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Homomorphism IN (Q, L) -Fuzzy Normal Ideals of a Ring

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ABSTRACT

In this paper, we study some of the properties of (Q, L) -fuzzy normal ideal of a ring and prove some results on these.

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KEY WORDS: (Q, L) -fuzzy subset, (Q, L) -fuzzy ideal, (Q, L) -fuzzy normal ideal, Q-level subset.

INTRODUCTION

After the introdution of fuzzy sets by L.A.Zadeh [12], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld [3] defined a fuzzy groups. Asok Kumer Ray [2] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan [10, 11] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of (Q, L) -fuzzy normal ideal of a ring and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, $\leq$) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L) -fuzzy subset** A of X is a function A : $X\times Q \rightarrow L$.

1.2 Definition: Let (R, +, $\cdot$) be a ring and Q be a non empty set. A (Q, L) -fuzzy subset A of R is said to be a **(Q, L) -fuzzy ideal (QLFI)** of R if the following conditions are satisfied:

1. A (x+y, q) $\geq$ A (x, q) $\wedge$ A (y, q),
2. A ($-$x, q) $\geq$ A (x, q),

3. A (xy, q) $\geq$ A (x, q) $\vee$ A (y, q), for all x and y in R and q in Q.

1.3 Definition: Let A and B be any two (Q, L) -fuzzy subsets of sets R and H, respectively. The product of A and B, denoted by A×B, is defined as A×B ={ ⟨ ((x, y), q), A×B ((x, y), q) ⟩ / for all x in R and y in H and q in Q }, where A×B ((x, y), q) = A (x, q) $\wedge$ B (y, q).

1.4 Definition: Let A be a (Q, L) -fuzzy subset in a set S, the **strongest (Q, L) -fuzzy relation** on S, that is a (Q, L) -fuzzy relation V with respect to A given by V ((x, y), q) = A (x, q) $\wedge$ A (y, q), for all x and y in S and q in Q.

1.5 Definition: Let (R, +, ·) be a ring and Q be a non-empty set. A (Q, L) -fuzzy ideal A of R is said to be a (Q, L) **-fuzzy normal ideal (**QLFNI) of R if A (xy, q) = A (yx, q), for all x and y in R and q in Q.

1.6 Definition: A (Q, L) -fuzzy subset A of a set X is said to be **normalized** if there exists an element x in X such that A (x, q) =1.

1.7 Definition: Let A be a (Q, L) -fuzzy subset of X. For α in L, a **Q-level subset** of A corresponding to α is the set A_α = { x$\in$X: A (x, q) $\geq \alpha$ }.

1.8 Definition: Let (R, +, ·) and (R^I, +, ·) be any two rings and Q be a non empty set. Let f : R $\rightarrow$ R^I be any function and A be a (Q, L) -fuzzy ideal in R, V be a (Q, L) -fuzzy ideal in f (R) = R^I, defined by V (y, q) = $\sup_{x\in f^{-1}(y)}$ A (x, q), for all x in R and y in R^I and q in Q. Then A is called a pre-image of V under f and is denoted by f^{-1} (V).

2-SOME PROPERTIES:

2.1 Theorem: *Let (R, +, ·) be a ring and Q be a non-empty set. If A and B are two (Q, L) -fuzzy normal ideals of R, then their intersection A$\cap$B is a (Q, L) -fuzzy normal ideal of R.*

Proof: Let C=A $\cap$B and C={⟨ (x, q), C (x, q) ⟩ / x in R and q in Q }, where C (x, q) = A (x, q) $\wedge$ B (x, q). Then, Clearly C is a (Q, L) -fuzzy ideal of R, since A and B are two (Q, L) -fuzzy ideals of R. And, C (xy, q) = A (xy, q) $\wedge$B (xy, q) = A (yx, q) $\wedge$B (yx, q) = C (yx, q). Therefore, C (xy, q) = C (yx, q), for all x and y in R and q in Q. Hence A$\cap$B is a (Q, L) -fuzzy normal ideal of the ring R.

