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A Generalized Fuzzy Inaccuracy Measure of Order α and Type β and Coding Theorems

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Abstract

In this paper, we propose a generalized fuzzy inaccuracy measure of order α and type β and studied its particular cases. Also, we developed some coding theorems. The results presented in this paper are not only new but several known measures are the particular cases of the proposed measure.

Keywords: - Fuzzy set, Shannon's inequality, Generalized Shannon's inequality, Coding theorems, Kerridge inaccuracy.

AMS subject Classification: 94A17, 94A24.

1. Introduction

Fuzzy sets play a significant role in many deployed system because of their capability to model non-statistical imprecision. Consequently, characterization and quantification of fuzziness are important issues that affect the management of uncertainty in many system models and designs.

The concept of Fuzzy Logic (FL) was conceived by Lotfi Zadeh [23] and presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership. This approach to set theory was not applied to control systems until the 70's due to insufficient small-computer capability prior to that time. Zadeh reasoned that people do not require precise, numerical information input, and yet they are capable of highly adaptive control. FL was conceived as a better method for sorting and handling data but has proven to be an excellent choice for many control system applications since it mimics human control logic. It can be built into anything from small, hand-held products to large computerized process control systems. It uses an imprecise but very descriptive language to deal with input data more like a human operator. It is very robust and forgiving of operator and data input and often works when first implemented with little or no tuning. There is a unique membership function associated with each input parameter.

Fuzzy Logic (FL) plays an important role in the context of Information theory. Klir G.J and B. Parviz [12] first made an attempt to apply Fuzzy set and Fuzzy logic in information theory, later on various researchers applied the concept of Fuzzy in information theoretic entropy function. Besides above applications of fuzzy logic in information theory there is a numerous literature present on the application of fuzzy logic in information theory.

A fuzzy set is represented as

$$A = \{x_i/\mu_A(x_i) : i = 1,2,\dots,n\},$$

where $\mu_A(x_i)$ gives the degree of belongingness of the element 'x_i' to the set 'A'. If every element of the set 'A' is '0' or '1', there is no uncertainty about it and a set is said to be a crisp set. On the other hand, a fuzzy set 'A' is defined by a characteristic function

$$\mu_A(x_i) = \{x_1, x_2, .., x_n\} \rightarrow [0,1].$$

The function $\mu_A(x_i)$ associates with each $x_i \in R^n$ grade of membership function.

A fuzzy set A^* is called a sharpened version of fuzzy set A if the following conditions are satisfied:

$$\mu_{A^*}(x_i) \leq \mu_A(x_i), \quad if \;\; \mu_A(x_i) \leq 0.5 \;\; for\, all\, i = 1,2,..,n$$

and

$$\mu_{A^*}(x_i) \geq \mu_A(x_i), \quad if \;\; \mu_A(x_i) \geq 0.5 \; for\, all\, i = 1,2,..,n$$

De Luca and Termini [14] formulated a set of properties and these properties are widely accepted as criterion for defining any fuzzy entropy. In fuzzy set theory, the entropy is a measure of fuzziness which expresses the amount of average ambiguity in making a decision whether an element belong to a set or not. So, a measure of average fuzziness is fuzzy set H(A) should have the following properties to be a valid entropy.

(Sharpness): $H(A)$ is minimum if and only if A is a crisp set
i.e., $\mu_A(x_i) = 0 \; or \; 1; \; \forall_i$

(Maximality): $H(A)$ is maximum if and only if A is most fuzzy set
i.e., $\mu_A(x_i) = \frac{1}{2} \;\; \forall_i$

(Resolution): $H(A^*) \leq H(A)$ where A^* is sharpened version of A.
(Symmetry): $H(A) = H(\overline{A})$, where $\overline{A}$ is the complement of set A
i.e. $\bar{\mu}_A(x_i) = 1 - \mu_A(x_i)$

The importance of fuzzy set comes from the fact that it can deal with imprecise and inexact information. Its application areas span from design of fuzzy controller to robotics and artificial intelligence.

2. Basic Concepts

Let X be discrete random variable taking on a finite number of possible values $X = (x_1, x_2, \ldots, x_n)$ with respective membership function $A = \{\mu_A(x_1), \mu_A(x_2), \ldots \mu_A(x_n)\} \rightarrow [0,1]$, $\mu_A(x_i)$ gives the elements the degree of belongingness x_i to the set A. The function $\mu_A(x_i)$ associates with each $x_i \in R^n$ a grade of membership to the set A and is known as membership function.

