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On Fuzzy Supra Boundary and Fuzzy Supra Semi Boundary

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Abstract

In this paper we introduce and study the concepts of fuzzy supra boundary and fuzzy supra semi boundary. Many fundamental properties and relations of these notions analogous with other existing fuzzy topological spaces are investigated. Counter examples have been given to support the strictness of the inequalities and relations. Fuzzy supra semi continuous functions and fuzzy supra irresolute functions are characterized in terms of fuzzy supra semi boundary.

Keywords: Fuzzy topological spaces, fuzzy supra topological spaces, fuzzy supra continuous functions.

1. Introduction

After the introduction of fuzzy topological spaces by Chang in 1968, several definitions of fuzzy boundary have been proposed (Warren in 1977, Pu and liu in 1980, Wu and Zheng in 1991, Cuchill-Ibanez and Tarres in 1997). Boundary generally marks the division of two contiguous properties. In many real life situations, boundary of spatial objects is not well defined due to the inherent fuzziness. In recent years, fuzzy boundary and many of its generalizations have been richly studied in existing fuzzy topological spaces. Properties of fuzzy boundary and fuzzy semi boundary were investigated by M. Athar and B. Ahmad [2] using the notion of Pu and Liu. Properties of intuitionistic fuzzy semi boundary were investigated by V. Thiripurasundari et al. [5] and A Manimaran et al. [4].

M. E. Abd El-Monsef et al. [2] introduced Fuzzy supra topological spaces as a natural generalizations of classical supra topological spaces introduced by A.S. Mashhour et al. In this paper, we introduce fuzzy supra boundary and fuzzy supra semi boundary generalizing the notion of Pu and Liu and investigate their fundamental properties supported by counter examples. Further we extend these notions in product related fuzzy supra topological spaces. Finally necessary conditions for fuzzy supra semi continuous functions and fuzzy supra irresolute functions are obtained in terms of fuzzy supra semi boundary.

2. Preliminaries

In order to make this paper self-contained, first we briefly recall certain definitions and results. Throughout the paper X, Y are non-empty sets.

Definition 2.1: A fuzzy set in X is a function from X into the closed unit interval I= [0, 1].

The collection of all fuzzy sets of X is denoted by I^X.

Definition 2.2: Let A, B and C be fuzzy sets in X. Then for all x ϵ X,

A = B iff A(x) = B(x),
A ≤ B iff A(x) ≤ B(x),
C = A $\vee$ B iff C(x) = max{A(x), B(x)},
C = A $\wedge$ B iff C(x) = min{A(x), B(x)},
B = A^c iff B(x) = 1- A(x).

The fuzzy sets 0_x and 1_x in X are defined as $0_x(x)=0$, $1_x(x)=1$, for all x ϵ X.

Definition 2.3: Let f : X→Y be a mapping. Let Aϵ I^X and Bϵ I^Y, then f(A) is a fuzzy set in Y, defined by

$$f(A)(y)=\begin{cases}\sup\{A(x): x \in f^{-1}(y)\}, & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{if } f^{-1}(y) = \phi\end{cases}$$

for all y ϵ Y and
$f^{-1}(B)$ is a fuzzy set in Y, defined by

$f^{-1}(B)(x) = B(f(x))$ for all x ϵ X.

Theorem 2.4 (see [1]): Let f : X→Y be a mapping, then
(1) $f^{-1}(B^c)=(f^{-1}(B))^c$, for any fuzzy set B in Y.
(2) $f(f^{-1}(B)) \leq B$, for any fuzzy set B in Y.
(3) $A \leq f^{-1}(f(A))$ for any fuzzy set A in X.
(4) $f^{-1}(\vee A_i) = \vee f^{-1}(A_i)$ and $f^{-1}(\wedge A_i) = \wedge f^{-1}(A_i)$, where $\{A_i\}$ is any family of fuzzy sets in Y.

Definition 2.5: A fuzzy topology T on a set X is a family of fuzzy sets in X such that
(1) 0_x, 1_x ϵ T;
(2) A, B ϵ T ⇒ A ∧ B ϵ T and
(3) A_i ϵ T ⇒ $\vee A_i$ ϵ T.

The pair (X, T) is called a fuzzy topological space (FTS). The elements of T are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.6: The Closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$Cl(A) = \wedge\{B: A \leq B, B \text{ is a fuzzy closed set in } X\}$,

$Int(A) = \vee\{B: B \leq A, B \text{ is a fuzzy open set in } X\}$.

Definition 2.7: [Pu and Liu, 1980] The boundary of a fuzzy set A in FTS (X, T) is denoted and defined as

$\partial(A) = Cl(A) \wedge Cl(A^c)$.

Definition 2.8: A collection T^* of fuzzy sets in a set X is called a fuzzy supra topology on X if the following conditions are satisfied:

(1) $0_x, 1_x \in T$ and

(2) $A_i \in T \Rightarrow \vee A_i \in T$.

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp. fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS(X) (resp. FSCS(X)).

Remark 2.9:

(1) Every FTS is a FSTS.

(2) If (X, T^*) is an associated FSTS with the FTS (X, T) (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS (X, T) is fuzzy supra open (closed) set in the FSTS (X, T^*).

Definition 2.10: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as

$Cl^*(A) = \wedge\{B: A \leq B, B \text{ is a fuzzy supra closed set in } X\}$,

$Int^*(A) = \vee\{B: B \leq A, B \text{ is a fuzzy supra open set in } X\}$.

Remark 2.11:

(1) The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A.

(2) The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set contained in A.

(3) If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X, then

$Int(A) \leq Int^*(A) \leq A \leq Cl^*(A) \leq Cl(A)$.

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in the following theorem.