2.2 Theorem: Let (R, +, ·) be a ring and Q be a non-empty set. The intersection of a family of (Q, L) -fuzzy normal ideals of R is a (Q, L) -fuzzy normal ideal of R.

Proof: Let { A_i }$_{i\in I}$ be a family of (Q, L) -fuzzy normal ideals of R and A = $\bigcap_{i\in I} A_i$.

Then for x and y in R and q in Q, clearly the intersection of a family of (Q, L) -fuzzy ideals of the ring R is a (Q, L) -fuzzy ideal of a ring R. Now, A (xy, q) = $\inf_{i \in I} A_i(xy, q) = \inf_{i \in I} A_i(yx, q)$ = A (yx, q). Therefore, A (xy, q) = A (yx, q), for all x and y in R and q in Q. Hence the intersection of a family of (Q, L) -fuzzy normal ideals of a ring R is a (Q, L) -fuzzy normal ideal of R.

2.3 Theorem: A (Q, L) -fuzzy ideal A of a ring R is normalized if and only if A (e, q) = 1, where e is the identity element of R and q in Q.

Proof: If A is normalized, then there exists x in R such that A (x, q) = 1, but by properties of a (Q, L) -fuzzy ideal A of R, A (x, q) ≤ A (e, q), for every x in R and q in Q. Since A (x, q) = 1and A (x, q) ≤ A (e, q), 1 ≤ A (e, q). But 1 ≥ A (e, q). Hence A (e, q) = 1. Conversely, if A (e, q) =1, then by the definition of normalized (Q, L) -fuzzy subset, A is normalized.

2.4 Theorem: Let A and B be (Q, L) -fuzzy ideals of the rings R and H, respectively. If A and B are (Q, L) -fuzzy normal ideals, then A×B is a (Q, L) -fuzzy normal ideal of R×H.

Proof: Let A and B be (Q, L) -fuzzy normal ideals of the rings R and H respectively. Clearly A×B is a (Q, L) -fuzzy ideal of R×H, since A and B are (Q, L) -fuzzy ideals R and H. Let x_1 and x_2 be in R, y_1 and y_2 be in H and q in Q. Then (x_1, y_1) and (x_2, y_2) are in R×H. Now, A×B $[(x_1, y_1)(x_2, y_2), q]$ = A×B $((x_1x_2, y_1y_2), q)$ = A $(x_1x_2, q) \wedge$ B (y_1y_2, q) = A $(x_2x_1, q) \wedge$ B (y_2y_1, q) = A×B $((x_2x_1, y_2y_1), q)$ = A×B $[(x_2, y_2)(x_1, y_1), q]$. Therefore, A×B $[(x_1, y_1)(x_2, y_2), q]$ = A×B $[(x_2, y_2)(x_1, y_1), q]$. Hence A×B is a (Q, L) -fuzzy normal ideal of R×H.

2.5 Theorem: Let A and B be (Q, L) -fuzzy subsets of the rings R and H, respectively. Suppose that e and e^{l} are the identity element of R and H, respectively. If A×B is a (Q, L) -fuzzy normal ideal of R×H, then at least one of the following two statements must hold.

(i) B (e^{l}, q) ≥ A (x, q), for all x in R and q in Q,

(ii) A (e, q) ≥ B (y, q), for all y in H and q in Q.

Proof: It is trivial.

2.6 Theorem: Let A and B be (Q, L) -fuzzy subsets of the rings R and H, respectively and A×B is a (Q, L) -fuzzy normal ideal of R×H. Then the following are true:

1. if A (x, q) ≤ B (e^{l}, q), then A is a (Q, L) -fuzzy normal ideal of R.
2. if B (x, q) ≤ A (e, q), then B is a (Q, L) -fuzzy normal ideal of H.
3. either A is a (Q, L) -fuzzy normal ideal of R or B is a (Q, L) -fuzzy normal ideal of H.

