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Land-Water Distribution and the Pear-Shape of the Earth

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Abstract

The orbital analysis of artificial Earth satellites have revealed the existence of a strong third harmonic in the gravitational potential of the Earth which gives its 'pear-shape' in the averaged meridional section. Various attempts to explain this pear-shape based on physical principles have proved to be inconclusive. In this study, a new approach based upon statistical correlation between variables has been advocated. It is shown that even though geoid highs and lows do not generally correspond to the locations of the continents and oceans, the average land-water distribution bears significant correlation with the geoid heights. Specifically, the distribution of water is directly correlated to the geoid heights whereas the distribution of land is inversely correlated with the latter. This analysis is consistent with the principle of mass loading and is also able to locate the 'stem' of the pear at the North pole. It is suggested that the correlation between land-water distribution and the pear-shape of the Earth is causal rather than coincidental.

1. INTRODUCTION

The shape of the Earth, like that of any other sizeable heavenly body, is determined largely by gravitation and centrifugal force of rotation. To the first approximation, owing to gravitational force alone, the Earth is spherical. To the second approximation, due to gravitation and centrifugal force of rotation, the Earth is an oblate spheroid, symmetrical about its axis of rotation. The surface of this oblate spheroid is referred to as the *reference ellipsoid*. Due to other factors such as inhomogeneities, internal convection and surface features, etc., however, the actual surface of the Earth departs slightly but significantly from the reference ellipsoid. The actual surface is called *geoid*, which is an equipotential surface. It coincides with the surface of the oceans, and extends under the continents to the level to which ocean water would settle if connected to the ocean by open channels such as the straights of Gibraltar, Dardanelles and Bosphorus.

Our knowledge about the actual shape of the Earth was greatly refined by the orbital analysis of the earliest artificial satellites. First, the orbital analysis of Sputnik 2 yielded a more accurate value of the oblateness of the Earth [1]. Next, the orbital analysis of Vanguard 1 uncovered the first indication of the departure of the Earth's figure from the reference ellipsoid: the averaged meridional section of the Earth was '*pear-shaped*' with the 'stem' of the pear at the North pole [2]. A sketch of this section (adapted from [3]) is shown in Fig. 1. The highest point of this geoid of 18 m is located at the North pole whereas the lowest point of the geoid of-28 m is at the South pole, thus affirming the position of the stem. It is to be reminded that the meridional section representing the pear-shape is averaged over all longitudes and that there are pronounced variations of the geoid over the globe. Whilst various attempts have been made to explain the pear-shape of the Earth, it may safely be stated that no generally accepted interpretation of this configuration has been put forward. Also puzzling is the question as to why is the stem of the pear at the North pole instead of the south? [4].

In order to explain the pear-shape of the Earth, various hypotheses have been made. Munk and McDonald [5] raised the possibility that density variations in the mantle, perhaps unrelated to the distribution of continents, are important in determining the gravitational coefficients of the lower order. Cook [6] speculated that the low-order harmonics could be undulations at the boundary between the solid mantle and the fluid metal core. Lee and McDonald [7] inferred that there might be a correlation between geoidal lows and high heat flow and vice-versa. More plausible theories have focused on the South polar region. King-Hele [3] speculated that alternate melting and solidifying of the south polar ice over the ages may have some bearing on the third harmonic of the geoid. Wang [8] proposed that the pear-shape of the Earth is the result of the asymmetrical distribution of land and sea and the re-distribution of sea water from melting of ice-caps world-wide. Kaula [9] asserted that the great Antarctic low is caused by a mass deficiency owing to insufficient aesthenospheric flow towards the South polar region. Khan and O'Keefe [10] suggested that a relationship between the Antarctic low and the great shrinkage of the Antarctic ice-cap around 4-5 million years ago is the principal cause of this harmonic. However, it may safely be stated that none of the above hypotheses seems to be entirely satisfactory. In this paper, we show that there exists a significant correlation between land-water distribution and the pear-shape of the Earth; and since the former is an independent quantity, it may well have created the latter.

