

## Stronger Forms of Fuzzy Generalized Preregular Continuous Functions in Fuzzy Topological Spaces

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### Abstract

In this paper, the stronger forms of fuzzy generalized preregular continuous functions namely, strongly fuzzy *fgpr*-continuous, perfectly *fgpr*-continuous and completely fuzzy *fgpr*-continuous functions have been introduced and studied in fuzzy topological spaces.

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**Keywords:** Strongly *fgpr*-continuous function, perfectly *fgpr*-continuous function and completely *fgpr*-continuous function.

### 1. Introduction

The concept of a fuzzy subset which was introduced and studied by L. A. Zadeh [26] in the year 1965. The subsequent research activities in this area and the related areas have found applications in many branches of science and engineering. C. L. Chang [5] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological

spaces. Many researchers like K.K> Azad [1, 2], M. Ferraro and D. H. Foster [6], A. Haydar Es [7], R. Lowen [8], S. R. Malghan and S. S. Benchalli [9, 10], A. S. Mashhour et. al [11, 12, 13, 14], M. N. Mukherjee and B. Ghosh [15], M. N. Mukherjee and S. P. Sinha [16], P. Sundaram [17, 18], R. H. Warren [21, 22], C. K. Wong [23, 24, 25], and many others have contributed to the development of fuzzy topological spaces.

In this paper, it is proved that every strongly *fgpr*-continuous function is *f*-continuous function and also every *f*-strongly continuous function is a strongly *fgpr*-continuous function. Also every perfectly *fgpr*-continuous function is *f*-continuous function and also every perfectly *fgpr*-continuous function is a *f*-perfectly continuous function. Further every completely *fgpr*-continuous function is *f*-continuous function and also every completely *fgpr*-continuous function is a *f*-completely continuous function. Also the image of regular fts is regular under *f*-continuous, *f*-open, *fgpr*-closed maps.

Let  $X, Y$  and  $Z$  be sets. Throughout the present chapter  $(X, \alpha)$ ,  $(Y, \beta)$  and  $(Z, \gamma)$  (or simply  $X, Y$  and  $Z$ ) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitly stated.

Before entering into our work we recall the following definitions, which are due to various authors.

## 2. Preliminaries

If  $A$  is a subset of  $X$  with a topology  $\tau$ , then the closure of  $A$  is denoted by  $\tau-cl(A)$  or  $cl(A)$ , the interior of  $A$  is denoted by  $\tau-int(A)$  or  $int(A)$  and the complement of  $A$  in  $X$  is denoted by  $A^c$ .

**Definition 2.1.** A fuzzy set  $\lambda$  in a fts  $(X, \tau)$  is called

- (i) a fuzzy pre-open set [4] if  $\lambda \leq cl(int(\lambda))$  and a fuzzy pre-closed set if  $cl(int(\lambda)) \leq \lambda$ ,
- (ii) fuzzy regular open set [1] if  $int(cl\lambda) = \lambda$  and a fuzzy regular closed set if  $cl(int(\lambda)) = \lambda$ .

**Definition 2.2.** A fuzzy set  $\lambda$  in a fts  $(X, \tau)$  is called

- (i) a fuzzy generalized closed set (briefly, *fg*-closed set)[4] if  $cl(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy open in  $X$ ,
- (ii) a fuzzy generalized preregular closed set [19] (briefly *fgpr*-closed fuzzy set) if  $pcl(\lambda) \subseteq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy regular open set in  $X$ .

The complement of the above mentioned closed sets are respective open sets.

