International Journal of Fuzzy Mathematics and Systems.
ISSN 2248-9940 Volume 2, Number 4 (2012), pp. 489-495

http://www.ripublication.com/ijfms.htm

Homomorphism in Intuitionistic Fuzzy Automata

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Abstract

In this paper we introduce some properties of homomorphism in Intuitionistic Fuzzy Automata.

AMS subject classification:
Keywords:

1. Introduction

The concept of intuitionistic fuzzy set was introduced by K.T. Atanassov [1], as a generalization of the notion of fuzzy set. Using the notion of intuitionistic fuzzy sets [1], it is possible to obtain intuitionistic fuzzy language [3]. J.B. Jun [2] defined a homomorphism in intuitionistic fuzzy automata. We introduce some properties of an intuitionistic fuzzy automata with homomorphism.

2. Preliminaries

2.1. Fuzzy Set [1]

Let a set 'E' be fixed. A Fuzzy set 'A' in 'E' is an object having the form $A = \{< x, \mu_A(x) > | x \in E\}$ where, the function $\mu_A(x) : E \to [0, 1]$ define the degree of membership of the element $x \in E$ to the set 'A' and for every $x \in E, 0 \leq \mu_A(x) \leq 1$.

2.2. Automata [4]

A non-deterministic finite Automaton is a triple $A = (Q, X, \delta)$ where Q is a finite set (the set of states), X is an alphabet and δ is a subset of $Q \times X \times Q$, called the set of transitions. Two transitions (p, a, q) and (p', a', q') are consecutive if $q = p'$.

Consider a word $a_0, a_1, \ldots, a_{n-1}$ with $a_i \in X$. A run α in A is sequence of states $q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2, \ldots, q_{n-1} \xrightarrow{a_{n-1}} q_n$.

2.3. Fuzzy Automata [3]

A Fuzzy Automaton is a triple $A = (Q, X, \delta)$, where Q is a nonempty set of states of A, X is a monoid (the input monoid of M), with identity e, and δ is a Fuzzy subset of $Q \times X \times Q$, i.e., a map $\delta : Q \times X \times Q \to [0, 1]$, such that $\forall q, p \in Q, \forall x, y \in X$.

$$\delta(q, e, p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

and

$$\delta(q, xy, p) = \vee\{\delta(q, x, r) \wedge \delta(r, y, p) : r \in Q\}$$

2.4. Intuitionistic Fuzzy Set [1]

Let a set 'E' be fixed. An Intuitionistic Fuzzy set 'A' in 'E' is an object having the form $A = \{< x, \mu_A(x), \gamma_A(x) > | x \in E\}$ where, the functions $\mu_A(x) : E \to [0, 1]$ and $\gamma_A(x) : E \to [0, 1]$ define the degree of membership and the degree of nonmembership of the element $x \in E$ to the set 'A', the subset of 'E' respectively, and for every $x \in E, 0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

2.5. Intuitionistic Fuzzy Automata [2]

An Intuitionistic Fuzzy Automaton is a triple $A = (Q, X, \delta)$, where Q is a set of states of A, X is a monoid (the input monoid of M with identity e), and δ is an Intuitionistic Fuzzy subset of $Q \times X \times Q$, such that $\forall q, p \in Q, \forall x, y \in X$.

$$\delta_1(q, e, p) = \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

$$\delta_2(q, e, p) = \begin{cases} 1 & \text{if } q \neq p \\ 0 & \text{if } q = p \end{cases}$$

$$\delta_1(q, xy, p) = \vee\{\delta_1(q, x, r) \wedge \delta_1(r, y, p) : r \in Q\},$$

and

$$\delta_2(q, xy, p) = \wedge\{\delta_2(q, x, r) \vee \delta_2(r, y, p) : r \in Q\}.$$

2.6. Homomorphism between Automata

Let $A_1 = (Q_1, X_1, \delta_1)$ and $A_2 = (Q_2, X_2, \delta_2)$ be two finite automata. A pair (α, β) of mappings, $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ is called a homomorphism, written $(\alpha, \beta) : A_1 \to A_2$, if

$$\alpha(\delta_1(q_1, a)) = \delta_2(\alpha(q_1), \beta(a)) \forall q_1 \in Q_1 \quad \text{and} \quad \forall a \in X_1.$$