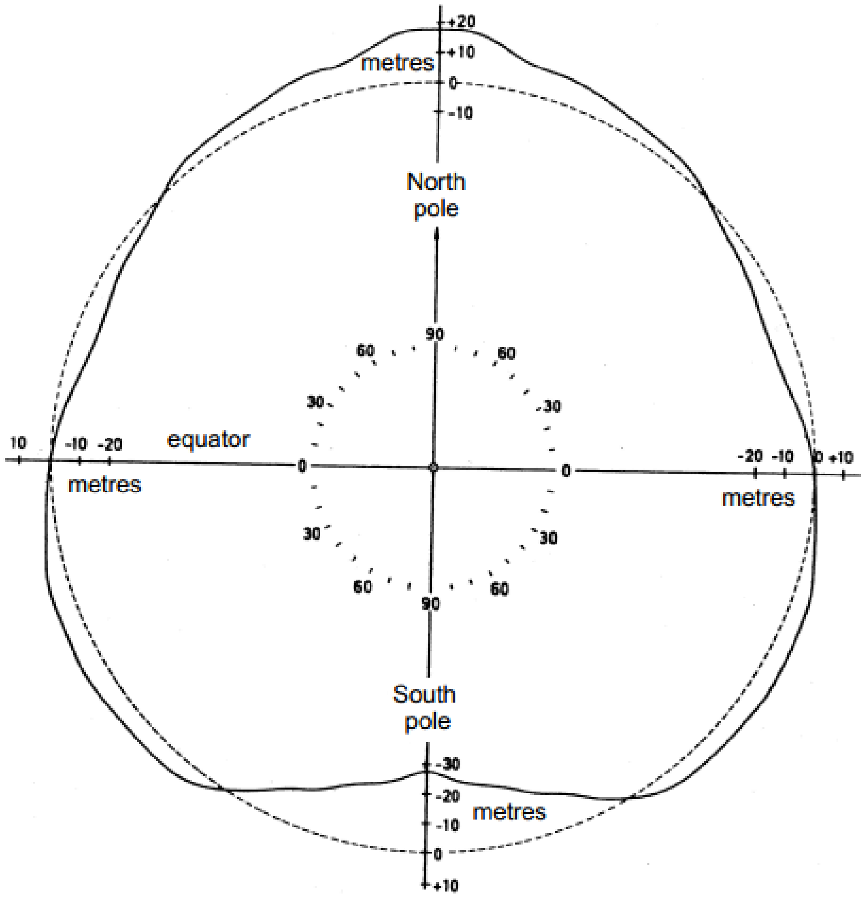

Fig. 1. Average meridional section of Earth's geoid showing its pear-shape.

2. POTENTIAL THEORY

The figure of the Earth is studied by solving Laplace's equation of gravitational potential theory at all points external to the Earth's surface. It is natural to use spherical polar coordinates (r, θ, ϕ) in a reference system with origin at the Earth's centre with the z-axis along the rotational axis of the Earth, the zenith angle (co-latitude) θ measured from the North pole, and the azimuth angle (longitude) ϕ measured from an arbitrary plane, usually the prime meridian. Laplace's equation for the gravitational potential V in this coordinate system is:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0 \qquad (1)$$

The solution to Eq. (1) is given by [11]:

$$V(r,\theta,\phi) = -\frac{GM}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n J_n P_n(cos\theta)\right] + U(r,\theta,\phi) \qquad (2)$$

where G is the universal gravitational constant, M the mass of the Earth, a the reference (equatorial) radius of the Earth, J_n's are dimensionless coefficients, Pn is the Legendre's function of the nth order and $U(r, \theta, \phi)$ consists of spherical harmonic terms. The last terms are disregarded when only latitudinal variations are considered. The lowest orders of the Legendre's polynomial expansion are the following:

$$P_1(cos\theta) = cos\theta \quad (3)$$

$$P_2(cos\theta) = \frac{1}{2}(3cos^2\theta - 1) \quad (4)$$

$$P_3(cos\theta) = \frac{1}{2}(5cos^3\theta - 3cos\theta) \quad (5)$$

$$P_4(cos\theta) = \frac{1}{8}(35cos^4\theta - 30cos^2\theta + 3) \quad (6)$$

and

$$P_5(cos\theta) = \frac{1}{8}(63cos^5\theta - 70cos^3\theta + 15cos\theta) \quad (7)$$

For our choice of coordinate system with the origin at the Earth's centre, J_1 is zero. Satellite orbital data indicate that J_2= 1.0826 × 10^{-3}; and J_3 =-2.53 × 10^{-6} [3]. The $P_n(cos\theta)$ are called 'zonal harmonics' and describe the average shape of the Earth's north-south section. $P_2(cos\theta)$ accounts for the oblateness of the Earth and $P_3(cos\theta)$ defines its 'pear-shape'.