**Definition 2.3.** Let  $X, Y$  be two fuzzy topological spaces. A function  $f : X \rightarrow Y$  is called

- (i) fuzzy continuous (briefly, *f*-continuous) [3] if  $f^{-1}(\lambda)$  is fuzzy open set in  $X$ , for every fuzzy open set  $\lambda$  of  $Y$ ,

- (ii) fuzzy strongly continuous (briefly, *fs*-continuous) [3] if  $f^{-1}(\lambda)$  is fuzzy open and fuzzy closed set in  $X$ , for every fuzzy set  $\lambda$  in  $Y$ ,
- (iii) fuzzy perfectly continuous (briefly, *fp*-continuous) [3] if  $f^{-1}(\lambda)$  is fuzzy open and fuzzy closed set in  $X$ , for every fuzzy open set  $\lambda$  in  $Y$ ,
- (iv) fuzzy completely continuous (briefly, *fc*-continuous) [15] if  $f^{-1}(\lambda)$  is fuzzy regular open set in  $X$ , for every fuzzy open set  $\lambda$  in  $Y$ .
- (v) fuzzy generalized preregular continuous (briefly, *fgpr*-continuous) [20] if  $f^{-1}(\lambda)$  is *fgpr*-open set in  $X$ , for every fuzzy open set  $\lambda$  in  $Y$ .
- (vi) fuzzy generalized preregular irresolute (briefly, *fgpr*-irresolute) [20] if  $f^{-1}(\lambda)$  is *fgpr*-open set in  $X$ , for every *fgpr*-open set  $\lambda$  in  $Y$ .

### 3. Stronger forms of fuzzy generalized preregular continuous functions

Now, the stronger forms of *fgpr*-continuous functions namely strongly fuzzy *fgpr*-continuous, perfectly *fgpr*-continuous and completely *fgpr*-continuous functions have been introduced and studied.

Now we introduce the following

**Definition 3.1.** A function  $f : X \rightarrow Y$  is called strongly fuzzy generalized preregular continuous (briefly strongly *fgpr*-continuous) iff the inverse image of every *fgpr*-open set in  $Y$  is fuzzy open set in  $X$ .

**Theorem 3.2.** A function  $f : X \rightarrow Y$  is strongly *fgpr*-continuous iff the inverse image of every *fgpr*-closed set in  $Y$  is fuzzy closed set in  $X$ .

*Proof.* Assume that  $f$  is strongly *fgpr*-continuous. Let  $\lambda$  be *fgpr*-closed set in  $Y$ . Then  $1 - \lambda$  is *fgpr*-open set in  $Y$ . Since  $f$  is strongly *fgpr*-continuous,  $f^{-1}(1 - \lambda)$  is fuzzy open set in  $X$ . But  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$  and so  $f^{-1}(\lambda)$  is fuzzy closed set in  $X$ .

Conversely, suppose that the inverse image of every *fgpr*-closed set in  $Y$  is fuzzy closed set in  $X$ . Let  $\mu$  be *fgpr*-open set in  $Y$ , then  $1 - \mu$  is *fgpr*-closed set in  $Y$ . By hypothesis,  $f^{-1}(1 - \mu)$  is fuzzy closed set in  $X$ . Now  $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$  and so  $f^{-1}(\mu)$  is fuzzy open set in  $X$ . Hence  $f$  is strongly *fgpr*-continuous. ■

**Theorem 3.3.** Every strongly *fgpr*-continuous function is a *f*-continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be strongly *fgpr*-continuous function. Let  $\mu$  be fuzzy open set in  $Y$ , and so  $\mu$  is *fgpr*-open set in  $Y$ . Then  $f^{-1}(\mu)$  is fuzzy open set in  $X$ . Hence  $f$  is *f*-continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.4.** Let  $X = Y = \{a, b, c\}$  and the fuzzy sets  $\lambda, \mu$  and  $\gamma$  be defined as follows.  $\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c}$ ,  $\mu = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c}$ ,  $\gamma = \frac{1}{a} + \frac{0.8}{b} + \frac{0.8}{c}$ . Consider  $\tau = \{0, 1, \lambda, \mu\}$  and  $\sigma = \{0, 1, \mu\}$ . Then  $(X, \tau)$  and  $(Y, \sigma)$  are fts. Define  $f : X \rightarrow Y$  by  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is  $f$ -continuous but not strongly  $fgpr$ -continuous as the fuzzy set  $\gamma$  is  $fgpr$ -closed in  $Y$  and  $f^{-1}(\gamma) = \gamma$  is not fuzzy closed set in  $X$ .