Theorem 2.12 (see [2]): For any fuzzy sets A and B in a FSTS (X, T^*),
(1) $A \in FSCS(X) \Leftrightarrow Cl^*(A) = A$,
$A \in FSOS(X) \Leftrightarrow Int^*(A) = A$;
(2) $A \le B, \Rightarrow Cl^*(A) \le Cl^*(B)$ and $Int^*(A) \le Int^*(B)$;
(3) $Cl^*(Cl^*(A)) = Cl^*(A)$ and $Int^*(Int^*(A)) = Int^*(A)$;
(4) $Cl^*(A \vee B) \ge Cl^*(A) \vee Cl^*(B)$;
(5) $Cl^*(A \wedge B) \le Cl^*(A) \wedge Cl^*(B)$;
(6) $Int^*(A \vee B) \ge Int^*(A) \vee Int^*(B)$;
(7) $Int^*(A \wedge B) \le Int^*(A) \wedge Int^*(B)$;
(8) $Cl^*(A^c) = (Int^*(A))^c$, $Int^*(A^c) = (Cl^*(A))^c$.

3. Fuzzy supra boundary and Fuzzy supra semi boundary

We define fuzzy supra boundary generalizing the notion of Pu and Liu.

Definition 3.1: The fuzzy supra boundary of a fuzzy set A in FSTS (X, T^*) is denoted and defined as $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c)$.

Clearly $\partial^*(A)$ is a FSCS in X.

Proposition 3.2: If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X, then $\partial^*(A) \le \partial(A)$.

Proof: $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) \le Cl(A) \wedge Cl(A^c) = \partial(A)$.
The above inequality is irreversible as seen in the following example.

Example 3.3: Let X = {a, b} be a set with a fuzzy topology $T = \{0_x, \{a_{.2}, b_{.5}\}, 1_x\}$. Let

$T^* = \{0_x, \{a_{.2}, b_{.5}\}, \{a_{.5}, b_{.1}\}, \{a_{.5}, b_{.5}\}, 1_x\}$ be a fuzzy supra topology on X associated with T. For $A = \{a_{.5}, b_{.2}\}$, fuzzy boundary of A is $\partial(A) = Cl(A) \wedge Cl(A^c) = \{a_{.8}, b_{.5}\} \wedge 1_x = \{a_{.8}, b_{.5}\}$ and fuzzy supra boundary of A is $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) = \{a_{.5}, b_{.5}\} \wedge \{a_{.5}, b_{.9}\} = \{a_{.5}, b_{.5}\}$,

Thus, $\partial^*(A) \ngeq \partial(A)$.

Some basic properties of fuzzy supra boundary are given in the following proposition.

Proposition 3.4: Let A and B be any two fuzzy sets in a FSTS (X, T^*), then
(1) $\partial^*(A^c) = \partial^*(A)$;
(2) $A \in FSCS(X) \Rightarrow \partial^*(A) \le A$ and
$A \in FSOS(X) \Rightarrow \partial^*(A) \le A^c$;
(3) if $A \le B$, then
$B \in FSCS(X) \Rightarrow \partial^*(A) \le B$, and
$B \in FSOS(X) \Rightarrow \partial^*(B) \le A^c$;

(4) $(\partial^*(A))^c = Int^*(A) \vee Int^*(A^c)$.

Proof:
(1) $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) = Cl^*(A^c) \wedge Cl^*(A\ ) = \partial^*(A^c)$.
(2) $A \in FSCS(X) \Rightarrow Cl^*(A) = A \Rightarrow \partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) = A \wedge Cl^*(A^c\ ) \leq A$.
Again,
$A \in FSOS(X) \Rightarrow Cl^*(A^c) = A^c \Rightarrow \partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) = Cl^*(A) \wedge A^c \leq A^c$.

(3) $A \leq B \Rightarrow Cl^*(A) \leq Cl^*(B)$,
$B \in FSCS(X) \Rightarrow Cl^*(B) = B \Rightarrow \partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) \leq Cl^*(B) \wedge Cl^*(A^c) = B \wedge Cl^*(A^c\ ) \leq B$.

Again,
$B \in FSOS(X) \Rightarrow Cl^*(B^c) = B^c \Rightarrow \partial^*(B) = Cl^*(B) \wedge Cl^*(B^c) = Cl^*(B) \wedge B^c \leq B^c \leq A^c$.

(4) $(\partial^*(A))^c = (Cl^*(A) \wedge Cl^*(A^c))^{\,c} = (Cl^*(A))^{\,c} \vee (Cl^*(A^c))^{\,c} = Int^*(A^c) \vee Int^*(A)$.

The inequalities (2) and (3) of the above proposition are irreversible as shown in the following examples.

Example 3.5: Let $X = \{a, b, c\}$ be a set with a fuzzy supra topology $T^* = \{0_x, \{a_0, b_{.4}, c_{.6}\}, \{a_{.5}, b_{.3}, c_{.8}\}, \{a_{.6}, b_{.8}, c_{.2}\}, \{a_{.5}, b_{.4}, c_{.8}\}, \{a_{.6}, b_{.8}, c_{.8}\}, \{a_{.6}, b_{.8}, c_{.6}\}, 1_x\}$. Let $A = \{a_{.6}, b_{.8}, c_{.6}\}$, $B = \{a_{.4}, b_{.2}, c_{.4}\}$ be two fuzzy sets in X, then calculations give
$\partial^*(A) = \{a_{.4}, b_{.2}, c_{.4}\} < A$, but A is not in FSCS(X) and
$\partial^*(B) = \{a_{.4}, b_{.2}, c_{.4}\} < B^c$, but B is not in FSOS(X).

Example 3.6: In the FSTS (X, T^*) of the above example, for fuzzy sets $A = \{a_{.4}, b_{.2}, c_{.4}\}$, $B = \{a_{.4}, b_{.3}, c_{.4}\}$, $C = \{a_0, b_{.2}, c_{.2}\}$, calculations give
$\partial^*(A) = \{a_{.4}, b_{.2}, c_{.4}\} \leq B$, where $A \leq B$, but B is not in FSCS(X),
$\partial^*(B) = \{a_1, b_{.6}, c_{.4}\} \leq C^c$, where $C \leq B$, but B is not in FSOS(X).