Denote

$$X = \begin{bmatrix} x_1 & x_2 & \ldots & x_2 \\ \mu_A(x_1) & \mu_A(x_2) & \cdots & \mu_A(x_n) \end{bmatrix} \tag{2.1}$$

We call the scheme (2.1) as a finite fuzzy information scheme. Every finite scheme describes a state of uncertainty.

Let a finite source of n source symbols encoded using alphabet of D symbols, then it has been shown by Feinstein [4] that there is a uniquely decipherable/ instantaneous code with lengths $l_1, l_2 \ldots, l_n$ iff the following Kraft inequality is satisfied

$$\sum_{i}^{n} D^{-l_i} \leq 1 \tag{2.2}$$

Let

$$\Gamma_n = \left\{ P = (p_1, p_2, \ldots, p_n);\ p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \right\}, n \geq 2$$

be a set of complete probability distributions.

For $P \in \Gamma_n$, Shannon's measure of information [19] is defined as

$$H(P) = -\sum_{i=1}^{n} p_i log p_i \tag{2.3}$$

The measure (2.3) has been generalized by various authors and has found applications in various disciplines such as economics, accounting, crime, physics, etc.

Many fuzzy measures have been discussed and derived by Kapur [9], Lowen [13], Pal and Bezdek [16] etc.

The basic noiseless coding theorems give the lower bound for the mean codeword length of a uniquely decipherable code in terms of Shannon's [19] measure of entropy. Kapur [10] has established relationship between probabilistic entropy and coding. But, there are situations where probabilistic measure of entropy does not work. To tackle such situations, instead of taking the probability, the idea of fuzziness was explored.

De Luca and Termini [14] introduced a measure of fuzzy entropy corresponding to measure Shannon's [19] information theoretic entropy and is given by

$$H(A) = -\sum_{i}^{n} \left[\mu_A(x_i) \log \mu_A(x_i) + \left(1 - \mu_A(x_i)\right) log\left(1 - \mu_A(x_i)\right) \right] \tag{2.4}$$

3. Generalization of Shannon's inaccuracy measure

Sharma and Mittal [18] generalized (2.3) in the following form:

$$H(A;\ \alpha,\beta) = \frac{1}{2^{1-\beta} - 1}\left[\left(\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}\}\right)^{\frac{\beta-1}{\alpha-1}} - 1\right],$$

$$\alpha,\beta > 0,\ \ \alpha \neq \beta, \alpha \neq 1 \neq \beta \qquad (3.1)$$

For $A, Q \in \Gamma_n$, then fuzzy Kerridge inaccuracy can be defined as

$$H(A,Q) = -\sum_{i=1}^{n}\{\mu_A(x_i) + (1-\mu_A(x_i))\} log\{\mu_B(x_i) + (1-\mu_B(x_i))\} \qquad (3.2)$$

There is well known relation between $H(P)$ and $H(A,Q)$ which is given by

$$H(P) \leq H(A,Q) \qquad (3.3)$$

The relation (3.3) is known as Shannon inequality and its importance is well known in coding theory.

In the literature of information theory, there are two approaches to extend the relation (3.3) for other measures. Nath and Mittal [15] extended the relation (3.3) in the case of entropy of type β.

Using the method of Nath and Mittal [15], Van derLubbe [22] generalizes (3.3) in the case of Renyi's entropy. On the other hand, using the method of Campbell, Lubbe [22] generalized (3.3) for the case of entropy of type β. Using these generalizations, coding theorems are proved by these authors for these measures.

The objective of this communication is to generalize (3.3) using the method of Campbell for (3.1) and give its application in coding theory.