**Theorem 3.5.** Every  $f$ -strongly continuous function is a strongly  $fgpr$ -continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be  $f$ -strongly continuous function. Let  $\lambda$  be  $fgpr$ -open set in  $Y$ . And then  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$  as  $f$  is  $f$ -strongly continuous function. Hence  $f$  is strongly  $fgpr$ -continuous function.

The converse of the above theorem need not be true as seen from the following example. ■

**Example 3.6.** Let  $X = Y = \{a, b, c\}$  and the fuzzy sets  $\lambda$  and  $\mu$  be defined as follows.  $\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c}$ ,  $\mu = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c}$ . Consider  $\tau = \{0, 1, \lambda\}$  and  $\sigma = \{0, 1, \mu\}$ . Then  $(X, \tau)$  and  $(Y, \sigma)$  are fts. Define  $f : X \rightarrow Y$  by  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is strongly  $fgpr$ -continuous but not strongly  $f$ -continuous as the fuzzy set  $\mu$  in  $Y$  is such that  $f^{-1}(\mu) = \mu$  is fuzzy open but not fuzzy closed set in  $X$ .

**Theorem 3.7.** If  $f : X \rightarrow Y$  be strongly  $fgpr$ -continuous and  $g : Y \rightarrow Z$  is strongly  $fgpr$  continuous. Then the composition map  $g \circ f : X \rightarrow Z$  is strongly  $fgpr$  continuous function.

*Proof.* Let  $\mu$  be  $fgpr$ -open set in  $Z$ . Then  $g^{-1}(\mu)$  is fuzzy open set in  $Y$ , since  $g$  is strongly  $fgpr$ -continuous. And therefore  $g^{-1}(\mu)$  is  $fgpr$ -open set in  $Y$ . Also since  $f$  is strongly  $fgpr$ -continuous,  $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$  is fuzzy open set in  $X$ . Hence  $g \circ f$  is strongly  $fgpr$ -continuous function. ■

**Theorem 3.8.** Let  $f : X \rightarrow Y$ ,  $g : Y \rightarrow Z$  be maps such that  $f$  is strongly  $fgpr$ -continuous and  $g$  is  $fgpr$ -continuous then  $g \circ f : X \rightarrow Z$  is  $f$ -continuous.

*Proof.* Let  $\lambda$  be a fuzzy closed set in  $Z$ . Then  $g^{-1}(\lambda)$  is  $fgpr$ -closed set in  $Y$ . Since  $g$  is  $fgpr$ -continuous. And since  $f$  is strongly  $fgpr$ -continuous,  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is fuzzy closed set in  $X$ . Hence  $g \circ f$  is  $f$  continuous. ■

**Theorem 3.9.** If  $f : X \rightarrow Y$  be strongly  $fgpr$ -continuous and  $g : Y \rightarrow Z$  is  $fgpr$ -irresolute, then the composition map  $g \circ f : X \rightarrow Z$  is strongly  $fgpr$ -continuous.

*Proof.* Let  $\lambda$  be  $fgpr$ -open set in  $Z$ . Then  $g^{-1}(\lambda)$  is  $fgpr$ -open set in  $Y$  since  $g$  is

$fgpr$ -irresolute. And then  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is fuzzy open set in  $X$ . Also since  $f$  is strongly  $fgpr$ -continuous. Hence  $g \circ f$  is strongly  $fgpr$ -continuous. ■

**Definition 3.10.** A functions  $f : X \rightarrow Y$  is called perfectly  $fgpr$ -continuous (briefly perfectly  $fgpr$ -continuous) if the inverse image of every  $fgpr$ -open set in  $Y$  is both fuzzy open and fuzzy closed set in  $X$ .