Remark 3.7: In [3], M. Athar and B. Ahmad in (4) of Proposition 1 of section 3, mentioned that for fuzzy sets A and B in a FTS (X, T), if $A \leq B$ and B is fuzzy open, then $\partial(A) \leq B^c$. But this is not true which is shown in the following example.

Example 3.8: Let $X = \{a, b\}$ be a set with a fuzzy topology $T = \{0_x, \{a_{.8}, b_{.5}\}, 1_x\}$. For fuzzy sets $A = \{a_{.5}, b_{.5}\}$, $B = \{a_{.8}, b_{.5}\}$. Here $A \leq B$ and B fuzzy open but $\partial(A) = 1_x \nleq B^c$.
Same kind of error is seen in [4, 5].
More properties of fuzzy supra boundary are given in the following proposition.

Proposition 3.9: Let A be any fuzzy set in a FSTS (X, T^*), then
(1) $\partial^*(Int^*(A)) \leq \partial^*(A)$;
(2) $\partial^*(Cl^*(A)) \leq \partial^*(A)$;
(3) $\partial^*(A) = Cl^*(A) — Int^*(A)$;
(4) $Int^*(A) \leq A — \partial^*(A)$;

(5) $A \vee \partial^*(A) \leq Cl^*(A)$;
(6) $\partial^*(\partial^*(A)) \leq \partial^*(A)$.

Proof :
(1) $\partial^*(Int^*(A)) = Cl^*(Int^*(A)) \wedge Cl^*((Int^*(A))^c)$
$= Cl^*(Int^*(A)) \wedge Cl^*(Cl^*(A^c))$
$\leq Cl^*(A) \wedge Cl^*(A^c)$
$= \partial^*(A)$.

(2) $\partial^*(Cl^*(A)) = Cl^*(Cl^*(A)) \wedge Cl^*((Cl^*(A))^c)$
$= Cl^*(A) \wedge Cl^*(Int^*(A^c))$
$\leq Cl^*(A) \wedge Cl^*(A^c)$
$= \partial^*(A)$.

(3) $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c) = Cl^*(A) \wedge (Int^*(A))^c = Cl^*(A) — Int^*(A)$.
(4) $A — \partial^*(A) = A \wedge (Cl^*(A) \wedge Cl^*(A^c))^c = A \wedge ((Cl^*(A))^c \vee (Cl^*(A^c))^c) = A \wedge (Int^*(A^c) \vee Int^*(A)) = (A \wedge Int^*(A^c)) \vee (A \wedge Int^*(A)) = (A \wedge (Int^*(A^c)) \vee Int^*(A) \geq Int^*(A)$.
(5) $A \vee \partial^*(A) = A \vee (Cl^*(A) \wedge Cl^*(A^c)) = (A \vee Cl^*(A)) \wedge (A \vee Cl^*(A^c)) = Cl^*(A) \wedge (A \vee Cl^*(A^c)) \leq Cl^*(A)$.
(6) Since $\partial^*(A)$ is a FSCS, so $Cl^*(\partial^*(A)) = \partial^*(A)$
$\partial^*(\partial^*(A)) = Cl^*(\partial^*(A)) \wedge Cl^*((\partial^*(A)^c)) = \partial^*(A) \wedge Cl^*((\partial^*(A)^c)) \leq \partial^*(A)$.

In general, the converses of inequalities in the above proposition are not true as seen in the following example.

Example 3.10: For fuzzy sets $A = \{a_{.4}, b_{.2}, c_{.4}\}$, $B = \{a_{.6}, b_{.8}, c_{.7}\}$, $P = \{a_{.5}, b_{.3}, c_{.8}\}$, $Q = \{a_{.8}, b_{.7}, c_{.3}\}$ in the FSTS in example 3.5, calculations give
$\partial^*(A) = \{a_{.4}, b_{.2}, c_{.4}\} \nleq 0_x = \partial^*(Int^*(A))$,
$\partial^*(B) = \{a_{.4}, b_{.2}, c_{.4}\} \nleq 0_x = \partial^*(Cl^*(B))$,
$Int^*(A) = 0_x \ngeq \{a_{.4}, b_{.2}, c_{.4}\} = A — \partial^*(A)$,
$P \vee \partial^*(P) = \{a_{.5}, b_{.7}, c_{.8}\} \ngeq 1_x = Cl^*(P)$,
$\partial^*(Q) = 1_x \nleq 0_x = \partial^*(\partial^*(Q)$.

Remark 3.11 (see [3]): For fuzzy sets A and B in a FST (X, T), on has the following relations
(1) $\partial(A \vee B) \leq \partial(A) \vee (B)$;
(2) $\partial(A \wedge B) \leq (\partial(A) \wedge Cl(B)) \vee (Cl(A) \wedge \partial(B)) \leq \partial(A) \vee \partial(B)$.

In case of fuzzy supra boundary $\partial^*(A \vee B)$, $\partial^*(A) \vee \partial^*(B)$, $\partial^*(A \wedge B)$, $(\partial^*(A) \wedge Cl^*(B)) \vee (Cl^*(A) \wedge \partial^*(B))$, $\partial^*(A) \wedge \partial^*(B)$ behave arbitrarily as seen in the following example.