Proof: Let A×B be a (Q, L) -fuzzy normal ideal of R×H and x, y in R and e^{l} in H.

Then (x, e^{l}) and (y, e^{l}) are in R×H. Clearly A×B is a (Q, L) -fuzzy ideal of R×H. Now, using the property that A (x, q) ≤ B (e^{l}, q), for all x in R, clearly A is a (Q, L) -fuzzy ideal of R. Now, A (xy, q) = A (xy, q) ∧ μ_B ($e^{l}e^{l}$, q) = A×B (((x y), (e^{l} e^{l})), q) = A×B [(x, e^{l}) (y, e^{l}), q] = A×B [(y, e^{l}) (x, e^{l}), q] = A×B [((yx), ($e^{l}e^{l}$)), q] = A (yx, q) ∧ B ($e^{l}e^{l}$, q) = A (yx, q). Therefore, A (xy, q) = A (yx, q), for all x and y in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of R. Thus (i) is proved. Now, using the property that B (x, q) ≤ A (e, q), for all x in H, let x and y in H and e in R. Then (e, x) and (e, y) are in R×H. Clearly B is a (Q, L) -fuzzy ideal of H. Now, B (xy, q) = B (xy, q) ∧A (ee, q) = A (ee, q) ∧ B (xy, q) = A×B (((ee), (xy)), q) = A×B [(e, x) (e, y), q] = A×B [(e, y) (e, x), q] = A×B [((ee), (yx)), q] = A (ee, q) ∧ B (yx, q) = B (yx, q). Therefore, B (xy, q) = B (yx, q), for all x and y in H and q in Q. Hence B is a (Q, L) -fuzzy normal ideal of H. Thus (ii) is proved. (iii) is clear.

2.7 Theorem: Let A be a (Q, L) -fuzzy subset of a ring R and V be the strongest (Q, L) -fuzzy relation of R with respect to A. Then A is a (Q, L) -fuzzy normal ideal of R if and only if V is a (Q, L) -fuzzy normal ideal of R×R.

Proof: Suppose that A is a (Q, L) -fuzzy normal ideal of R. Then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R and q in Q. Clearly V is a (Q, L) -fuzzy ideal of R×R. We have, V (xy, q) = V [(x_1, x_2) (y_1, y_2), q]= V ((x_1y_1, x_2y_2), q) = A ((x_1y_1), q) ∧ A ((x_2y_2), q) = A ((y_1x_1), q) ∧ A ((y_2x_2), q) = V ((y_1x_1, y_2x_2), q) = V [(y_1, y_2) (x_1, x_2), q] = V (yx, q). Therefore, V (xy, q) = V (yx, q), for all x and y in R×R and q in Q. This proves that V is a (Q, L) -fuzzy normal ideal of R×R. Conversely, assume that V is a (Q, L) -fuzzy normal ideal of R×R, then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R, we have A (x_1y_1, q) ∧ A (x_2y_2, q) = V ((x_1y_1, x_2y_2), q) = V [(x_1, x_2) (y_1, y_2), q] = V (xy, q) = V (yx, q) = V [(y_1, y_2) (x_1, x_2), q] = V ((y_1x_1, y_2x_2), q) = A (y_1x_1, q) ∧ A (y_2x_2, q). If we put $x_2 = y_2 = e$, where e is the identity element of R. We get, A ((x_1y_1), q) = A (y_1x_1, q), for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of R.

2.8 Theorem: Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The homomorphic image of a (Q, L) -fuzzy normal ideal of R is a (Q, L) -fuzzy normal ideal of R^{l}.

Proof: Let f : R → R^{l} be a homomorphism. Let A be a (Q, L) -fuzzy normal ideal of R. We have to prove that V is a (Q, L) -fuzzy normal ideal of f (R) = R^{l}. Now, for f (x) and f (y) in R^{l}, we have clearly V is a (Q, L) -fuzzy ideal of a ring f (R) = R^{l}, since A is a (Q, L) -fuzzy ideal of a ring R. Now, V (f (x) f (y), q) = V (f (xy), q) ≥ A (xy, q) = A (yx, q) ≤ V (f (yx), q) = V (f (y) f (x), q), which implies that V (f (x) f (y), q) = V (f (y) f (x), q). Hence V is a (Q, L) -fuzzy normal ideal of the ring R^{l}.