2.7. Homomorphism between Fuzzy Automata [3]

Let $A_1 = (Q_1, X_1, \mu_1)$ and $A_2 = (Q_2, X_2, \mu_2)$ be ffsms. A pair (α, β) of mappings, $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ is called a homomorphism, written $(\alpha, \beta) : A_1 \to A_2$, if

$$\mu_1(q, x, p) \leq \mu_2(\alpha(q), \beta(x), \alpha(p)) \forall q, p \in Q_1$$

and

$$\forall x \in X_1.$$

The pair (α, β) is called a strong homomorphism if

$$\mu_2(\alpha(q), \beta(x), \alpha(p)) = \vee\{\mu_1(q, x, t) | t \in Q_1, \alpha(t) = \alpha(p)\} \forall q, p \in Q_1$$

and $\forall x \in X_1$.

2.8. Homomorphism between Intuitionistic Fuzzy Automata [2]

Let $A_1 = (Q_1, X_1, \mu_1, \gamma_1)$ and $A_2 = (Q_2, X_2, \mu_2, \gamma_2)$ be iffsms. A pair (α, β) of mappings, $\alpha : Q_1 \to Q_2$ and $\beta : X_1 \to X_2$ is called an intuitionistic fuzzy homomorphism, written $(\alpha, \beta) : A_1 \to A_2$, if

$$\mu_1(q, x, p) \leq \mu_2(\alpha(q), \beta(x), \alpha(p))$$

$$\gamma_1(q, x, p) \geq \gamma_2(\alpha(q), \beta(x), \alpha(p)) \forall p, q \in Q_1$$

and $\forall x \in X_1$.

The pair (α, β) is called a strong intuitionistic fuzzy homomorphism if

$$\mu_2(\alpha(q), \beta(x), \alpha(p)) = \vee\{\mu_1(q, x, t) | t \in Q, \alpha(t) = \alpha(p)\}$$

$$\gamma_2(\alpha(q), \beta(x), \alpha(p)) = \wedge\{\gamma_1(q, x, t) | t \in Q, \alpha(t) = \alpha(p)\} \forall q, p \in Q_1$$

and $\forall x \in X_1$.

3. Some properties of Homomorphism in Inutitionistic Fuzzy Automata

Lemma 3.1. Let $A_1 = (Q_1, X_1, \mu_1, \gamma_1)$ and $A_2 = (Q_2, X_2, \mu_2, \gamma_2)$ be two iffsms. Let $(\alpha, \beta) : A_1 \to A_2$ be a strong intuitionistic homomorphism. Then $\forall q, r, \in Q_1, \forall x \in X_1$, if

$$\mu_2(\alpha(q), \beta(x), \alpha(r)) > 0$$

$$\gamma_2(\alpha(q), \beta(x), \alpha(r)) < 1,$$

then $\exists t \in Q_1$ such that

$$\mu_1(q, x, t) > 0, \gamma_1(q, x, t) < 1$$

and $\alpha(t) = \alpha(r)$. Furthermore, $\forall p \in Q$ if $\alpha(p) = \alpha(q)$, then

$$\mu_1(q, x, t) \geq \mu_1(p, x, r)$$

and

$$\gamma_1(q, x, t) \leq \gamma_1(p, x, r).$$

Proof. Let $p, q, r \in Q_1, x \in X_1$, and

$$\mu_2(\alpha(q), \beta(x), \alpha(r)) > 0$$

$$\gamma_2(\alpha(q), \beta(x), \alpha(r)) < 1.$$

Then

$$\vee\{\mu_1(q, x, s) | s \in Q_1, \alpha(s) = \alpha(r)\} > 0$$
$$\wedge\{\gamma_1(q, x, s) | s \in Q_1, \alpha(s) = \alpha(r)\} < 1.$$

