

## **The Application of Delphi Adapted (Weighted) Bidirectional Associative Memories (DABAM) in the Analysis of Women Empowerment through Capabilities**

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### **Abstract**

A new fuzzy technique Delphi Adapted BAM is proposed in this paper and is used to analyze the empowerment of women by adopting Amartya Sen's capability approach. DABAM is proposed to function as a multiple expert system, in that it can be used to combine any number of expert's views into one relational matrix. To create a balance between views of experts, who might have different experiences in the problem under analysis, each expert is given a weight as in Delphi method. The first section of this paper gives an introduction to what is done in the paper and the second section explains the dynamics of BAM. Section three explains the technique DABAM and in the fourth section DABAM is adapted to investigate the empowerment of women through their capabilities. Section five derives the conclusion.

**Keywords:** Bidirectional associative memory, Fuzzy Delphi method, Delphi adapted BAM, women empowerment, capability approach.

### **1. Introduction**

The Delphi method (Dalkey & Helmer, 1963) is a proven tool for collective decision making (Linstone & Turoff, 2002) for a situation in which decision needs to be made by a group of experts who might have divergent views on the topic. This method can also be called a prediction method based on expert judgment. The Delphi method is characterized by the properties like anonymity, feedback, statistical and convergence. It tries to achieve a consensus among the experts. In order to account for the amount

of fuzziness in group decision making Murray, Pipino & Gigch (1985) proposed Fuzzy Delphi method. Since then the method has found many applications.

Bi-directional associative memories (BAM) was introduced by Kosko (1988) to produce two-way associative search for stored associations and as a model BAM has been applied to analyze problems by passing information through  $M$  (forward recall) and  $M^t$  (backward recall) in which the factors that attribute to the problem can be classified into antecedent and consequent sets. In this paper, the novelty of Fuzzy Delphi Method in bringing a consensus is combined with Bidirectional associative memories so that the new technique thus obtained can function as a multiple expert system.

## 2. Bidirectional Associative Memories (BAM)

### 2.1. Neuron Fields

A group of neurons forms a field. Neural networks contain many fields of neurons.  $F_x$  denotes a neuron field which contains  $n$  neurons and  $F_y$  denotes a neuron field which contains  $p$  neurons.

### 2.2. Neuronal Dynamical Systems

The neuronal dynamical system is described by a system of first order differential equations that govern the time evaluation of the neuronal activations or membrane

potentials.  $\dot{X}_i = g_i ( X, Y, \dots )$  ,  $\dot{Y}_j = h_j ( X, Y, \dots )$  where  $x_i$  and  $y_j$  denote respectively the activation time function of the  $i^{\text{th}}$  neuron in  $F_x$  and the  $j^{\text{th}}$  neuron in  $F_y$ . The over dot denotes time differentiation,  $g_i$  and  $h_j$  are functions of  $X, Y$  etc. and  $X(t) = (x_1(t), \dots, x_n(t))$ ,  $Y(t) = (y_1(t), \dots, y_p(t))$

Define the state of the neuronal dynamical system at time  $t$ . Additive bivalent Models describe asynchronous and stochastic behavior. At each moment each neuron can randomly decide whether to change state, or whether to omit a new signal given its current activation.

The BAM is a non- adaptive, additive, bivalent neural network.

### 2.3. Bivalent Additive BAM

In neural literature, the discrete version of the earlier equations is often referred to as the Bidirectional Associative Memories or BAMs. A discrete additive BAM with threshold signal functions, arbitrary thresholds and inputs, an arbitrary but a constant synaptic connection matrix  $M$  and discrete time steps  $K$  are defined by the equations

$$x_i^{k+1} = \sum_j^p S_j(y_j^k)m_{ij} + I_i$$

$$y_j^{k+1} = \sum_i^n S_i(y_i^k)m_{ij} + I_j$$

where  $m_{ij} \in M$ .  $S_i$  and  $S_j$  are the signal functions. They represent binary or bipolar threshold functions.

#### 2.4. Synaptic connection Matrices

Let us suppose that the field  $F_x$  with  $n$  neurons is synaptically connected to the field  $F_y$  with  $p$  neurons. Let  $m_{ij}$  be a synapse where the axon from the  $i^{\text{th}}$  neuron in  $F$  terminates,  $m_{ij}$  can be positive, negative or zero. The synaptic matrix  $M$  is a  $n \times p$  matrix of real numbers whose entries are the synaptic efficacies  $m_{ij}$ . The matrix  $M$  describes the forward projections from the neuronal field  $F_x$  to the neuronal field  $F_y$ . Similarly,  $M^T$ , a  $p \times n$  synaptic matrix and describes the backward projections  $F_y$  to  $F_x$ .