2.9 Theorem: Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The homomorphic pre-image of a (Q, L) -fuzzy normal ideal of R^{l} is a (Q, L) -fuzzy normal ideal of R.

Proof: Let f : R → R^l be a homomorphism. Let V be a (Q, L) -fuzzy normal ideal of f (R) = R^l. We have to prove that A is a (Q, L) -fuzzy normal ideal of R. Let x and y in R and q in Q. Then, clearly A is a (Q, L) -fuzzy ideal of the ring R, since V is a (Q, L) -fuzzy ideal of the ring R^l. Now, A (xy, q) = V (f (xy), q) = V (f (x) f (y), q) = V (f (y) f (x), q) = V (f (yx), q) = A (yx, q), which implies that A (xy, q) = A (yx, q), for x and y in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of the ring R.

2.10 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic image of a (Q, L) -fuzzy normal ideal of R is a (Q, L) -fuzzy normal ideal of R^l.

Proof: Let f : R → R^l be an anti-homomorphism. Let A be a (Q, L) -fuzzy normal ideal of R. We have to prove that V is a (Q, L) -fuzzy normal ideal of f (R) = R^l. For f (x) and f (y) in R^l, clearly V is a (Q, L) -fuzzy ideal of R^l, since A is a (Q, L) -fuzzy ideal of R. Now, V (f (x) f (y), q) = V (f (yx), q) ≥ A (yx, q) = A (xy, q) ≤ V (f (xy), q) = V (f (y) f (x), q), which implies that V (f (x) f (y), q) = V (f (y) f (x), q). Hence V is a (Q, L) -fuzzy normal ideal of the ring R^l.

2.11 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic pre-image of a (Q, L) -fuzzy normal ideal of R^l is a (Q, L) -fuzzy normal ideal of R.

Proof: Let f : R → R^l be anti-homomorphism. Let V be a (Q, L) -fuzzy normal ideal of f (R) = R^l. We have to prove that A is a (Q, L) -fuzzy normal ideal of R. Let x and y in R and q in Q, we have clearly A is a (Q, L) -fuzzy ideal of R, since V is a (Q, L) -fuzzy ideal of R^l. Now, A (xy, q) = V (f (xy), q) = V (f (y) f (x), q) = V (f (x) f (y), q) = V (f (yx), q) = A (yx, q), which implies that A (xy, q) = A (yx, q), for x and y in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of the ring R.

2.12 Theorem: Let A be a (Q, L) -fuzzy normal ideal of a ring H and f is a isomorphism from a ring R onto H. Then A◦f is a (Q, L) -fuzzy normal ideal of R.

Proof: Let x and y in R and A be a (Q, L) -fuzzy normal ideal of a ring H. Then clearly (A◦f) is a (Q, L) -fuzzy ideal of the ring R. Then we have, (A◦f) (xy, q) = A (f (xy), q) = A (f (x) f (y), q) = A (f (y) f (x), q) = A (f (yx), q) = (A◦f) (yx, q), which implies that (A◦f) (xy, q) = (A◦f) (yx, q), for x and y in R and q in Q. Hence A◦f is a (Q, L) -fuzzy normal ideal of the ring R.

2.13 Theorem: Let A be a (Q, L) -fuzzy normal ideal of a ring H and f is an anti-isomorphism from a ring R onto H. Then A◦f is a (Q, L) -fuzzy normal ideal of R.