4. Generalized inaccuracy measure of Order α and type β

For $A, Q \in \Gamma_n$, defines a measure of inaccuracy, denoted by $H(A,Q;\alpha,\beta)$ as

$$H(A,Q;\alpha,\beta) = \frac{1}{2^{1-\beta} - 1}\left[\left(\sum_{i=1}^{n}\{\mu_A(x_i) + (1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i) + (1-\mu_B(x_i))\}^{(\frac{\alpha-1}{\alpha})}\right)^{(\frac{\beta-1}{\alpha-1})\alpha} - 1\right]$$

$$\alpha,\beta > 0,\ \ \alpha \neq \beta, \alpha \neq 1 \neq \beta \qquad (4.1)$$

Since $H(A,Q;\alpha,\beta) \neq H(A;\ \alpha,\beta)$, we will not interpret (4.1) as a measure of inaccuracy. But $H(A,Q;\alpha,\beta)$ is a generalization of the measure of inaccuracy defined in (3.1). In spite of the fact that $H(A,Q;\alpha,\beta)$ is not a measure of inaccuracy in its usual sense, its study is justified because it leads to meaningful new measures of length. In the following theorem, we will determine a relation between (3.1) and (4.1) of the type (3.3).

Since (4.1) is not a measure of inaccuracy in its usual sense, we will call the generalized relation as Pseudo-generalization of the Shannon inequality.

Theorem 4.1. If $A, Q \in \Gamma_n$, then it holds that

$$H(A;\ \alpha,\beta) \neq H(A,Q;\alpha,\beta) \qquad (4.2)$$

Under the condition

$$\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\alpha-1}\{\mu_B(x_i)+(1-\mu_B(x_i))\}\le 1 \qquad (4.3)$$

And equality holds if

$$\{\mu_B(x_i)+(1-\mu_B(x_i))\}=\frac{\{\mu_A(x_i)+(1-\mu_A(x_i))\}}{\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}},\quad i=1,2,\ldots,n.$$

Proof. (a) If $0<\alpha<1<\beta$.
By Shisha [20] Holder's inequality, we get

$$\left(\sum_{i=1}^{n}X_i^p\right)^{\frac{1}{p}}\left(\sum_{i=1}^{n}Y_i^q\right)^{\frac{1}{q}}\le\sum_{i=1}^{n}X_iY_i \qquad (4.4)$$

for all $X_i, Y_i>0, i=1,2,\ldots,n$ and

$$\frac{1}{p}+\frac{1}{q}=1,\qquad p<1(\neq 0), q<0\ or\ q<1(\neq 0), p<0$$

We see that equality holds if and only if there exists a positive constant c such that:

$$X_i^p=cY_i^q$$

Making the substitutions

$$p=\frac{\alpha-1}{\alpha}, q=1-\alpha$$

$$X_i=\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i)+(1-\mu_B(x_i))\},$$

$$Y_i=\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha}{1-\alpha}}$$

In (4.4) we get

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i)+(1-\mu_B(x_i))\}^{(\frac{\alpha-1}{\alpha})}\right]^{\frac{\alpha}{\alpha-1}}\left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\right]^{\frac{1}{1-\alpha}}$$

$$\le\sum_{i=1}^{n}\{\mu_A^{\alpha-1}(x_i)+(1-\mu_A(x_i))^{\alpha-1}\}\{\mu_B(x_i)+(1-\mu_B(x_i))\},;\ \alpha>0,\alpha\neq 1$$

Using the condition (4.3), we get

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i)+(1-\mu_B(x_i))\}^{(\frac{\alpha-1}{\alpha})}\right]^{\frac{\alpha}{\alpha-1}}\left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\right]^{\frac{1}{1-\alpha}}\le 1$$

$$\alpha>0,\alpha\neq 1 \qquad (4.5)$$

Since $\alpha<1$, (4.5) becomes

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i)+(1-\mu_B(x_i))\}^{(\frac{\alpha-1}{\alpha})}\right]^{\alpha} \geq \left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\right] \quad (4.6)$$

Raising both sides of (4.6) with $(\beta-1)/(\alpha-1)<0$, we get

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}}\{\mu_B(x_i)+(1-\mu_B(x_i))\}^{(\frac{\alpha-1}{\alpha})}\right]^{\alpha(\frac{\beta-1}{\alpha-1})} \leq \left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\right]^{(\frac{\beta-1}{\alpha-1})} \quad (4.7)$$

Using (4.7) and the fact that $\beta>1$, we get (4.2).

(b) $\alpha>1,\beta>1; 0<\alpha<1;\ \beta>1\ (\alpha<\beta\ or\ \beta<\alpha); 0<\beta<1<\alpha.$

The proof follows on the similar lines.