#### 2.5. Unidirectional Networks

These kinds of networks occur when a neuron synoptically interconnects to itself. The matrix  $N$  is  $n \times n$  square matrix.

#### 2.6. Bidirectional Networks

A network is said to be a bidirectional network if  $M = N^T$  and  $N = M^T$ .

#### 2.7. Bidirectional Associative Memories

When the activation dynamics of the neuronal fields  $F_x$  and  $F_y$  lead to the overall stable behavior, the bi-directional networks are called as Bi-directional Associative Memories or BAM. A unidirectional network also defines a BAM if  $M$  is symmetric, that is,  $M = M^t$ . In the next section, we proceed on to give more details about this BAM.

#### 2.8. Additive Activation Models

An additive activation model is defined by a system of  $n + p$  coupled first-order differential equations that interconnects the fields  $F_x$  and  $F_y$  through the constant synaptic matrices  $M$  and  $N$  described earlier.

$$\begin{aligned} \dot{x}_i &= -A_i x_i + \sum_j^p S_j(y_j^k) m_{ji} + I_i \\ \dot{y}_j &= -A_j y_j + \sum_i^n S_i(x_i^k) m_{ij} + I_j \end{aligned}$$

$S_i(x_i)$  and  $S_j(y_j)$  denote respectively the signal function of the  $i^{\text{th}}$  neuron in the field  $F_x$  and the signal function of the  $j^{\text{th}}$  neuron in the field  $F_y$ . Discrete additive activation models correspond to neurons with threshold signal functions. The neurons can assume only two values **ON** and **OFF**. **ON** represents the signal value + 1 and **OFF** represents 0 or -1 (-1 when the representation is bipolar). At each moment each neuron define a random variable that can assume the value **ON** (+1) or **OFF** (0 or -1). The network is often assumed to be deterministic and state changes are synchronous ie an entire field of neurons is updated at a time. In case of simple asynchrony only one neuron makes a state change decision at a time. When the subsets represent the entire fields  $F_x$  and  $F_y$  synchronous state change results.

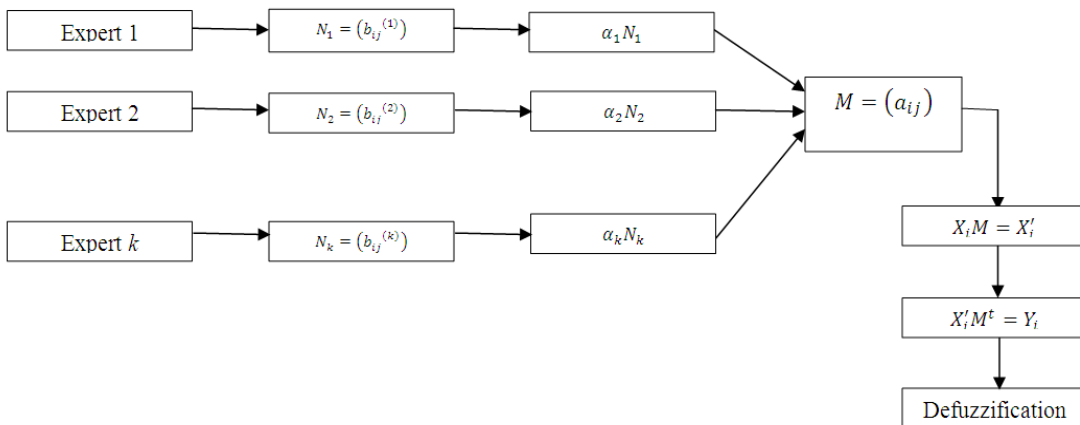
In a real life problem the entries of the constant synaptic matrix  $M$  depends upon the investigator's feelings. The synaptic matrix is given a weightage according to their feelings.

**3. Methodology**

**Delphi Adapted (weighted) BAM**

Form synaptic connection matrices  $N_1 = (b_{ij}^{(1)})$ ,  $N_2 = (b_{ij}^{(2)})$ , .....  $N_k = (b_{ij}^{(k)})$ , where  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, p$ , which represent opinions of  $k$  experts about causal relationship between the neuron field  $F_x$  with  $n$  neurons and the neuron field  $F_y$  with  $p$  neurons.