Proof: Let x and y in R and A be a (Q, L) -fuzzy normal ideal of a ring H. Then clearly (A◦f) is a (Q, L) -fuzzy ideal of the ring R. Then we have, (A◦f) (xy, q) = A (f (xy), q) = A (f (y) f (x), q) = A (f (x) f (y), q) = A (f (yx), q) = (A◦f) (yx, q), which implies that (A◦f) (xy, q) = (A◦f) (yx, q), for x and y in R and q in Q. Hence A◦f is a

(Q, L) -fuzzy normal ideal of the ring R.

2.14 Theorem: Let A be a (Q, L) -fuzzy normal ideal of a ring R, then the pseudo (Q, L) -fuzzy coset $(aA)^p$ is a (Q, L) -fuzzy normal ideal of the ring R, for a in R.

Proof: Let A be a (Q, L) -fuzzy normal ideal of a ring R. For every x and y in R and q in Q, we have, clearly $(aA)^p$ is a (Q, L) -fuzzy ideal of the ring R and ($(aA)^{p)}$ (xy' q) = p (a) A (xy, q) = p (a) A (yx, q) = ($(aA)^p$) (yx' q). Therefore, ($(aA)^{p)}$ (xy' q) = ($(aA)^{p)}$ (yx' q), for x and y in R and q in Q. Hence $(aA)^p$ is a (Q, L) -fuzzy normal ideal of the ring R.

2.15 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. Then for α in L such that $\alpha \leq$ A (e, q), A_α is a ideal of R.

Proof: For all x and y in $A_{\alpha,}$ we have, A (x, q) $\geq \alpha$ and A (y, q) $\geq \alpha$. Now, A (x−y, q) $\geq$ A (x, q) $\wedge$ A (y, q) $\geq \alpha \wedge \alpha = \alpha$, which implies that, A (x−y, q) $\geq \alpha$. And, A (xy, q) $\geq$ A (x, q) $\vee$ A (y, q) $\geq \alpha \vee \alpha = \alpha$, which implies that, A (xy, q) $\geq \alpha$. Therefore, A (x−y, q) $\geq \alpha$, A (xy, q) $\geq \alpha$, we get x−y and xy in A_α. Hence A_α is a ideal of R.

2.16 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. Then two Q-level ideals $A_{\alpha 1}$, $A_{\alpha 2}$ and α_1 and α_2 in L and $\alpha_1 \leq$ A (e, q), $\alpha_2 \leq$ A (e, q) with $\alpha_2 < \alpha_1$ of A are equal if and only if there is no x in R such that $\alpha_1 >$ A (x, q) $> \alpha_2$.

Proof: Assume that $A_{\alpha 1} = A_{\alpha 2}$. Suppose there exists an $x \in R$ such that $\alpha_1 >$ A (x, q) $> \alpha_2$. Then $A_{\alpha 1} \subseteq A_{\alpha 2}$, which implies that x belongs to $A_{\alpha 2}$, but not in $A_{\alpha 1}$. This is a contradiction to $A_{\alpha 1} = A_{\alpha 2}$. Therefore, there is no $x \in R$ such that $\alpha_1 >$ A (x, q) $> \alpha_2$. Conversely, if there is no $x \in R$ such that $\alpha_1 >$ A (x, q) $> \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$.

2.17 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. The intersection of two Q-level ideals of A in R is also a Q-level ideal of A in R.

Proof: For $\alpha_1$and α_2 in L, $\alpha_1 \leq$ A (e, q) and $\alpha_2 \leq$ A (e, q). **Case (i) :** If $\alpha_1 <$ A (x, q) $< \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a Q-level ideal of A. **Case (ii) :** If $\alpha_1 >$ A (x, q) $> \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cap A_{\alpha 2} = A_{\alpha 1,}$ but $A_{\alpha 1}$ is a Q-level ideal of A. **Case (iii) :** If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2.}$ In all cases, intersection of any two Q-level ideals is also a Q-level ideal of A.

2.18 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. If α_i in L, $\alpha_i \leq$A (e, q) and $A_{\alpha i}$, i in I, is a collection of Q-level ideals of A, then their intersection is also a Q-level ideal of A.

Proof: It is trivial.

