, τ) $\to$ (Y,σ) be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then the following statements are equivalent.

a. f is an intuitionistic fuzzy almost weakly generalized closed mapping,
b. f(A) $\subseteq$ wgint(f(int(cl(A)))) for every intuitionistic fuzzy preopen set A in X,
c. wgcl(f(cl(int(A)))) $\subseteq$ f(A) for every intuitionistic fuzzy preclosed set A in X.

Proof:
(a) $\Rightarrow$(b): Let A be an IFPOS in X. Then A $\subseteq$ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A)) is an IFWGOS in Y. Therefore f(A) $\subseteq$ f(int(cl(A)) $\subseteq$ wgint(f(int(cl(A)))).
(b) $\Rightarrow$(c): It can be proved by using the complement.
(c) $\Rightarrow$(a): Let A be an IFRCS in X. Then A is an IFPCS in X. By hypothesis, f(A) $\supseteq$ wgcl(f(cl(int(A)))) = wgcl(f(A)) $\supseteq$ f(A). This implies f(A) is an IFWGCS in Y. Hence f is an IFAWGCM.

Theorem 3.26: Let f: (X, τ) $\to$ (Y,σ) be a bijective mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then the following statements are equivalent.

a. f is an IFAWGOM,
b. f is an IFAWGCM,
c. f^{-1} is an IFAWG continuous mapping.

Proof:
(a) $\Rightarrow$(b): Obvious.
(b) $\Rightarrow$(c): Let A be an IFRCS in X. By hypothesis, f(A) is an IFWGCS in Y. That is $(f^{-1})^{-1}$(A) = f(A) is an IFWGCS in Y. Hence f^{-1} is an IFAWG continuous mapping.
(c) $\Rightarrow$(a): Let A be an IFROS in X. By hypothesis, $(f^{-1})^{-1}$(A) = f(A) is an IFWGOS in Y. Hence f is an IFAWGOM.

Theorem 3.27: Let f: (X, τ) $\to$ (Y,σ) and g: (Y,σ) $\to$ (Z,δ) be any two mappings. Then the following statements hold.

a. Let f: (X, τ) $\to$ (Y,σ) be an IFACM and g: (Y,σ) $\to$ (Z,δ) an IFWGCM. Then their composition gof: (X, τ) $\to$(Z,δ) is an IFAWGCM.
b. Let f: (X, τ) $\to$ (Y,σ) be an IFAWGCM and g: (Y,σ) $\to$ (Z,δ) an IFWG*CM. Then their composition gof: (X, τ) $\to$(Z,δ) is an IFAWGCM.
c. Let f: (X, τ) $\to$ (Y,σ) be an IFAWGCM and g: (Y,σ) $\to$ (Z,δ) an IFWGCM where (Y,σ) is an $IF_wT_{1/2}$ space. Then their composition gof: (X, τ) $\to$(Z,δ) is an IFAWGCM.

Proof:

a. Let A be an IFRCS in X. Since f is an IFACM, f(A) is an IFCS in Y. By hypothesis, g(f(A)) = gof(A) is an IFWGCS in Z. Hence gof: $(Y,\sigma) \rightarrow (Z,\delta)$ is an IFAWGCM.
b. Let A be an IFRCS in X. Since f is an IFAWGCM, f(A) is an IFWGCS in Y. By hypothesis, g(f(A)) = gof(A) is an IFWGCS in Z. Hence gof: $(Y,\sigma) \rightarrow (Z,\delta)$ is an IFAWGCM.
c. Let A be an IFRCS in X. By hypothesis, f(A) is an IFWGCS in Y. Since (Y,σ) is an $IF_wT_{1/2}$ space, f(A) is an IFCS in Y. Therefore g(f(A)) = gof(A) is an IFWGCS in Z, by hypothesis. Hence gof: $(Y,\sigma) \rightarrow (Z,\delta)$ is IFAWGCM.

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