Example 3.12: For fuzzy sets $A = \{a_{.8}, b_{.5}, c_{.3}\}$, $B = \{a_{.5}, b_{.7}, c_{.2}\}$, $C = \{a_{.2}, b_{.5}, c_{.7}\}$, $D = \{a_{.5}, b_{.3}, c_{.8}\}$, $E = \{a_{.8}, b_{.7}, c_{.3}\}$, $F = \{a_{.2}, b_{.3}, c_{.7}\}$, in the FSTS in example 3.5, calculations give

(1) $\partial^*(A \vee B) = 1_x > \partial^*(A) \vee \partial^*(B) = \{a_1, b_{.7}, c_{.4}\}$, but
$\partial^*(C \vee D) = \{a_{.5}, b_{.6}, c_{.2}\} < \partial^*(C) \vee \partial^*(D) = \{a_1, b_{.7}, c_{.4}\}$;

(2) $\partial^*(C \wedge D) = 1_x > \partial^*(C) \vee \partial^*(D) = \{a_1, b_{.7}, c_{.4}\}$, but
$\partial^*(E \wedge F) = \{a_1, b_{.6}, c_{.4}\} < \partial^*(E) \vee \partial^*(F) = 1_x$;

(3) $\partial^*(E \wedge F) = \{a_1, b_{.6}, c_{.4}\} < \partial^*(E) \wedge \partial^*(F) = 1_x$, but
$\partial^*(C \wedge D) = 1_x > \partial^*(C) \wedge \partial^*(D) = \{a_{.5}, b_{.6}, c_{.2}\}$;

(4) $\partial^*(A \vee B) = 1_x > \partial^*(A \wedge B) = \{a_{.5}, b_{.6}, c_{.2}\}$, but
$\partial^*(C \vee D) = \{a_{.5}, b_{.6}, c_{.2}\} < \partial^*(C \wedge D) = 1_x$;

(5) $\partial^*(C \wedge D) = 1_x > (\partial^*(C) \wedge Cl^*(D)) \vee (Cl^*(C) \wedge \partial^*(D)) = \{a_1, b_{.7}, c_{.4}\}$, but
$\partial^*(E \wedge F) = \{a_1, b_{.6}, c_{.4}\} < (\partial^*(E) \wedge Cl^*(F)) \vee (Cl^*(E) \wedge \partial^*(F)) = 1_x$.

Before we discuss fuzzy supra semi boundary, we need some more definitions.

Definition 3.13: A fuzzy set A in a FSTS (X, T^*) is called a fuzzy supra semi open set (FSSOS) if there exist a fuzzy supra open set $\mu \in T^*$ such that $\mu \leq A \leq Cl^*(\mu)$.

The complement of fuzzy supra semi open set is called fuzzy supra semi closed set (FSSCS). The collection of all FSSOS (resp. FSSCS) of the FSTS (X, T^*) is denoted by FSSOS(X) (resp. FSSCS(X)).

Remark 3.14:

(1) Every fuzzy supra open(closed) set is fuzzy supra semi open(closed).
(2) Union of a family of fuzzy supra semi open sets is fuzzy supra semi open.
(3) Intersection of a family of fuzzy supra semi closed sets is fuzzy supra semi closed.

Definition 3.15: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra semi closure and fuzzy supra semi interior are denoted and defined respectively as

$sCl^*(A) = \wedge\{B: A \leq B$, B is a fuzzy supra semi closed set in X$\}$,
$sInt^*(A) = \vee\{B: B \leq A$, B is a fuzzy supra semi open set in X$\}$.

Remark 3.16: For any fuzzy set A in a FSTS (X, T^*),

(1) $sCl^*(A)$ is the smallest fuzzy supra semi closed set containing A.
(2) $sInt^*(A)$ FSTS is the largest fuzzy supra semi open set contained in A.
(3) $Int^*(A) \leq sInt^*(A) \leq A \leq sCl^*(A) \leq Cl^*(A)$.

Properties of fuzzy supra semi closure and fuzzy supra semi interior are summarized in the following proposition.

Proposition 3.17: For any fuzzy sets A and B in a FSTS (X, T^*),

(1) $A \in FSSCS(X) \Leftrightarrow sCl^*(A) = A$,
$A \in FSSOS(X) \Leftrightarrow sInt^*(A) = A$;

(2) $A \leq B, \Rightarrow sCl^*(A) \leq sCl^*(B)$ and $sInt^*(A) \leq sInt^*(B)$;
(3) $sCl^*(sCl^*(A)) = sCl^*(A)$ and $sInt^*(sInt^*(A)) = sInt^*(A)$;
(4) $sCl^*(A \vee B) \geq sCl^*(A) \vee sCl^*(B)$;
(5) $sCl^*(A \wedge B) \leq sCl^*(A) \wedge sCl^*(B)$;
(6) $sInt^*(A \vee B) \geq sInt^*(A) \vee sInt^*(B)$;
(7) $sInt^*(A \wedge B) \leq sInt^*(A) \wedge sInt^*(B)$;
(8) $sCl^*(A^c) = (sInt^*(A))^c$, $sInt^*(A^c) = (sCl^*(A))^c$.

Proof: (1)-(3), (8) proofs are straight forward.
(4) $A \leq A \vee B$, $B \leq A \vee B \Rightarrow sCl^*(A) \leq sCl^*(A \vee B)$, $sCl^*(B) \leq sCl^*(A \vee B)$. Hence $sCl^*(A) \vee sCl^*(B) \leq sCl^*(A \vee B)$.

(5) $A \geq A \wedge B$, $B \geq A \wedge B \Rightarrow sCl^*(A) \geq sCl^*(A \wedge B)$, $sCl^*(B) \geq sCl^*(A \wedge B)$. Hence $sCl^*(A) \wedge sCl^*(B) \geq sCl^*(A \wedge B)$.

proofs of (6), (7) are similar to that of (4) and (5).

In the above proposition (4)- (7), the equality may not hold as shown in the following example.

Example 3.18: Let $X = \{a, b\}$ be a set with a fuzzy supra topology $T^* = \{0_x, \{a_{.5}, b_{.1}\}, \{a_{.2}, b_{.5}\}, \{a_{.5}, b_{.5}\}, 1_x\}$. Then

$FSSOS(X) = \{0_x, 1_x\} \cup \{\{a_{.5}, b_y\}: 0.1 \leq y \leq 0.5\} \cup \{\{a_x, b_{.5}\}: 0.2 \leq x \leq 0.5\}$,

$FSSCS(X) = \{0_x, 1_x\} \cup \{\{a_{.5}, b_y\}: 0.5 \leq y \leq 0.9\} \cup \{\{a_x, b_{.5}\}: 0.5 \leq x \leq 0.8\}$.

