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A Note on K-Domination in Fuzzy Graphs

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Abstract

A set D⊆ V of a fuzzy graph G = (V,σ, μ) is a fuzzy k-dominating set of G if for every node u ∊ V-D there exist at least k strong arcs (u,v) for v ∊ D. The minimum fuzzy cardinality of a fuzzy k-dominating set in G is called the fuzzy k-dominating number of G. In this Paper, a fuzzy k-dominating set is defined and also an algorithm to find the k-dominating set for a fuzzy graph is formulated. Some important results are also discussed.

Key Words: Fuzzy k-dominating set, Strong arcs, Strength of connectedness, Neighborhoods.

Introduction
Generally, Graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. When there is a vagueness in the description of the relationship between objects, It is natural that we need to design a Fuzzy Graph Model. Rosenfield(1975) introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Somasundaram A. and Somasundaram S. discussed about domination in fuzzy graph. In this Paper, a fuzzy k-dominating set is defined and also an algorithm to find the k-dominating set for a fuzzy graph is formulated.

Preliminaries
Let us see the following basic definitions in a fuzzy graph.

Definition 1: Fuzzy graph
Let V be a finite non-empty set. A fuzzy graph G = (σ , μ) is a pair of functions

$\sigma: V\rightarrow[0,1]$ and $\mu : V\times V\rightarrow[0,1]$ where $\mu(u, v) \leq \sigma(u) \Lambda\ \sigma(v)$ for $u, v \in V$.

Definition 2: Underlying (crisp) graph
The underlying (crisp) graph of $G = (\sigma , \mu)$ is denoted by $G^* = (V,E)$ where $V= \{u \in V / \sigma(u)>0\}$ and $E=\{(u , v) \in V \times V / \mu(u, v) >0\}$.

Definition 3: Maximum Degree of a node
The maximum degree of a node u in a fuzzy graph is defined as (G) = max{d(u)/ for all u in V(G)} where d(u) denotes the degree of the node u.

Definition 4: The order p and size q
The order p and size q of the fuzzy graph $G = (\sigma , \mu)$ are defined by $p= \Sigma\, \sigma(v)$ and $q= \Sigma\, \mu (u ,v)$ $,v \in V$ and $(u, v) \in E$.

Definition 5: Strength of the connectedness between two nodes.
The strength of the connectedness between two nodes u,v in a fuzzy graph G is

$\mu^{\infty} (u,v) = \sup\{ \mu^k(u,v):k=1,2,3..\}$ where

$\mu^k(u,v)= \{\mu(u,u_1)\wedge \mu(u,u_2)\wedge....\mu(u_{k-1},v)\}$.

Definition 6: Complement of a fuzzy graph
The complement of a fuzzy graph $G=(\sigma , \mu)$ is a fuzzy graph $G=(\sigma ,\bar{\mu})$ where $\sigma = \sigma$ and $\bar{\mu} (u ,v)=\sigma(u) \Lambda \sigma(v)-\mu(u ,v)$
for all u , v in V.

Definition 7: Neighborhood of a node
Let G be a fuzzy graph. The neighborhood of a node v in V is defined by $N(v)= \{u \in V / \mu(u,v) = \sigma(u) \Lambda \sigma(v)\}$. The scalar cardinality of N(v) is the neighborhood degree of a node v which is denoted by $d_N(v)$.

Definition 8: Strong Arc
An Arc e=(x,y) of a fuzzy graph is called Strong Arc if $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$. If $\mu(u,v)=0$ for every v in V, then u is called an isolated node.

Definition 9: Fuzzy dominating set
Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then $D \subseteq V$ is said to be a fuzzy dominating set of G if for every $v \in V - D$ there exist u in D such that $\mu(u,v) = \sigma(u) \Lambda \sigma(v)$.

Definition 10: Fuzzy domination number
The minimum fuzzy cardinality of a fuzzy dominating set is called the fuzzy domination number of G and is denoted by $\gamma_f(G)$.

Definition 11: Fuzzy Cardinality
The fuzzy cardinality of a fuzzy subset D of V is $|D|_f = \sum_{v \in D} \sigma(v)$.