- a. Assign each expert a weight  $\alpha_n$  ranging from 1 to 5 according to his/her expertise in the problem under analysis.
- b. Multiply the weight with the corresponding synaptic connection matrix  $M_n = \alpha_n N_n = \alpha_n (b_{ij}^{(n)}) = (\alpha_n b_{ij}^{(n)}) = (a_{ij}^{(n)})$  where  $\alpha_n$  is a constant,  $n = 1, 2, \dots, k$ .
- c. Form fuzzy triangular number  $(P_{ij}, M_{ij}, O_{ij})$ . Here  $P_{ij} = \min\{a_{ij}^{(n)}\}_{1 \leq n \leq k}$ ,  $M_{ij} = \frac{1}{k} \sum_{n=1}^k a_{ij}^{(n)}$ , and  $O_{ij} = \max\{a_{ij}^{(n)}\}_{1 \leq n \leq k}$  where  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, p$  and  $a_{ij}^{(n)} \in M_n \forall n = 1, 2, \dots, k$
- d. Obtain the combined synaptic connection matrix  $M = (a_{ij})$  where  $a_{ij} = \frac{P_{ij} + M_{ij} + O_{ij}}{3}$  for all  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, p$ .
- e. Choose different input vectors keeping different nodes in ON state to obtain the dynamical system.



**Fig. 1** DABAM as a multiple expert system

#### **4. Application of DABAM to the problem**

As traditional methods of analyzing the empowerment of women have not adequately enlightened the researchers, an attempt here is made to adopt Sen's capability approach. Capability of a person reflects a person's freedom to choose between different ways of living (Sen, 2003). Already many studies have been carried out using this approach. Alkire and Black (1997) chose Life, Knowledge and appreciation of beauty, Work and play, Friendship, Self-integration, Coherent self-determination, Transcendence, and Other species as capabilities for their study. Nussbaum (2003) selected the following capabilities Life, Bodily health, Bodily integrity, Senses, imagination, and thought, Emotions, Practical reason, Affiliation, Other species, Play, Control over one's environment. To analyze the gender inequality in Western societies Robeyns (2003) had another list of fifteen attributes. The following list of capabilities is chosen by the authors of this paper after analyzing the life of twenty successful Indian women and is identified to contribute to women empowerment.

The list of capabilities are selected as the nodes of the domain space

- C<sub>1</sub> – Mental well-being and Resilience
- C<sub>2</sub> – Bodily integrity
- C<sub>3</sub> – Creative Imagination
- C<sub>4</sub> – Emotion stability
- C<sub>5</sub> – Social sensitivity
- C<sub>6</sub> – Recreational activity
- C<sub>7</sub> – Bodily control
- C<sub>8</sub> – Growth of self-regulation
- C<sub>9</sub> – Self-reorganization

The following attributes related with empowerment of women are taken as nodes of the range space:

- P<sub>1</sub> – Freedom of movement
- P<sub>2</sub> – Economic Independence
- P<sub>3</sub> – Education
- P<sub>4</sub> – Decision making power
- P<sub>5</sub> – Vocational preference
- P<sub>6</sub> – Property rights
- P<sub>7</sub> – Respect
- P<sub>8</sub> – Creating opportunities
- P<sub>9</sub> – Dignified treatment
- P<sub>10</sub> – Equality
- P<sub>11</sub> – Self-esteem
- P<sub>12</sub> – Right to privacy
- P<sub>13</sub> – Change in social system and culture
- P<sub>14</sub> – Freedom to express own thought

The Delphi process of obtaining the relational matrix is given in the following table