For fuzzy sets $A = \{a_{.5}, b_{.2}\}$, $B = \{a_{.2}, b_{.5}\}$, $C = \{a_{.5}, b_{.8}\}$, $D = \{a_{.8}, b_{.5}\}$, $E = \{a_{.5}, b_1\}$,, $F = \{a_1, b_{.5}\}$,, $P = \{a_{.5}, b_0\}$, $Q = \{a_0, b_{.5}\}$, in X, calculations give

$sCl^*(C \vee D) = 1_x > sCl^*(C) \vee sCl^*(D) = \{a_{.8}, b_{.8}\}$,
$sCl^*(E \wedge F) = \{a_{.5}, b_{.5}\} < sCl^*(E) \wedge sCl^*(F) = 1_x$,
$sInt^*(P \vee Q) = \{a_{.5}, b_{.5}\} > sInt^*(P) \vee sInt^*(Q) = 0_x$,
$sInt^*(A \wedge B) = 0_x < sInt^*(A) \wedge sInt^*(B) = \{a_{.2}, b_{.2}\}$.

Definition 3.19: The fuzzy supra semi boundary of a fuzzy set A in FSTS (X, T^*) is denoted and defined as $s\partial^*(A) = sCl^*(A) \wedge sCl^*(A^c)$.

Clearly $s\partial^*(A)$ is a FSSCS in X.

Proposition 3.20: Let A be any fuzzy set in a FSTS (X, T^*), then $s\partial^*(A) \leq \partial^*(A)$.

Proof: $s\partial^*(A) = sCl^*(A) \wedge sCl^*(A^c) \leq Cl^*(A) \wedge Cl^*(A^c) = \partial^*(A)$.

The converse of the above inequality is not true as seen in the following example.

Example 3.21: Choose $A = \{a_{.1}, b_{.6}\}$ in the FSTS (X, T^*) in example 3.18. Then calculations give

$s\partial^*(A) = \{a_{.5}, b_{.6}\} \ngeq \partial^*(A) = \{a_{.5}, b_{.9}\}$.

In the following proposition, we note that almost all the properties related to fuzzy supra semi boundary are analogous to their counter parts in fuzzy supra boundary and hence proofs are omitted.

Proposition 3.22: : Let A and B be any two fuzzy sets in a FSTS (X, T^*), then
(1) $s\partial^*(A^c) = s\partial^*(A)$;
(2) A ϵ FSSCS(X) $\Rightarrow s\partial^*(A) \leq A$ and
A ϵ FSSOS(X) $\Rightarrow s\partial^*(A) \leq A^c$;

(3) if $A \leq B$, then
B ϵ FSSCS(X) $\Rightarrow s\partial^*(A) \leq B$, and
B ϵ FSSOS(X) $\Rightarrow s\partial^*(B) \leq A^c$;
(4) $(s\partial^*(A))^c = sInt^*(A) \vee sInt^*(A^c)$;
(5) $s\partial^*(sInt^*(A)) \leq s\partial^*(A)$;
(6) $s\partial^*(sCl^*(A)) \leq s\partial^*(A)$;
(7) $s\partial^*(A) = sCl^*(A) — sInt^*(A)$;
(8) $sInt^*(A) \leq A — s\partial^*(A)$;
(9) $A \vee s\partial^*(A) \leq sCl^*(A)$;
(10) $s\partial^*(s\partial^*(A)) \leq s\partial^*(A)$.

In general, the converses of inequalities of the above proposition are not true as shown in the following example.

Example 3.23: Choose A = $\{a_{.9}, b_{.5}\}$, B = $\{a_{.1}, b_{.5}\}$, C = $\{a_{.5}, b_{.5}\}$, D = $\{a_{.6}, b_{.9}\}$, E = $\{a_{.9}, b_0\}$ in the FSTS (X, T^*) in example 3.18. Then calculations give
$s\partial^*(A) = \{a_{.5}, b_{.5}\} \leq A$, but A is not a FSSCS,
$s\partial^*(B) = \{a_{.5}, b_{.5}\} \leq B^c$, but B is not a FSSOS,
$s\partial^*(C) = \{a_{.5}, b_{.5}\} \leq D$, $C \leq D$, but D is not a FSSCS,
$s\partial^*(D) = \{a_{.5}, b_{.5}\} \leq B^c$, $B \leq D$, but D is not a FSSOS,
$s\partial^*(sInt^*(B)) = 0_x \ngeq s\partial^*(B) = \{a_{.5}, b_{.5}\}$,
$s\partial^*(sCl^*(A)) = 0_x \ngeq s\partial^*(A) = \{a_{.5}, b_{.5}\}$,
$sInt^*(B) = 0_x \ngeq B — s\partial^*(B) = \{a_{.1}, b_{.5}\}$,
$A \vee s\partial^*(A) = \{a_{.9}, b_{.5}\} \ngeq sCl^*(A) = 1_x$,
$s\partial^*(s\partial^*(E)) = 0_x \ngeq s\partial^*(E) = 1_x$.

Remark 3.24 : For fuzzy sets A and B in a FSST (X, T^*), $s\partial^*(A \vee B)$, $s\partial^*(A) \vee s\partial^*(B)$, $s\partial^*(A \wedge B)$, $(s\partial^*(A) \wedge sCl^*(B)) \vee (sCl^*(A) \wedge s\partial^*(B))$, $s\partial^*(A) \wedge s\partial^*(B)$ behave arbitrarily as seen in the following example.