5. Coding Theorems

We will now give an application of Theorem 4.1 in coding theory. Let a finite set of n-input symbols with probabilities $p_1,p_2,\dots,p_n$ be encoded in terms of symbols taken from the alphabet $\{a_1,a_2,\dots,a_n\}$, then there always exist a uniquely decipherable code with length $l_1,l_2,\dots,l_n$ iff

$$\sum_{i=1}^{n}D^{-l_i}\leq 1, \quad (5.1)$$

If

$$L=\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}\,l_i \quad (5.2)$$

is the average codeword length, then for a code which satisfies (5.1), it has been shown that Feinstein [4],

$$L\geq H(P) \quad (5.3)$$

With equality iff $l_i=-log\{\mu_A(x_i)+(1-\mu_A(x_i))\}, i=1,2,\dots,n$ and that by suitable encoded into words of long sequences, the average length can be arbitrary close to H(P). this is Shannon's noiseless coding theorem.

By considering Renyi [17] entropy a coding theorem and analogous to the above noiseless coding theorem has been established by Campbell [2] and the authors obtained bounds for it in terms of

$$H_\alpha(P) = \frac{1}{1-\alpha} log \sum_{i=1}^{n} \{\mu_A(x_i) + (1-\mu_A(x_i))\}^\alpha, \qquad \alpha > 0(\neq 1)$$

Kieffer [11] defined a class rules and showed $H_\alpha(P)$ is the best decision rule for deciding which of the two sources can be coded with expected cost of sequences of length n when n → ∞, where the cost of encoding a sequence is assumed to be a function of length only. Further Jelinek [7] showed that coding with respect to Campbell [2] mean length is useful in minimizing the problem of buffer overflow which occurs when the source symbol are being produced at a fixed rate and the code words are stored temporarily in a finite buffer.

Hooda and Bhaker [6] consider the following generalization of Campbell [2] mean length:

$$L^\beta(t) = \frac{1}{t} log \left[\frac{\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\right\} D^{tl_i}}{\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\right\}}\right], \quad \beta \geq 1$$

And proved

$$H_\alpha^\beta(P) \leq L^\beta(t) < H_\alpha^\beta(P) + 1, \qquad \alpha > 0, \alpha \neq 1, \beta \geq 1$$

Under the condition

$$\sum_{i=1}^{n}\left\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\right\} D^{tl_i} \leq \sum_{i=1}^{n}\left\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\right\}$$

Where $H_\alpha^\beta(P)$ is generalized entropy of order $\alpha = 1/1 + t$ and type β studied by Aczel and Daroczy [1] and Kapur [8]. It may be seen that the mean codeword length (5.2) had been generalized parametrically and their bounds had been studied in terms of generalized measures of entropies. Here we give the following another generalization of (5.2) and study its bounds in terms of generalized entropy of order α and type β.

In this paper we study some coding theorems by considering a new function depending on parameters α and β. Our motivation for studying this new function is that it generalizes some entropy function already existing in literature.

We define the measure of length $L(\alpha,\beta)$ by:

$$L(\alpha,\beta) = \frac{1}{2^{1-\beta} - 1}\left[\left(\sum_{i=1}^{n}\{\mu_A(x_i) + (1-\mu_A(x_i))\}^{\frac{\alpha^2-\alpha+1}{\alpha}} D^{l_i\left(\frac{1-\alpha}{\alpha}\right)}\right)^{\left(\frac{\beta-1}{\alpha-1}\right)\alpha} - 1\right] \tag{5.4}$$

where

$$\alpha,\beta > 0, \alpha \neq \beta, \alpha \neq 1 \neq \beta.$$

Also, we have used the condition

$$\sum_{i=1}^{n}\{\mu_A^{\alpha-1}(x_i)+(1-\mu_A(x_i))^{\alpha-1}\}\,D^{l_i}\leq 1 \tag{5.5}$$

To find bounds. It may be seen that in the case when $\alpha = 1$, then (5.5) reduces to Kraft Inequality (5.1).