Since Q_1 is finite, $\exists t \in Q_1$ such that $\alpha(t) = \alpha(r)$ and

$$\mu_1(q, x, t) = \vee\{\mu_1(q, x, s) | s \in Q_1, \alpha(s) = \alpha(r)\} > 0$$

$$\gamma_1(q, x, t) = \wedge\{\gamma_1(q, x, s) | s \in Q_1, \alpha(s) = \alpha(r)\} < 1$$

suppose $\alpha(p) = \alpha(q)$. Then

$$\begin{aligned}
\mu_1(q, x, t) &= \mu_2(\alpha(q), \beta(x), \alpha(r)) \\
&= \mu_2(\alpha(p), \beta(x), \alpha(r)) \\
&\geq \mu_1(p, x, r) \\
\gamma_1(q, x, t) &= \gamma_2(\alpha(q), \beta(x), \alpha(r)) \\
&= \gamma_2(\alpha(p), \beta(x), \alpha(r)) \\
&\leq \gamma_1(p, x, r)
\end{aligned}$$

■

Definition 3.2. Let $A_1 = (Q_1, X_1, \mu_1, \gamma_1)$ and $A_2 = (Q_2, X_2, \mu_2, \gamma_2)$ be two iffsms. Let $(\alpha, \beta) : A_1 \to A_2$ be an Intuitionistic fuzzy homomorphism. Define $\beta^* : X_1^* \to X_2^*$ by $\beta^*(\lambda) = \lambda$ and $\beta^*(ua) = \beta^*(u)\beta(a) \forall u \in X_1^*, a \in X_1$.

Theorem 3.3. Let $A_1 = (Q_1, X_1, \mu_1, \gamma_1)$ and $A_2 = (Q_2, X_2, \mu_2, \gamma_2)$ be two iffsms. Let $(\alpha, \beta) : A_1 \to A_2$ be an Intuitionistic fuzzy homomorphism. Then

$$\mu_1^*(q, x, p) \leq \mu_2^*(\alpha(q), \beta^*(x), \alpha(p))$$
$$\gamma_1^*(q, x, p) \geq \mu_2^*(\alpha(q), \beta^*(x), \alpha(p))$$

$\forall q, p \in Q_1$ and $x \in X_1^*$.

Proof. Let $q, p \in Q_1$ and $x \in X_1^*$. We prove the result by induction on $|x| = n$. If $n = 0$, then $x = \lambda$ and $\beta^*(x) = \beta^*(\lambda) = \lambda$. Now if $q = p$, then

$$\mu_1^*(q, \lambda, p) = 1 = \mu_2^*(\alpha(q), \lambda, \alpha(p))$$
$$\gamma_1^*(q, \lambda, p) = 0 = \gamma_2^*(\alpha(q), \lambda, \alpha(p))$$

If $q \neq p$, then

$$\mu_1^*(q, \lambda, p) = 0 \leq \mu_2^*(\alpha(q), \lambda, \alpha(p))$$
$$\gamma_1^*(q, \lambda, p) = 1 \geq \gamma_2^*(\alpha(q), \lambda, \alpha(p)).$$

Suppose now the result is true $\forall y \in X^*$ such that $|y| = n - 1, n > 0$. Let $x = ya$ and $y \in X_1^*, a \in X_1$ and $|y| = n - 1$. Now

$$\begin{aligned}
\mu_1^*(q, x, p) &= \mu_1^*(q, ya, p) \\
&= \vee\{\mu_1^*(q, y, r) \wedge \mu_1^*(r, a, p) | r \in Q_1\} \\
&\leq \vee\{\mu_2^*(\alpha(q), \beta^*(y), \alpha(r)) \\
&\quad \wedge \mu_2^*(\alpha(r), \beta(a), \alpha(p)) | r \in Q_1\} \\
&\leq \vee\{\mu_2^*(\alpha(q), \beta^*(y), r') \\
&\quad \wedge \mu_2^*(r', \beta^*(a), \alpha(p)) | r' \in Q_2\} \\
&= \mu_2^*(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) \\
&= \mu_2^*(\alpha(q), \beta^*(ya), \alpha(p)) \\
&= \mu_2^*(\alpha(q), \beta^*(x), \alpha(p)).
\end{aligned}$$

and

$$\begin{aligned}
\gamma_1^*(q, x, p) &= \mu_1^*(q, ya, p) \\
&= \wedge\{\gamma_1^*(q, y, r) \vee \mu_1^*(r, a, p) | r \in Q_1\} \\
&\geq \wedge\{\gamma_2^*(\alpha(q), \beta^*(y), \alpha(r)) \\
&\quad \vee \gamma_2^*(\alpha(r), \beta(a), \alpha(p)) | r \in Q_1\} \\
&\geq \wedge\{\gamma_2^*(\alpha(q), \beta^*(y), r') \\
&\quad \vee \gamma_2^*(r', \beta^*(a), \alpha(p)) | r' \in Q_2\} \\
&= \gamma_2^*(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) \\
&= \gamma_2^*(\alpha(q), \beta^*(ya), \alpha(p)) \\
&= \gamma_2^*(\alpha(q), \beta^*(x), \alpha(p)).
\end{aligned}$$