Example 3.25: Let X = {a, b, c} be a set with a fuzzy supra topology $T^* = \{0_x, \{a_0, b_{.4}, c_{.6}\}, \{a_{.5}, b_{.3}, c_{.8}\}, \{a_{.5}, b_{.4}, c_{.8}\}, 1_x\}$. Then
FSSOS(X) = $\{0_x, 1_x\} \cup \{\{a_x, b_y, c_z\}: 0 \leq x \leq 1, 0.4 \leq y \leq 1, 0.6 \leq z \leq 1\} \cup \{\{a_x, b_y, c_z\}: 0.5 \leq x \leq 1, 0.3 \leq y \leq 1, 0.8 \leq z \leq 1\} \cup \{\{a_x, b_y, c_z\}: 0.5 \leq x \leq 1, 0.4 \leq y \leq 1, 0.8 \leq z \leq 1\}$,

FSSCS(X) = $\{0_x, 1_x\} \cup \{\{a_x, b_y, c_z\}: 0 \le x \le 1, 0 \le y \le 0.6, 0 \le z \le 0.4\} \cup \{\{a_x, b_y, c_z\}: 0 \le x \le 0.5, 0 \le y \le 0.7, 0 \le z \le 0.2\} \cup \{\{a_x, b_y, c_z\}: 0 \le x \le 0.5, 0 \le y \le 0.6, 0 \le z \le 0.2\}$.

For fuzzy sets $A = \{a_{.8}, b_{.5}, c_{.3}\}$, $B = \{a_{.5}, b_{.7}, c_{.2}\}$, $C = \{a_{.2}, b_{.5}, c_{.7}\}$, $D = \{a_{.5}, b_{.3}, c_{.8}\}$, $E = \{a_{.8}, b_{.7}, c_{.3}\}$, $F = \{a_{.2}, b_{.3}, c_{.7}\}$ in X, calculations give

(1) $s\partial^*(A \vee B) = 1_x > s\partial^*(A) \vee s\partial^*(B) = \{a_{.8}, b_{.7}, c_{.3}\}$, but
$s\partial^*(C \vee D) = \{a_{.5}, b_{.5}, c_{.2}\} < s\partial^*(C) \vee s\partial^*(D) = \{a_{.8}, b_{.7}, c_{.3}\}$;

(2) $s\partial^*(C \wedge D) = 1_x > s\partial^*(C) \wedge s\partial^*(D) = \{a_{.5}, b_{.5}, c_{.2}\}$, but
$s\partial^*(E \wedge F) = \{a_{.2}, b_{.3}, c_{.3}\} < s\partial^*(E) \wedge s\partial^*(F) = 1_x$,

(3) $s\partial^*(C \wedge D) = 1_x > s\partial^*(C) \vee s\partial^*(D) = \{a_{.8}, b_{.7}, c_{.3}\}$, but
$s\partial^*(E \wedge F) = \{a_{.2}, b_{.3}, c_{.3}\} < s\partial^*(E) \vee s\partial^*(F) = 1_x$;

(4) $s\partial^*(A \vee B) = 1_x > s\partial^*(A \wedge B) = \{a_{.5}, b_{.5}, c_{.2}\}$, but
$s\partial^*(C \vee D) = \{a_{.5}, b_{.5}, c_{.2}\} < s\partial^*(C \wedge D) = 1_x$;

(5) $s\partial^*(C \wedge D) = 1_x > (s\partial^*(C) \wedge sCl^*(D)) \vee (sCl^*(C) \wedge s\partial^*(D)) = \{a_{.8}, b_{.7}, c_{.3}\}$, but
$s\partial^*(E \wedge F) = \{a_{.2}, b_{.3}, c_{.3}\} < (s\partial^*(E) \wedge sCl^*(F)) \vee (sCl^*(E) \wedge s\partial^*(F)) = 1_x$.

4. Boundary in product related fuzzy supra topological spaces

First we recall certain definitions and results.

Definition 4.1: (K.K. Azad, 1981): Let $A \in I^X$ and $B \in I^Y$, then by $A \times B$ we denote the product fuzzy set of A and B in $X \times Y$ for which $(A \times B)(x, y) = \min\{A(x), B(y)\}$, for every $(x, y) \in X \times Y$.

Theorem 4.2: (see [1]): Let $A \in I^X$ and $B \in I^Y$, then
(1) $(A \times B) = (A \times 1) \wedge (1 \times B)$;
(2) $(A \times B)^c = 1 - (A \times B) = (A^c \times 1) \vee (1 \times B^c)$, where 1 stands for the unit fuzzy set.

Definition 4.3: A FSTS (X, T_1^*) is product related to another FSTS (Y, T_2^*), if for any $A \in I^X$ and $B \in I^Y$, whenever $\lambda^c \not\ge A$ and $\mu^c \not\ge B$ imply $(\lambda^c \times 1) \vee (1 \times \mu^c) \ge A \times B$, where $\lambda \in T_1^*$ and $\mu \in T_2^*$, there exist $\lambda_1 \in T_1^*$ and $\mu_1 \in T_2^*$ such that $\lambda_1^c \ge A$ or $\mu_1^c \ge B$ and $(\lambda_1^c \times 1) \vee (1 \times \mu_1^c) = (\lambda^c \times 1) \vee (1 \times \mu^c)$.

Lemma 4.4: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. Then for any $A \in$ FSCS(X) and $B \in$ FSCS(Y), $A \times B$ is a FSCS in the fuzzy supra product space of $X \times Y$.

Proof: $A \in$ FSCS(X), $B \in$ FSCS(Y) $\Rightarrow A^c \times 1, 1 \times B^c \in$ FSOS$(X \times Y)$.
By (2) of theorem 4.2,
$(A \times B)^c = (A^c \times 1) \vee (1 \times B^c) \in$ FSOS$(X \times Y)$.

Hence $A \times B \in FSCS(X \times Y)$.

Lemma 4.5: For $A, B \in I^X$ and $C, D \in I^Y$ one has
$(A \wedge B) \times (C \wedge D) = (A \times D) \wedge (A \times B)$.

Proof: For each $(x, y) \in X \times Y$.

$(A \wedge B) \times (C \wedge D)(x, y)$
$= \min\{(A \wedge B)(x), (C \wedge D)(y)\}$
$= \min\{\min\{A(x), B(x)\}, \min\{C(y), D(y)\}\}$
$= \min\{\min\{A(x), D(y)\}, \min\{B(x), C(y)\}\}$
$= \min\{(A \times D)(x, y), (B \times C)(x, y)\}$
$= (A \times D) \wedge (A \times B)(x, y)$.