Let us now define fuzzy k-dominating set and fuzzy k-domination number of a fuzzy graph.

Definition 12: Fuzzy k-domination set
A set $D \subseteq V$ of a fuzzy graph $G = (V,\sigma,\mu)$ is a fuzzy k-dominating set of G if for every node $u \in V$-D there exist at least k strong arcs (u,v) for $v \in D$. The minimum fuzzy cardinality of a fuzzy k-dominating set in G is called the fuzzy k-dominating number γ_{fk} of G.

ALGORITHM
An algorithm to find the fuzzy k-dominating set of a fuzzy graph G is given below:
For a given fuzzy graph $G = (V,\sigma,\mu)$

- Find the strength of connectedness for all arcs (u,v) of G.
- Remove the weak arcs and G_1 be the resulting fuzzy graph.
- Choose the node n_1 in $V(G_1)$ such that degree of $n_1=\Delta(G_1)$ and collect the neighbors of n_1 as N_1.
- Choose the node n_2 in $V(G_1)-\{n_1\}$ such that degree of $n_2= \Delta(G_2)$ [Here $G_2= G_1-\{n_1\}$] and collect the neighbors of n_2 as N_2.
- Repeat previous step until all the remaining nodes are isolated.
- For $k = \min\{ |N_1| , |N_2|\ldots\}$, the remaining isolated vertices form the k-dominating set for the fuzzy graph $G = (V,\sigma,\mu)$.

Proposition 1:
Let n, k be positive integers and G be a fuzzy graph such that (G) $\geq \frac{n+1}{n} k - 1$.
Then, $\gamma_{fk}(G) \leq \frac{np}{n+1}$.

Proof:
Let $G= (\sigma,\mu)$ be a fuzzy graph with the underlying crisp graph $G^* = (V,E)$ where
$\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, $E \subseteq V \times V$
are mapping. Assume that $|V| = p$.

Let $v_1,v_2,\ldots,v_{n+1}$ be a partition of V(G) into n+1 subsets. Now, Consider the fuzzy spanning sub graph $H(\sigma',\mu')$ of $G= (\sigma,\mu)$ such that $\sigma' = \sigma'$ and $\mu' \leq \mu$.Then
$\sigma'(u) = \sigma'(u)$ for u in V and $\mu'(e) \leq \mu(e)$ for every e in E.

Define $E(H) = E(G) - \bigcup_{i=1}^{n+1} E(\langle v_i \rangle)$ and
$\langle v_i \rangle$ is the sub graph induced by the node v_i.

For every node x in V, We have (by [2])

$$(d)_H(x) \geq \frac{n}{n-1} (d)_G(x)$$
$$= \frac{n}{n-1}(\frac{n}{n-1} K - 1))$$
$$= k - -\frac{n}{n+1} = k(\text{ k is very small})$$
$$\Rightarrow (d)_H(x) \geq k. \text{---------------------} \quad (1)$$

Assume without loss of generality that $|V1| = \max_{1\leq i\leq n+1} |Vi|$. From(1)
$V(G) - V_1$ is a fuzzy k-dominating set.

$\gamma_{fk}(G) = | V(G) - V_1| \leq |V(G)| - |V_1|$

$= p - \frac{p}{n+1}$

$= \frac{np+p-p}{n+1} = \frac{np}{n+1}$

Hence $\gamma_{fk}(G) \leq \frac{np}{n+1}$.

Proposition 2:
In a fuzzy graph G if (G) $\geq 2k - 1$
then, $\gamma_{fk}(G) \leq \frac{p}{2}$.

Proposition 3:
If $G(\sigma, \mu)$ is a fuzzy graph and k an integer such that $1\leq k\leq \boldsymbol{\delta}(G)-1$ then $\gamma_{fk+1}(G) \leq \frac{n+\gamma_{fk}(G)}{2}$ Here n is the number of nodes of a fuzzy graph G.

Conclusion

In this Paper, a fuzzy k-dominating set is defined and also an algorithm to find the k-dominating set for a fuzzy graph is formulated. Some important results regarding the fuzzy k-domination number of a fuzzy graph are discussed.

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