2.19 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. The union of two Q-level ideals of A in R is also a Q-level ideal of A in R.

Proof: Let $\alpha_1$and α_2 be in L, $\alpha_1 \leq$ A (e, q) and $\alpha_2 \leq$ A (e, q). **Case (i) :** If $\alpha_1 <$ A (x, q) $< \alpha_2$, then $A_{\alpha 2} \subseteq A_{\alpha 1}$. Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 1}$, but $A_{\alpha 1}$ is a Q-level ideal of A. **Case (ii) :** If $\alpha_1 >$ A (x, q) $> \alpha_2$, then $A_{\alpha 1} \subseteq A_{\alpha 2}$. Therefore, $A_{\alpha 1} \cup A_{\alpha 2} = A_{\alpha 2}$, but $A_{\alpha 2}$ is a Q-level ideal of A. **Case (iii) :** If $\alpha_1 = \alpha_2$, then $A_{\alpha 1} = A_{\alpha 2}$. In all cases, union of any two Q-level ideal is also a Q-level ideal of A.

2.20 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. If α_i in L, $\alpha_i \leq$A (e, q) and $A_{\alpha i}$, i in I, is a collection of Q-level ideals of A, then their union is also a Q-level ideal of A.

Proof: It is trivial.

2.21 Theorem: The homomorphic image of a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R is a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R^{l}.

Proof: Let f : R → R^{l} be a homomorphism. Let V = f (A), where A is a (Q, L) -fuzzy ideal of the ring R. Clearly V is a (Q, L) -fuzzy ideal of the ring R^{l}. Let x and y in R and q in Q, implies f (x) and f (y) in R^{l}. Let A_α is a Q-level ideal of A. That is, A (x, q) $\geq \alpha$ and A (y, q) $\geq \alpha$; A (x−y, q) $\geq \alpha$, A (xy, q) $\geq \alpha$. We have to prove that f (A_α) is a Q-level ideal of V. Now, V (f (x), q) $\geq$ A (x, q) $\geq \alpha$, which implies that V (f (x), q) $\geq \alpha$; and V (f (y), q) $\geq$ A (y, q) $\geq \alpha$, which implies that V (f (y), q) $\geq \alpha$ and V (f (x) − f (y), q) = V (f (x− y), q) $\geq$ A (x− y, q) $\geq \alpha$, which implies that V (f (x) − f (y), q) $\geq \alpha$. Also, V (f (x) f (y), q) = V (f (xy), q) $\geq$ A (xy, q) $\geq \alpha$, which implies that V (f (x) f (y), q) $\geq \alpha$. Therefore, V (f (x) − f (y), q) $\geq \alpha$ and μ_V (f (x) f (y), q) $\geq \alpha$. Hence f (A_α) is a Q-level ideal of a (Q, L) -fuzzy ideal V of the ring R^{l}.

2.22 Theorem: The homomorphic pre-image of a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R^{l} is a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R.

Proof: Let f : R → R^{l} be a homomorphism. Let V = f (A), where V is a (Q, L) -fuzzy ideal of the ring R^{l}. Clearly A is a (Q, L) -fuzzy ideal of the ring R. Let f (x) and f (y) in R^{l}, implies x and y in R and q in Q. Let f (A_α) is a Q-level ideal of V. That is, V (f (x), q) $\geq \alpha$ and V (f (y), q) $\geq \alpha$; V (f (x) − f (y), q) $\geq \alpha$, V (f (x) f (y), q) $\geq \alpha$. We have to prove that A_α is a Q-level ideal of A. Now, A (x, q) = V (f (x), q) $\geq \alpha$, implies that A (x, q) $\geq \alpha$; A (y, q) = V (f (y), q) $\geq \alpha$, implies that A (y, q) $\geq \alpha$ and A (x−y, q) = V (f (x−y), q) = V (f (x) − f (y), q) $\geq \alpha$, which implies that A (x−y, q) $\geq \alpha$. Also, A (xy, q) = V (f (xy), q) = V (f (x) f (y), q) $\geq \alpha$, which implies that A (xy, q) $\geq \alpha$. Therefore, A (x−y, q) $\geq \alpha$, A (xy, q) $\geq \alpha$. Hence, A_α is a Q-level ideal of a (Q, L) -fuzzy ideal A of R.