Theorem 4.6: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's such that X is product related to Y. Then $A \in I^X$ and $B \in I^Y$
(1) $Cl^*(A \times B) = Cl^*(A) \times Cl^*(B)$;
(2) $Int^*(A \times B) = Int^*(A) \times Int^*(B)$.

Proof: (1) $A \leq Cl^*(A)$, $B \leq Cl^*(B) \Rightarrow A \times B \leq Cl^*(A) \times Cl^*(B)$; by lemma 4.4, $Cl^*(A) \times Cl^*(B)$ is a FSCS in $X \times Y$. Hence $Cl^*(A \times B) \leq Cl^*(A) \times Cl^*(B)$.

Let $\lambda_i \in T_1^*$ and $\mu_j \in T_2^*$, then
$Cl^*(A \times B) = \inf\{(\lambda_i \times \mu_j)^c : (\lambda_i \times \mu_j)^c \geq A \times B\}$
$= \inf\{(\lambda_i^c \times 1) \vee (1 \times \mu_j^c): (\lambda_i^c \times 1) \vee (1 \times \mu_j^c) \geq A \times B\}$
$= \inf\{(\lambda_i^c \times 1) \vee (1 \times \mu_j^c): \lambda_i^c \geq A \text{ or } \mu_j^c \geq B\}$
$= \min(\inf\{(\lambda_i^c \times 1) \vee (1 \times \mu_j^c): \lambda_i^c \geq A\}, \inf\{(\lambda_i^c \times 1) \vee (1 \times \mu_j^c): \mu_j^c \geq B\})$
$\geq \min(\inf\{\lambda_i^c \times 1: \lambda_i^c \geq A\}, \inf\{1 \times \mu_j^c: \mu_j^c \geq B\})$
$= \min(\inf\{\lambda_i^c : \lambda_i^c \geq A\} \times 1, 1 \times \inf\{\mu_j^c: \mu_j^c \geq B\})$
$= \min(Cl^*(A) \times 1, 1 \times Cl^*(B))$
$= (Cl^*(A) \times 1) \wedge (1 \times Cl^*(B))$
$= (Cl^*(A) \wedge 1) \times (1 \wedge Cl^*(B))$ [By lemma 4.5]
$= Cl^*(A) \times Cl^*(B))$.

(2) follows from (1) as $(Cl^*(A))^c = Int^*(A^c)$

Theorem 4.7: : Let (X, T_1^*) and (Y, T_2^*) be two FSTS's such that X is product related to Y. Then for $A \in I^X$ and $B \in I^Y$
$\partial^*(A \times B) = (\partial^*(A) \times Cl^*(B)) \vee (Cl^*(A) \times \partial^*(B))$.

Proof: $\partial^*(A \times B)$
$= Cl^*(A \times B) \wedge Cl^*((A \times B)^c)$
$= Cl^*(A \times B) \wedge (Int^*((A \times B))^c$
$= (Cl^*(A) \times Cl^*(B)) \wedge (Int^*(A) \times Int^*(B))^c$

$= (Cl^*(A) \times Cl^*(B)) \wedge ((Int^*(A))^c \times 1) \vee (1 \times (Int^*(B))^c)$
$= (Cl^*(A) \times Cl^*(B)) \wedge (Cl^*(A^c) \times 1) \vee (1 \times Cl^*(B^c))$
$= ((Cl^*(A) \times Cl^*(B)) \wedge (Cl^*(A^c) \times 1)) \vee ((Cl^*(A) \times Cl^*(B)) \wedge (1 \times Cl^*(B^c))$
$= ((Cl^*(A) \wedge Cl^*(A^c)) \times (1 \wedge Cl^*(B))) \vee ((Cl^*(A) \wedge 1)) \times (Cl^*(B^c) \wedge Cl^*(B)))$
$= (\partial^*(A) \times Cl^*(B)) \vee (Cl^*(A) \times \partial^*(B))$.

Corollary 4.8: Let (X_k, T_k^*), k=1, 2, 3,..., n, be a family of FSTS's. If each A_k is a fuzzy set in X_k, then

$$\partial^*(\prod_{k=1}^{n} A_k) = (\partial^*(A_1) \times Cl^*(A_2) \times \ldots \times Cl^*(A_n))$$
$$\vee (Cl^*(A_1) \times \partial^*(A_2) \times Cl^*(A_3) \times \ldots \times Cl^*(A_n))$$
$$\vee \ldots \vee (Cl^*(A_1) \times Cl^*(A_2) \times \ldots \times Cl^*(A_{n-1}) \times \partial^*(A_n)).$$

Lemma 4.9: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. Then for any $A \in$ FSSCS(X) and $B \in$ FSSCS(Y), $A \times B$ is a FSSCS in the fuzzy supra product space of $X \times Y$.

Theorem 4.10: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's such that X is product related to Y. Then $A \in I^X$ and $B \in I^Y$
(1) $sCl^*(A \times B) = sCl^*(A) \times sCl^*(B)$;
(2) $sInt^*(A \times B) = sInt^*(A) \times sInt^*(B)$.

Theorem 4.11: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's such that X is product related to Y. Then $A \in I^X$ and $B \in I^Y$

$$s\partial^*(A \times B) = (s\partial^*(A) \times sCl^*(B)) \vee (sCl^*(A) \times s\partial^*(B)).$$

Corollary 4.12: Let (X_k, T_k^*), k=1, 2, 3,..., n, be a family of FSTS's. If each A_k is a fuzzy set in X_k, then

$$s\partial^*(\prod_{k=1}^{n} A_k) = (s\partial^*(A_1) \times sCl^*(A_2) \times \ldots \times sCl^*(A_n))$$
$$\vee (sCl^*(A_1) \times s\partial^*(A_2) \times sCl^*(A_3) \times \ldots \times sCl^*(A_n))$$
$$\vee \ldots \vee (sCl^*(A_1) \times sCl^*(A_2) \times \ldots \times sCl^*(A_{n-1}) \times s\partial^*(A_n)).$$

5. Fuzzy supra continuous functions and Fuzzy supra irresolute functions

Definition 5.1(see [2]): Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A mapping $f : X \to Y$ is called fuzzy supra continuous if $f^{-1}(A) \in T_1^*$ for each $A \in T_2^*$.