The last term in Eq. (2) represents spherical harmonic solutions containing Associated Legendre functions which describe the global geoid heights from the reference ellipsoid. Since the launch of the first satellites, geoid height estimations have improved continuously. Today, the EGM 2008 model [12] is widely used by researchers. Recently, the GRIM5-S1 model [13] has further improved the geoid height accuracies. Fig. 2 (adapted from [15]) is a world-wide geoid map based on the latest model. A close inspection reveals striking extensions of the geoid highs and lows in generally north-south directions. There is a broad low from central Eurasia down to south-eastern Indian Ocean, with an intense low over the Indian sub-continent and north Indian Ocean. There is another broad low over North America which branches out southwards into eastern Pacific Ocean and western Atlantic Ocean. There is an intense high in the western Pacific Ocean; another intense high over north Atlantic Ocean; and a third extensive high over the waters between Africa and Antarctica. The deepest part of the geoid of -105 m is located on north Indian Ocean, just south-west of Cape Comorin, whereas the highest part of the geoid of 85 m is found under New Guinea.

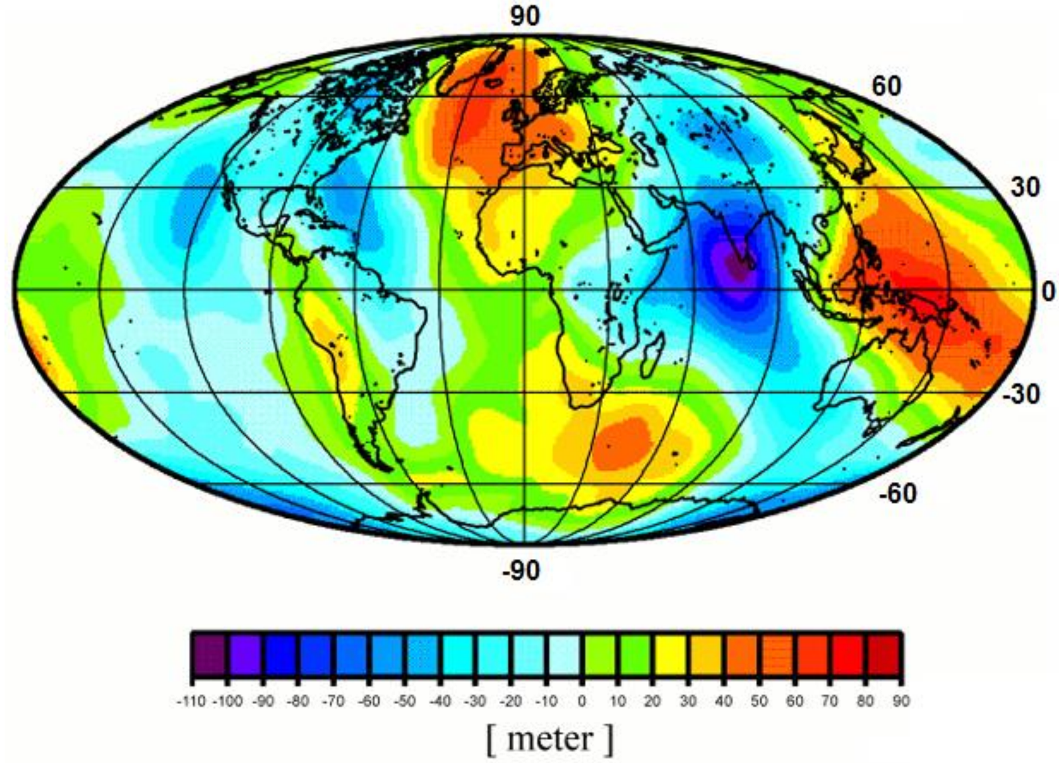

Fig. 2. Geoid map of the world based on latest satellite orbital data.

3. LAND-WATER DISTRIBUTION AND THE PEAR-SHAPE OF THE EARTH

At a first glance, the geoid highs and lows bear no particular correlation with continents and oceans, except for the polar regions. This has led the authors Munk and McDonald [5] Cook [6] and Stacey [15] to conclude that there is no correlation of geoid with continentality. Now, lowering of sea level by *mass loading* is a well-defined principle in geophysics (cf. [16]). If one assumes that continents represent mass loading, then geoid lows are expected to be located under the continents. On the other hand, if one associates water bodies to the absence of mass loading, then geoid highs will be expected on the ocean surfaces. The Antarctic low and the Arctic Ocean high are readily explained this way. However, other than these two instances, continents and oceans do not correspond to geoid lows and highs exactly.