	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>
C <sub>1</sub>	(1.2,1.67,2)	(0,0.1,0.3)	(1.5,1.97,2.8)	(2.4,3.47,4)	(0.6,0.87,1.2)	(-4,-3.27,-1.8)	(0,0.1,0.3)
C <sub>2</sub>	(0,0.1,0.3)	(-2.8,-2.3,-1.8)	(2.4,2.93,3.2)	(1.2,1.47,1.6)	(1.2,1.73,2)	(-4,-3.67,-3)	(1.8,1.93,2)
C <sub>3</sub>	(1.2,1.87,2.4)	(-4,-3.3,-2.7)	(2.4,3.47,4)	(1.5,1.97,2.4)	(0,0.1,0.3)	(-4,-3.67,-3)	(1.5,2.5,3.2)
C <sub>4</sub>	(0.3,0.63,0.8)	(0,0.5,1.3)	(1.5,2.5,3.2)	(1.8,2.6,3.2)	(1.5,2.1,2.4)	(0,0.23,0.4)	(1.6,1.8,2)
C <sub>5</sub>	(1.5,2.23,2.8)	(3,3.4,4)	(0.9,1.5,2)	(1.5,1.97,2.4)	(1.6,1.8,2)	(3,3.67,4)	(1.8,2.33,2.8)
C <sub>6</sub>	(0,0.1,0.3)	(0.6,1,1.2)	(1.5,1.83,2)	(0,0,0)	(-4,-3.3,-2.7)	(-4,-3.4,-3)	(1.5,1.83,2)
C <sub>7</sub>	(0.9,1.9,2.4)	(1.2,1.33,1.6)	(1.5,2.23,3.2)	(1.2,4,3.2)	(0,0,0)	(0,0,0)	(3,3.67,4)
C <sub>8</sub>	(3,3.4,0.6)	(0.6,1.4,2)	(0.8,1.06,1.2)	(3,3.4,4)	(2,2.03,2.1)	(0,0,0)	(3,3.67,4)
C <sub>9</sub>	(3,3.67,4)	(0.8,0.97,1.2)	(1.2,1.53,1.8)	(3,3.4,4)	(1.5,1.833,2)	(0,0,0)	(3,3.67,4)

	P <sub>8</sub>	P <sub>9</sub>	P <sub>10</sub>	P <sub>11</sub>	P <sub>12</sub>	P <sub>13</sub>	P <sub>14</sub>
C <sub>1</sub>	(0.6,0.73,0.8)	(1.5,1.833,2)	(1.5,1.967,2.4)	(3,3.4,4)	(-2.1,-1.367,0)	(-4,-2.067,0.8)	(0,0,0)
C <sub>2</sub>	(0,0.13,0.4)	(0.8,0.633,2)	(0,0.133,0.4)	(2.7,3.3,4)	(0,0.27,0.8)	(-4,-3.267,-2.8)	(-2,-1.467,0)
C <sub>3</sub>	(1.5,1.7,2)	(0.8,1.467,2)	(0,0.133,0.4)	(1.8,2.067,2.4)	(-4,-1.4,3)	(1.2,2.267,3.2)	(3,3.67,4)
C <sub>4</sub>	(0,0,0)	(0.8,0.967,1.2)	(0,0.133,0.4)	(1.2,2.267,3.2)	(-1.5,0.833,2)	(1.8,2.067,2.4)	(1.5,1.967,2.4)
C <sub>5</sub>	(0,0.13,0.4)	(0.8,1.2,1.6)	(3,3.4,4)	(2,2.63,3.2)	(0,0.133,0.4)	(3,3.4,4)	(0.9,1.77,2.8)
C <sub>6</sub>	(0,0,0)	(0,0,0)	(-4,-2.3,0.3)	(1.2,1.47,1.6)	(0,0.1,0.3)	(0,0.1,0.3)	(0.8,1.43,2)
C <sub>7</sub>	(-4,-3.67,-3)	(3,3.67,4)	(0.4,0.7,0.9)	(1.5,2.23,3.2)	(0,0.2,0.6)	(0,0,0)	(0.4,0.9,1.5)
C <sub>8</sub>	(0.8,1.033,1.5)	(0.9,2.57,3.6)	(2.2,5,2.8)	(1.5,3.17,4)	(2.2,17,2.4)	(1.2,1.33,1.6)	(1.5,1.83,2)
C <sub>9</sub>	(1.5,1.56,1.6)	(1.2,2.27,3.2)	(1.6,2.13,2.4)	(2.1,3.1,4)	(2.4,2.73,3)	(1.6,1.8,2)	(0.8,1.2,1.6)