Theorem 5.1. If $l_i, i = 1,2,\dots,n$ are the lengths of codewords satisfying (5.5), then

$$H(A;\ \alpha,\beta)\leq\ L(\alpha,\beta)<D^{1-\beta}H(A;\ \alpha,\beta)+\frac{1-D^{1-\beta}}{1-2^{1-\beta}}. \tag{5.6}$$

Proof. In (4.2), choose $Q = (q_1, q_2, \dots, q_n)$ where

$$\mu_B = D^{-l_i} \tag{5.7}$$

With choice of Q, (4.2) becomes

$$H(A;\ \alpha,\beta)\leq\frac{1}{2^{1-\beta}-1}\left[\left(\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}^{\frac{\alpha^2-\alpha+1}{\alpha}}D^{l_i\left(\frac{1-\alpha}{\alpha}\right)}\right)^{(\frac{\beta-1}{\alpha-1})\alpha}-1\right]$$

$H(A;\alpha,\beta)\leq L(°\alpha,\beta)$ which proves the first part of (5.6).

The equality holds iff

$$D^{-l_i}=\frac{\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}}{\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}}, i = 1,2,\dots,n$$

which is equivalent to

$$l_i=-log\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}+log\left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\right], i=1,2,.,n \tag{5.8}$$

Choose all l_i such that

$$-log\frac{\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}}{\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}}\leq l_i<-log\frac{\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}}{\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}}+1$$

Using the above relation, it follows that

$$D^{-l_i}>\frac{\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}}{\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\,D} \tag{5.9}$$

We now have two possibilities:

If $\alpha > 1$, (5.9) gives us

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}^{\frac{\alpha^2-\alpha+1}{\alpha}}D^{l_i\frac{1-\alpha}{\alpha}}\right]^{\alpha}>\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}\,D^{1-\alpha} \tag{5.10}$$

Now consider two cases:

Let $0 < \beta < 1$

Raising both sides of (5.10) with $(\beta - 1)/(\alpha - 1)$ we get

$$\left[\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}^{\frac{\alpha^2-\alpha+1}{\alpha}}D^{l_i\frac{1-\alpha}{\alpha}}\right]^{\alpha\left(\frac{\beta-1}{\alpha-1}\right)} < \left[\sum_{i=1}^{n}\left\{\mu_A^{\alpha}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha}\right\}\right]^{\frac{\beta-1}{\alpha-1}} D^{1-\beta} \qquad (5.11)$$

Since $2^{1-\beta} - 1 > 0$ for $\beta < 1$, we get from (5.11) the right hand side in (5.6).

Let $\beta > 1$. The proof follows similarly.

$0 < \alpha < 1$, the proof follows on the same lines.

Particular cases.

Since $D \geq 2$, we have

$$\frac{1-D^{1-\beta}}{1-2^{1-\beta}} \geq 1$$

It follows then the upper bound of $L(\alpha,\beta)$ in (5.6) is greater than unity.

If $\beta = \alpha$ (5.6) becomes:

$$H(A;\,\alpha) \leq L(\alpha) < D^{1-\alpha}H(A;\,\alpha) + \frac{1-D^{1-\alpha}}{1-2^{1-\alpha}}.$$

$where$

$$H(A;\,\alpha) = \frac{1}{2^{1-\alpha}-1}\left[\sum_{i=1}^{n}\{\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\}-1\right], \quad \alpha > 0, \alpha \neq 1$$

Be the Havrda and Charvat [5] entropy. Later on it also studied by Daroczy [3] and Tsallis [21] entropy.

And

$$L(\alpha) = \frac{1}{2^{1-\alpha}-1}\left[\left\{\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}^{\frac{\alpha^2-\alpha+1}{\alpha}}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right\}^{\alpha}-1\right], \quad \alpha > 0, \alpha \neq 1$$

Be the new codeword length.

If $\beta \to 1$ then (5.6) becomes

$$H(A;\,\alpha) \leq L(\alpha) < H(A;\,\alpha) + logD$$

where

$$H_{\alpha}(P) = \frac{1}{1-\alpha}log\sum_{i=1}^{n}\{\mu_A(x_i)+(1-\mu_A(x_i))\}^{\alpha}$$

Be the Renyi [17] Entropy and

$$L(\alpha) = \frac{\alpha}{1-\alpha}log\sum_{i=1}^{n}\{\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\}^{\frac{\alpha^2-\alpha+1}{\alpha}}D^{-l_i\left(\frac{\alpha}{1-\alpha}\right)}, \qquad \alpha > 0(\neq 1)$$

Be the new codeword length.

If $\beta = \alpha$ and $\alpha \to 1$ then (5.6) becomes

$$\frac{H(A)}{logD} \leq \frac{L}{logD} < \frac{H(A)}{logD} + 1$$

Which is the Shannon [19] Classical noiseless coding theorem.

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