**Theorem 3.11.** A map  $f : X \rightarrow Y$  is perfectly  $fgpr$ -continuous iff the inverse image of every  $fgpr$ -closed set in  $Y$  is both fuzzy open and fuzzy closed set in  $X$ .

*Proof.* Assume that  $f$  is perfectly  $fgpr$ -continuous. Let  $\lambda$  be  $fgpr$ -closed set in  $Y$ . Then  $1 - \lambda$  is  $fgpr$ -open set. And therefore  $f^{-1}(1 - \lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . But  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$  and so  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ .

Conversely, suppose that the inverse image of every  $fgpr$ -closed set in  $Y$  is both fuzzy open and fuzzy closed set in  $X$ . Let  $\lambda$  be  $fgpr$ -open set in  $Y$ . Then  $1 - \lambda$  is  $fgpr$ -closed set in  $Y$ . By hypothesis,  $f^{-1}(1 - \lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . But  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ . Therefore  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . Hence  $f$  is perfectly  $fgpr$ -continuous. ■

**Theorem 3.12.** Every perfectly  $fgpr$ -continuous function is a  $f$ -continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be perfectly  $fgpr$ -continuous. Let  $\lambda$  be a fuzzy open set in  $Y$ , and so  $\lambda$  is  $fgpr$ -open set in  $Y$ . Since  $f$  is perfectly  $fgpr$ -continuous, then  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . That is  $f^{-1}(\lambda)$  is fuzzy open set in  $X$ . Hence  $f$  is  $f$ -continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.13.** In the Example 3.4, the function  $f$  is  $f$ -continuous but not perfectly  $fgpr$ -continuous as the fuzzy set  $1 - \gamma = \frac{0}{a} + \frac{0.2}{b} + \frac{0.2}{c}$  is  $fgpr$ -open set in  $Y$  and  $f^{-1}(1 - \gamma) = 1 - \gamma$  which is not both fuzzy open and fuzzy closed set in  $X$ .

**Theorem 3.14.** Every perfectly  $fgpr$ -continuous function is a  $f$ -perfectly continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be perfectly  $fgpr$ -continuous. Let  $\lambda$  be fuzzy open set in  $Y$ , then  $\lambda$  be  $fgpr$ -open set in  $Y$ . Since  $f$  is perfectly  $fgpr$ -continuous. Then  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . And hence  $f$  is  $f$ -perfectly continuous function. ■

The converse of the above theorem need not true as seen from the following example.

**Example 3.15.** Let  $X = Y = \{a, b, c\}$  and the fuzzy sets  $\lambda, \mu, \gamma$  and  $\delta$  be defined

as follows.  $\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c}$ ,  $\mu = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c}$ ,  $\gamma = \frac{0}{a} + \frac{0.1}{b} + \frac{0.2}{c}$  and  $\delta = \frac{1}{a} + \frac{0.8}{b} + \frac{0.8}{c}$ . Consider  $\tau = \{0, 1, \lambda, \mu, \gamma\}$  and  $\sigma = \{0, 1, \mu\}$ . Then  $(X, \tau)$  and  $(Y, \sigma)$  are fts. Define  $f : X \rightarrow Y$  by  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is perfectly  $fgpr$ -continuous function. As the fuzzy set  $\mu$  is fuzzy open set in  $Y$  and its inverse image  $f^{-1}(\mu) = \mu$  is both fuzzy open and fuzzy closed set in  $X$ . But  $f$  is not perfectly  $fgpr$ -continuous. As the fuzzy set  $\delta$  is  $fgpr$ -open set in  $Y$  and  $f^{-1}(\delta) = \delta$  is not both fuzzy open and fuzzy closed set in  $X$ .