The relational matrix thus obtained is given by  $M$

$$M = \begin{matrix} & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{matrix} & \left( \begin{matrix} 1.6 & 0.1 & 2.1 & 3.3 & 0.9 & -3 & 0.1 & 0.7 & 1.8 & 2 & 3.5 & -1.2 & -1.8 & 0 \\ 0.1 & -2.3 & 2.8 & 1.4 & 1.6 & -3.6 & 1.9 & 0.2 & 1.1 & 0.2 & 3.3 & 0.36 & -3.4 & -1.2 \\ 1.8 & -3.3 & 3.3 & 2 & 0.1 & -3.6 & 2.4 & 1.7 & 1.4 & 0.2 & 2.1 & -0.8 & 2.2 & 3.6 \\ 0.6 & 0.6 & 2.4 & 2.5 & 2 & 0.2 & 1.8 & 0 & 1 & 0.2 & 2.2 & 0.4 & 2.1 & 2 \\ 2.2 & 3.5 & 1.46 & 2 & 1.8 & 3.6 & 2.3 & 0.2 & 1.2 & 3.5 & 2.6 & 0.2 & 3.5 & 1.8 \\ 0.1 & 0.9 & 1.8 & 0 & 3.3 & -3.5 & 1.8 & 0 & 0 & -2 & 1.4 & 0.1 & 0.1 & 1.4 \\ 1.7 & 1.4 & 2.3 & 2.2 & 0 & 0 & 3.6 & 3.6 & 3.6 & 0.7 & 2.3 & 0.3 & 0 & 0.9 \\ 3.5 & 1.3 & 1 & 3.5 & 2 & 0 & 3.6 & 1 & 2.4 & 2.4 & 2.9 & 2.2 & 1.4 & 1.8 \\ 3.6 & 1 & 1.5 & 3.5 & 1.7 & 0 & 3.6 & 1.6 & 2.2 & 2 & 3 & 2.7 & 1.8 & 1.2 \end{matrix} \right) \end{matrix}$$

Consider an input vector (1 0 0 0 0 0 0 0) where the first attribute *mental well-being and resilience* is kept in ON state. We proceed as follows to obtain the limit point of the dynamical system:

$$\begin{aligned}
 S(X_k) &= (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \\
 S(X_k) \cdot M &= (1.6 \ 0.1 \ 2.1 \ 3.3 \ 0.9 \ -3 \ 0.1 \ 0.7 \ 1.8 \ 2 \ 3.5 \ -1.2 \ -1.8 \ 0) \\
 &\rightarrow (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0) = y_{k+1} \\
 S(y_{k+1}) \cdot M^T &= (6.8 \ 4.7 \ 4.1 \ 4.7 \ 4.6 \ 1.4 \ 4.5 \ 6.4 \ 6.5) \\
 &\rightarrow (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1) = X_{k+2} \\
 S(X_{k+2}) \cdot M &= (5.2 \ 1.1 \ 3.6 \ 6.8 \ 2.6 \ -3 \ 3.7 \ 2.3 \ 4 \ 4 \ 6.5 \ 1.5 \ 0 \ 1.2) \\
 &\rightarrow (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0) = y_{k+3} \\
 S(y_{k+3}) \cdot M^T &= (6.8 \ 4.7 \ 4.1 \ 4.7 \ 4.6 \ 1.4 \ 4.5 \ 6.4 \ 6.5) \\
 &\rightarrow (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)
 \end{aligned}$$

The binary pair  $\{(00010000001000), (100000001)\}$  gives the limit point for the dynamical system. The following table gives different limit points when we take different input vectors.

Input vector	Limit point
(100000000)	(00010000001000), (100000001)
(010000000)	(00100000001000), (110000000)
(001000000)	(001000000000001), (001100000)
(000100000)	(000100100000000), (000100111)
(000010000)	(100000100000000), (000010011)
(000001000)	(00000010001000), (000001011)
(000000100)	(00000010100000), (000000110)
(000000010)	(100000100000000), (000000011)
(000000001)	(100000100000000), (000000011)

### 5. Conclusion

Inferences can be made from the list of limit points enumerated in the table above. For the first input vector, the limit point implies that when  $C_1$  is kept in ON state, it pushes  $C_9$  also to ON state and they, together, affect  $P_4$  and  $P_{11}$  in the range space. That is, the capabilities Mental well-being and Resilience and Self-reorganization affect Decision making power and Self-esteem of women which are vital for their empowerment.

It can be observed that when  $C_4$  is kept in ON state, it turns ON four more attributes in the domain space itself, which is the maximum, and two nodes  $P_4$  and  $P_7$  in the range space. Therefore  $C_4$  (Emotion stability) plays a crucial role in women empowerment.

Next in the list are  $C_5$  and  $C_6$  which, when kept in ON state, push three nodes in the domain and two in the range space to ON state. Therefore they, Social sensitivity

and Recreational activity assume the next importance. Similar inferences can be made from the table.

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