In this paper, we reveal the finding that there is actually a greater correlation between land-water distribution and geoid of the Earth than what prima facie evidence suggests. First, the three great geoid highs are all located largely over ocean waters. And even though the most intense geoid low is also found over the water body of north Indian Ocean, it is not far from the centroid of the 'world continent' comprising Africa, Eurasia and Australasia. Yet another facet of correlation between land-water distribution and geoid height is uncovered if one considers the distribution of land-water averaged over all longitudes. This is perfectly logical since the pear-shape itself represents the north-south section averaged over the longitudes.

In this study, we examine three variables and analyze their latitudinal/co-latitudinal variations:

(1) The average geoid height h (in metres) as a function of the co-latitude θ;

(2) The third harmonic of Legendre function $P_3(cos\theta)$ defined by Eq. (5); and

(3) The percentage of (*water –land*) as a function of co-latitude w.

It may be well to explain the third quantity a little further: Technically, it is the percentage of excess of water over land. We have:

$$w = (water - land)\% = water\% - land\% \tag{8}$$

Since

$$land\% + water\% = 100 \tag{9}$$

we can also write:

$$w = 2water\% - 100 \tag{10}$$

and

$$w = 100 - 2land\% \tag{11}$$

Eq. (10) shows that w is, alternatively, a measure of the percentage of water. Eq. (11) further shows that w is also a negative measure of the percentage of land. Thus there exists three interchangeable definitions of w, all of which represents the land-water distribution of the Earth as a function of the co-latitude θ.

In Fig. 3, three variables are plotted against the co-latitude θ: (1) h multiplied by 5; (2) $P_3(cos\theta)$ magnified by 100; and (3) w. The values for h are taken from Ref. [3] and those for w are calculated from Ref. [17]. The multiplication factors of the first two quantities are chosen such that their values fall within the same range. The third quantity was defined in such a way that it too falls within the same range. This allows one to better visualize the similarities or dissimilarities between the three variables. (The justification of multiplying the first two variables and translating the third will soon become apparent.) The striking resemblance between the first two variables and their curious similarity with the third suggest the possibility of causal relationship among the three variables.