■

Theorem 3.4. Let $A_1 = (Q_1, X_1, \mu_1, \gamma_1)$ and $A_2 = (Q_2, X_2, \mu_2, \gamma_2)$ be two iffsms. Let $(\alpha, \beta) : A_1 \to A_2$ be an strong homomorphism. Then α is one-one if and only if

$$\mu_1^*(q, x, p) = \mu_2^*(\alpha(q), \beta^*(x), \alpha(p))$$
$$\gamma_1^*(q, x, p) \neq \gamma_2^*(\alpha(q), \beta^*(x), \alpha(p)$$

$\forall q, p \in Q_1$ and $x \in X_1^*$.

Proof. Suppose α is one-one. Let $p, q \in Q_1$ and $x \in X_1^*$. Let $|x| = n$. We prove the result by induction on n. Let $n = 0$. Then $x = \lambda$ and $\beta^*(\lambda) = \lambda$. Now $\alpha(q) = \alpha(p)$ iff $q = p$. Hence

$$\begin{aligned}\mu_1^*(q, \lambda, p) = 1, \gamma_1^*(q, \lambda, p) = 0 \quad \text{iff}\\ \mu_2^*(\alpha(q), \beta^*(\lambda), \alpha(p)) = 1,\\ \gamma_2^*(\alpha(q), \beta^*(\lambda), \alpha(p)) = 0.\end{aligned}$$

Suppose the result is true $\forall y \in X_1^*, |y| = n - 1, n > 0$. Let $x = ya, |y| = n - 1, y \in X_1^*, a \in X_1$. Then

$$\begin{aligned}\mu_2^*(\alpha(q), \beta^*(x), \alpha(p)) &= \mu_2^*(\alpha(q), \beta(ya), \alpha(p))\\ &= \mu_2^*(\alpha(q), \beta^*(y)\beta(a), \alpha(p)\\ &= \vee\{\mu_2^*(\alpha(q), \beta^*(y), \alpha(r))\\ &\quad \wedge \mu_2^*(\alpha(r), \beta(a), \alpha(p))|r \in Q_1\}\\ &= \vee\{\mu_1^*((q, y, r) \wedge \mu_1(r, a, p)|r \in Q_1\}\\ &= \mu_1^*(q, ya, p)\\ &= \mu_1^*(q, x, p).\end{aligned}$$

and

$$\begin{aligned}\gamma_2^*(\alpha(q), \beta^*(x), \alpha(p)) &= \gamma_2^*(\alpha(q), \beta(ya), \alpha(p))\\ &= \gamma_2^*(\alpha(q), \beta^*(y)\beta(a), \alpha(p))\\ &= \wedge\{\gamma_2^*(\alpha(q), \beta^*(y), \alpha(r))\\ &\quad \vee \gamma_2^*(\alpha(r), \beta(a), \alpha(p))|r \in Q_1\}\\ &= \wedge\{\gamma_1^*((q, y, r) \wedge \gamma_1(r, a, p)|r \in Q_1\}\\ &= \gamma_1^*(q, ya, p)\\ &= \gamma_1^*(q, x, p).\end{aligned}$$

Conversely, let $q, p \in Q_1$ and let $\alpha(q) = \alpha(p)$. Then

$$\begin{aligned}1 = \mu_2^*(\alpha(q), \lambda, \alpha(p)) = \mu_1^*(q, \lambda, p)\\ 0 = \gamma_2^*(\alpha(q), \lambda, \alpha(p)) = \gamma_1^*(q, \lambda, p).\end{aligned}$$

Hence $q = p$, i.e., α is one-one. ∎

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