**Theorem 3.16.** Every perfectly  $fgpr$ -continuous function is strongly  $fgpr$ -continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be perfectly  $fgpr$ -continuous. Let  $\lambda$  be  $fgpr$ -open set in  $Y$ . Then  $f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . Therefore  $f^{-1}(\lambda)$  is fuzzy open set in  $X$ . Hence  $f$  is strongly  $fgpr$ -continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.17.** In the Example 3.6, the function  $f$  is strongly  $fgpr$ -continuous but not perfectly  $fgpr$ -continuous as the fuzzy set  $\mu$  is  $fgpr$ -open set in  $Y$  and  $f^{-1}(\mu) = \mu$  is fuzzy open but not fuzzy closed set in  $X$ .

**Theorem 3.18.** Let  $f : X \rightarrow Y$ ,  $g : Y \rightarrow Z$  be two perfectly  $fgpr$ -continuous function then  $g \circ f : X \rightarrow Z$  is perfectly  $fgpr$ -continuous function.

*Proof.* Let  $\lambda$  be  $fgpr$ -open fuzzy in  $Z$ . Then  $g^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $Y$ , since  $g$  is perfectly  $fgpr$ -continuous. Therefore  $g^{-1}(\lambda)$  is  $fgpr$ -open set in  $Y$ . Also since  $f$  is perfectly  $fgpr$ -continuous,  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . Hence  $g \circ f$  is perfectly  $fgpr$ -continuous function. ■

**Theorem 3.19.** Let  $f : X \rightarrow Y$  be perfectly  $fgpr$ -continuous and  $g : Y \rightarrow Z$  be  $fgpr$ -irresolute functions then  $g \circ f : X \rightarrow Z$  is perfectly  $fgpr$ -continuous functions.

*Proof.* Let  $\lambda$  be  $fgpr$  open set in  $Z$ . Then  $g^{-1}(\lambda)$  is  $fgpr$ -open set in  $Y$ , since  $g$  is  $fgpr$ -irresolute functions. Also since  $f$  is perfectly  $fgpr$ -continuous,  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is both fuzzy open and fuzzy closed set in  $X$ . Hence  $g \circ f$  is perfectly  $fgpr$ -continuous functions. ■

**Definition 3.20.** A map  $f : X \rightarrow Y$  is said to be completely fuzzy generalized preregular continuous (briefly completely  $fgpr$ -continuous) if the inverse image of every  $fgpr$ -open set in  $Y$  is fuzzy regular open set in  $X$ .

**Theorem 3.21.** A map  $f : X \rightarrow Y$  is completely  $fgpr$ -continuous iff the inverse image of every  $fgpr$ -closed set in  $Y$  is fuzzy regular closed set in  $X$ .

*Proof.* Suppose  $f$  is completely  $fgpr$ -continuous. Let  $\lambda$  be  $fgpr$ -closed set in  $Y$ . Then  $1 - \lambda$  is  $fgpr$ -open set in  $Y$ . Therefore  $f^{-1}(1 - \lambda)$  is fuzzy regular open set in  $X$ . Now  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ . Therefore  $f^{-1}(\lambda)$  is fuzzy regular closed set in  $X$ .

Conversely, assume that the inverse image of every  $fgpr$ -closed set in  $Y$  is fuzzy regular closed set in  $X$ . Let  $\lambda$  be  $fgpr$ -open set in  $Y$ . Then  $1 - \lambda$  is  $fgpr$ -closed set in  $Y$ . By hypothesis,  $f^{-1}(1 - \lambda)$  is fuzzy regular closed set in  $X$ . Now  $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ . And therefore  $f^{-1}(\lambda)$  is fuzzy regular open set in  $X$ . Hence  $f$  is completely  $fgpr$ -continuous function. ■

**Theorem 3.22.** Every completely  $fgpr$ -continuous function is a  $f$ -continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be a completely  $fgpr$ -continuous function. Let  $\lambda$  be fuzzy open set in  $Y$ . Then  $\lambda$  is  $fgpr$ -open set in  $Y$ . And then  $f^{-1}(\lambda)$  is both fuzzy regular open set in  $X$ , and therefore  $f^{-1}(\lambda)$  is fuzzy open set in  $X$ . Hence  $f$  is  $f$ -continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.23.** In the Example 3.4, the function  $f$  is  $f$ -continuous but not completely  $fgpr$ -continuous as the fuzzy set  $1 - \gamma = \frac{0}{a} + \frac{0.2}{b} + \frac{0.2}{c}$  is  $fgpr$ -open set in  $Y$  and  $f^{-1}(1 - \gamma) = 1 - \gamma$  which is not both fuzzy regular open set in  $X$ .