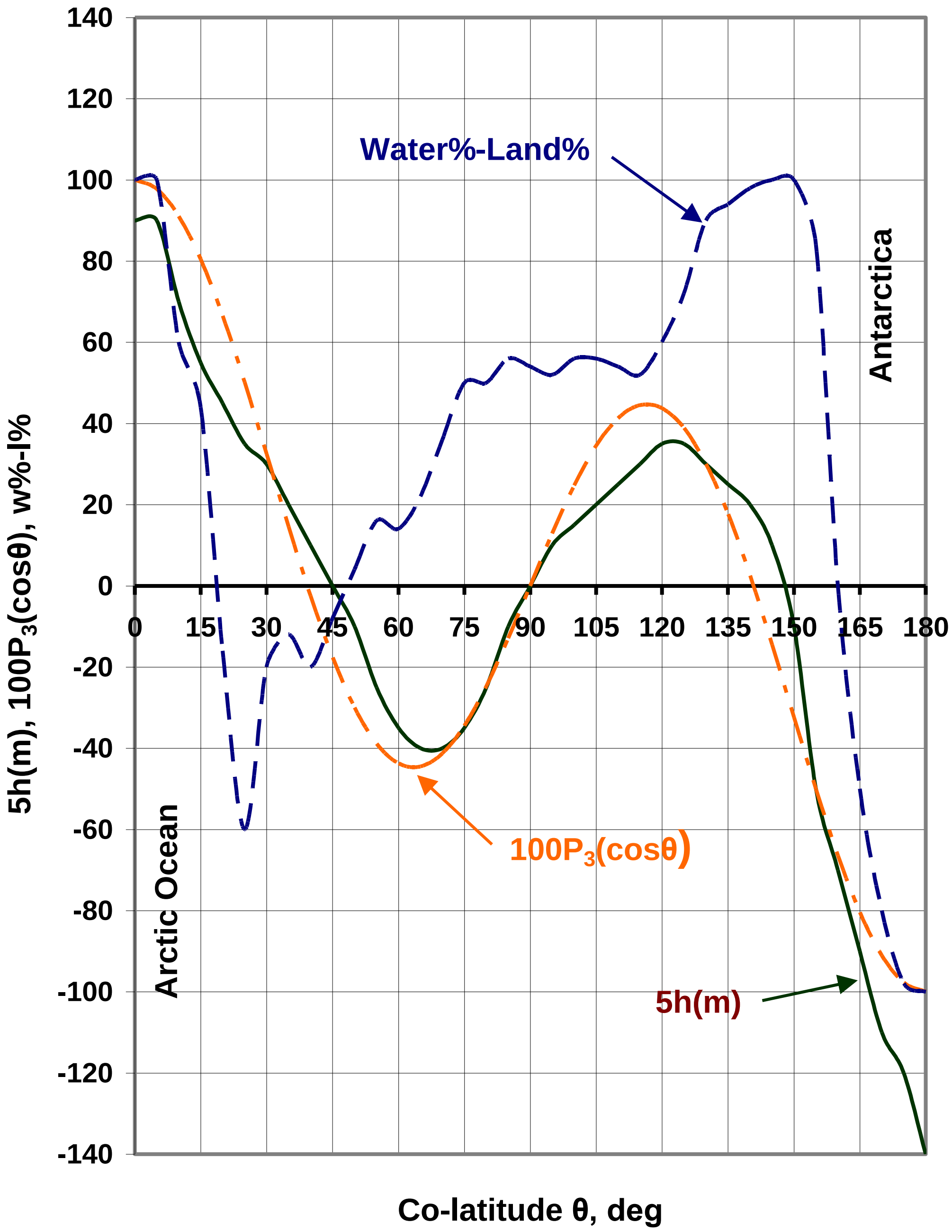

Fig. 3. Co-latitudinal variations of geoid height, third harmonic of Legendre function and land-water distribution of the Earth.

The *correlation coefficient* (cc) denoted by r is a quantity which illustrates a measure of dependence of some sorts between two variables. Given n values of a set of two variables x and y, their cc is defined by [18]

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)}\sqrt{(\sum y^2 - n\bar{y}^2)}} \tag{12}$$

where$\bar{x}$ and $\bar{y}$ are the average values of x and y respectively, and the summation Σ runs from 1 to n. We can now go to the business of calculating the cc's between the variables $5h$, $100P_3$ and w. Since the cc is independent of scale (multiplication by a constant) and origin (translation by a constant) [19], the cc's obtained will be the same as those between h, P_3 and w. Stated mathematically: $r\,(5h, 100P_3) = r\,(h, P_3)$; $r\,(w, 5h) = r\,(w, h)$; and $r\,(w, 100P_3) = r\,(w,P_3)$.

In this study, the values of the three variables for every 5^o of co-latitude θ were used as data points. The results are: $r(h, P_3) = .963$; $r(w, h) = .630$; and $r\,(w, P_3) = .516$. The very high positive value of $r(h, P_3)$ indicates that the geoid is mainly shaped by the third harmonic, i.e., it has a predominantly pear-shape. The significantly large positive value of $r(w, h)$ indicates that there is a significant inter-dependence between the variables w and h. By definition (10), this means that a positive distribution of water is directly correlated with the geoid height h; whereas by definition (11), it also means that a positive distribution of land is inversely correlated with the latter. Further, since w, representing the land-water distribution, is an independent variable, it is very likely that this quantity is responsible for that dependence and may well have created the geoid heights in the first place. Finally, $r\,(w, P_3)$ is slightly smaller than $r(w, h)$. This merely affirms that if the land-water distribution w created the geoid heights h, it created mostly, but not entirely, the third harmonic P_3. In short, the similarity between the land-water distribution, on one hand, and the geoid and the pear-shape of the Earth, on the other, may well be causal rather than coincidental.

We can add that the similarity between land-water distribution and the pear-shape of the Earth is fully consistent with the principle of mass loading. The South polar region, occupied by Antarctica, is entirely land and w is minimum. Because of the positive correlation between w and h or the negative correlation between *land%* and h, for that matter, the geoid heights are minimum there. The diametrically opposite North polar region, on the other hand, is entirely ocean and w is maximum. The geoid heights are maximum here because of the positive correlation between *water%* and h. Further, except for the South polar region, the Southern hemisphere is mostly ocean, where the main bulge of the 'pear' is found. Finally, according to either the positive correlation between *water%* and geoid heights or the negative correlation between *land%* and geoid heights, the 'stem of the pear' is located at the North pole.

4. DISCUSSION

The pear-shape of the Earth remains one of the unsolved problems of the Geophysical era. Attempts at the solution based on physical principles have been far from satisfactory. The difficulty is rooted in the incomplete knowledge of the Earth's mantle and the crust, complexities of the geophysical processes taking place there, geological history, and the uncertainties associated with them. Given these factors, it is safe to say that a satisfactory solution based on physical principles will remain elusive in the foreseeable future. Hence a different approach, a statistical one, was taken in this study. At least, a possible causality between the land-water distribution and the pear-shape of the Earth was established. It is hoped this finding will be one of the guiding principles for any future researchers attempting to find a solution to this outstanding problem in Geophysics.

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