2.23 Theorem: The anti-homomorphic image of a Q-level ideal of a Q, L) -fuzzy ideal of a ring R is a Q-level ideal of a (Q, L) -fuzzy ideal of a ring R^{l}.

Proof: Let f: $R \rightarrow R^{l}$ be an anti-homomorphism. Let V = f (A), where A is a (Q, L) -fuzzy ideal of R. Clearly V is a (Q, L) -fuzzy ideal of R^{l}. Let x and y in R and q in Q, implies f (x) and f (y) in R^{l}. Let A_{α} is a Q-level ideal of A. That is, A (x, q) ≥ α and A (y, q) ≥ α. A (y– x, q) ≥ α, A (yx, q) ≥ α. We have to prove that f (A_{α}) is a Q-level ideal of V. Now, V (f (x), q) ≥ A (x, q) ≥ α, which implies that V (f (x), q) ≥ α; V (f (y), q) ≥ A (y, q) ≥ α, which implies that V (f (y), q) ≥ α. Now, V (f (x) –f (y), q) = V (f (x) – f (y), q) = V (f (y–x), q) ≥ A (y–x, q) ≥ α, which implies that V (f (x) –f (y), q) ≥ α. Also, V (f (x) f (y), q) = V (f (yx), q) ≥ A (yx, q) ≥ α, which implies that V (f (x) f (y), q) ≥ α. Therefore, V (f (x) –f (y), q) ≥ α and V (f (x) f (y), q) ≥ α. Hence f (A_{α}) is a Q-level ideal of a (Q, L) -fuzzy ideal V of R^{l}.

2.24 Theorem: The anti-homomorphic pre-image of a Q-level ideal of a (Q, L) -fuzzy ideal of a ring R^{l} is a Q-level ideal of a (Q, L) -fuzzy ideal of a ring R.

Proof: Let f : R → R^{l} be an anti-homomorphism. Let V = f (A), where V is a (Q, L) -fuzzy ideal of the ring R^{l}. Clearly A is a (Q, L) -fuzzy ideal of the ring R. Let f (x) and f (y) in R^{l}, implies x and y in R and q in Q. Let f (A_{α}) is a Q-level ideal of V. That is, V (f (x), q) ≥ α and V (f (y), q) ≥ α; V (f (y) – f (x), q) ≥ α, V (f (y) f (x), q) ≥ α. We have to prove that A_{α} is a Q-level ideal of A. Now, A (x, q) = V (f (x), q) ≥ α, which implies that A (x, q) ≥ α; A (y, q) = V (f (y), q) ≥ α, which implies that A (y, q) ≥ α. Now, A (x–y, q) = V (f (x – y), q) = V (f (y) –f (x), q) = V (f (y) –f (x), q) ≥ α, which implies that A (x–y, q) ≥ α. Also, A (xy, q) = V (f (xy), q) = V (f (y) f (x), q) ≥ α, which implies that A (xy, q) ≥ α. Therefore, A (x–y, q) ≥ α and A (xy, q) ≥ α. Hence A_{α} is a Q-level ideal of a (Q, L) -fuzzy ideal A of R.

2.25 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. Then $a + A_{\alpha} = (a + A)_{\alpha}$, for every a in R, α in L.

Proof: Let A be a (Q, L) -fuzzy ideal of a ring R and let x in R. Now, x∈ $(a + A)_{\alpha}$ if and only if (a+A) (x, q) ≥ α if and only if A (x–a, q) ≥ α if and only if x–a∈ A_{α} if and only if x∈ a+ A_{α}. Therefore, a+ A_{α} = $(a + A)_{\alpha}$, for every x in R.

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