**Theorem 3.24.** Every completely  $fgpr$ -continuous function is a  $f$ -completely continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be completely  $fgpr$ -continuous. Let  $\lambda$  be fuzzy open set in  $Y$ . Then  $\lambda$  be  $fgpr$ -open set in  $Y$ . Then  $f^{-1}(\lambda)$  is fuzzy regular open set in  $X$ . Hence  $f$  is  $f$ -completely continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.25.** In the Example 3.15, the function  $f$  is completely  $f$ -continuous function, as the fuzzy set  $\mu$  is fuzzy open set in  $Y$  and its inverse image  $f^{-1}(\mu) = \mu$  is fuzzy regular open set in  $X$ . But  $f$  is not completely  $fgpr$ -continuous as the fuzzy set  $1 - \gamma = \frac{0}{a} + \frac{0.2}{b} + \frac{0.2}{c}$  is  $fgpr$ -open set in  $Y$  and  $f^{-1}(1 - \gamma) = 1 - \gamma$  which is not fuzzy regular open set in  $X$ .

**Theorem 3.26.** Every completely  $fgpr$ -continuous function is strongly  $fgpr$ -continuous function.

*Proof.* Let  $f : X \rightarrow Y$  be completely  $fgpr$ -continuous. Let  $\lambda$  be  $fgpr$ -open set in  $Y$ . Then  $f^{-1}(\lambda)$  is fuzzy regular open set in  $X$ . Therefore  $f^{-1}(\lambda)$  is fuzzy open set in  $X$ . Hence  $f$  is strongly  $fgpr$ -continuous function. ■

The converse of the above theorem need not be true as seen from the following example.

**Example 3.27.** In the Example 3.6, the function  $f$  is strongly  $fgpr$ -continuous but not completely  $fgpr$ -continuous as the fuzzy set  $\mu^c$  is  $fgpr$ -closed set in  $Y$  and its inverse image  $f^{-1}(\mu) = \mu$  is not fuzzy regular closed set in  $X$ .

**Theorem 3.28.** If  $f : X \rightarrow Y$  is completely  $fgpr$ -continuous and  $g : Y \rightarrow Z$  is  $fgpr$ -irresolute functions then  $g \circ f : X \rightarrow Z$  is completely  $fgpr$ -continuous functions.

*Proof.* Let  $\lambda$  be  $fgpr$ -open set in  $Z$ . Then  $g^{-1}(\lambda)$  is  $fgpr$ -open set in  $Y$ , since  $g$  is  $fgpr$ -irresolute function. Also since  $f$  is completely  $fgpr$ -continuous.  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is fuzzy regular open set in  $X$ . Hence  $g \circ f$  is completely  $fgpr$ -continuous functions. ■

**Theorem 3.29.** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two completely  $fgpr$ -continuous functions then  $g \circ f : X \rightarrow Z$  is completely  $fgpr$ -continuous functions.

*Proof.* Let  $\lambda$  be  $fgpr$ -open set in  $Z$ . Then  $g^{-1}(\lambda)$  is fuzzy regular open set in  $Y$ . Since  $g$  is completely  $fgpr$ -continuous,  $g^{-1}(\lambda)$  is fuzzy open set and then  $fgpr$ -open set in  $Y$ . Also since  $f$  is completely  $fgpr$ -continuous function,  $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$  is fuzzy regular open set in  $X$ . Hence  $g \circ f$  is completely  $fgpr$ -continuous functions. ■

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