Definition 5.2: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A mapping $f : X \to Y$ is called fuzzy supra semi continuous if $f^{-1}(A) \in$ FSSOS(X) for each $A \in T_2^*$.

Definition 5.3: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A mapping $f : X \to Y$ is called fuzzy supra irresolute if $f^{-1}(A) \in$ FSSOS(X) for each $A \in$ FSSOS(Y).

Theorem 5.4: Let $f : X \to Y$ be a mapping between two FSTS's. Then the following are equivalent:

(1) f is fuzzy supra semi continuous;
(2) $f^{-1}(B) \in FSSCS(X)$ for each $B \in FSCS(Y)$;
(3) $sCl^*(f^{-1}(B)) \leq f^{-1}(Cl^*(B))$ for each fuzzy set B in Y;
(4) $f(sCl^*(A)) \leq Cl^*(f(A))$ for each fuzzy set A in X;
(5) $f^{-1}(Int^*(B)) \leq sInt^*(f^{-1}(B))$ for each fuzzy set B in Y.

Proof: (1) ⇒ (2): Let $B \in FSCS(Y)$, then $B^c \in FSOS(Y)$. Since f is fuzzy supra semi continuous, so

$(f^{-1}(B))^c = f^{-1}(B^c) \in FSSOS(X)$. Hence $f^{-1}(B) \in FSSCS(X)$.

(2) ⇒ (1): Similar to above.
(2) ⇒ (3): Let B be any fuzzy set in Y, then $Cl^*(B) \in FSCS(Y)$. By (2), $f^{-1}(Cl^*(B)) \in FSSCS(Y)$. Hence $sCl^*(f^{-1}(B)) \leq sCl^*(f^{-1}(Cl^*(B))) = f^{-1}(Cl^*(B))$.
(3) ⇒ (4): Let A be any fuzzy set in X and B = f(A). By (3), $sCl^*(f^{-1}(B)) \leq f^{-1}(Cl^*(B))$. Hence $f(sCl^*(A)) \leq f(sCl^*(f^{-1}f(A))) = f(sCl^*(f^{-1}(B))) \leq f(f^{-1}(Cl^*(B))) \leq Cl^*(B) = Cl^*(f(A))$.
(4) ⇒ (2): Let $B \in FSCS(Y)$ and $A = f^{-1}(B)$. By (4), $f(sCl^*(A)) \leq Cl^*(f(A)) = Cl^*(f(f^{-1}(B))) \leq Cl^*(B) = B$ and $sCl^*(A) \leq f^{-1}f(sCl^*(A)) \leq f^{-1}(B) = A$ implies $sCl^*(A) = A$. Hence $A = f^{-1}(B) \in FSSCS(X)$.
(5) ⇒ (3): Let B be any fuzzy set in Y. By (5) $f^{-1}(Int^*(B^c)) \leq sInt^*(f^{-1}(B^c)) \Rightarrow f^{-1}((sCl^*(B))^c) \leq sInt^*(f^{-1}(B)^c) \Rightarrow (f^{-1}(sCl^*(B)))^c \leq (sCl^*(f^{-1}(B)))^c \Rightarrow sCl^*(f^{-1}(B)) \leq f^{-1}(Cl^*(B))$.

(3) ⇒ (5): Similar to above.

Theorem 5.5: Let f : X→Y be a mapping between two FSTS's. Then the following are equivalent:
(1) f is fuzzy supra continuous;
(2) $f^{-1}(B) \in FSCS(X)$ for each $B \in FSCS(Y)$;
(3) $Cl^*(f^{-1}(B)) \leq f^{-1}(Cl^*(B))$ for each fuzzy set B in Y;
(4) $f(Cl^*(A)) \leq Cl^*(f(A))$ for each fuzzy set A in X;
(5) $f^{-1}(Int^*(B)) \leq Int^*(f^{-1}(B))$ for each fuzzy set B in Y.

Theorem 5.6: Let f : X→Y be a mapping between two FSTS's. Then the following are equivalent:
(1) f is fuzzy supra irresolute;
(2) $f^{-1}(B) \in FSSCS(X)$ for each $B \in FSSCS(Y)$;
(3) $sCl^*(f^{-1}(B)) \leq f^{-1}(sCl^*(B))$ for each fuzzy set B in Y;
(4) $f(sCl^*(A)) \leq sCl^*(f(A))$ for each fuzzy set A in X;
(5) $f^{-1}(sInt^*(B)) \leq sInt^*(f^{-1}(B))$ for each fuzzy set B in Y.

The following theorems give necessary conditions for fuzzy supra continuous functions and fuzzy supra irresolute functions in terms of fuzzy supra boundary.

Theorem 5.7: Let f : X→Y be a fuzzy supra continuous function. Then
$\partial^*(f^{-1}(B)) \leq f^{-1}(\partial^*(B))$ for each fuzzy set B in Y.

Proof:
$\partial^*(f^{-1}(B)) = Cl^*(f^{-1}(B)) \wedge Cl^*((f^{-1}(B))^c)$
$\leq f^{-1}(Cl^*(B)) \wedge Cl^*(f^{-1}(B^c))$
$\leq f^{-1}(Cl^*(B)) \wedge f^{-1}(Cl^*(B^c))$
$= f^{-1}(Cl^*(B) \wedge Cl^*(B^c))$
$= f^{-1}(\partial^*(B))$.

Theorem 5.8: Let f : X→Y be a fuzzy supra semi continuous function. Then
$s\partial^*(f^{-1}(B)) \leq f^{-1}(\partial^*(B))$ for each fuzzy set B in Y.

Theorem 5.9: Let f : X→Y be a fuzzy supra irresolute function. Then
$s\partial^*(f^{-1}(B)) \leq f^{-1}(s\partial^*(B))$ for each fuzzy set B in Y.

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