International Journal of Oceans and Oceanography
ISSN 0973-2667 Volume 10, Number 2 (2016), pp. 205-217

http://www.ripublication.com

Turbulent Mixing Model based on Similarity Theory

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Abstract

This paper considers the similarity theory from [5] modified for calculating turbulence properties in the upper ocean. This modification of similarity theory implies using the integral function of stratification instead of heat flow, which lets us apply it to dealing with high thermal lag environments. Prior model formulations based on this theory are cited and their respective drawbacks discussed. What is proposed is a new scheme for turbulence factor calculation proceeding from similarity theory. In order to restrain the turbulence factor's linear growth under neutral stratification, the study uses a function obtained from the model of planetary boundary layer, based on the equation of turbulence kinetic energy balance. Thus, the new version of the scheme will provide us with results which are in full compliance with the commonly accepted boundary layer theory, under indifferent stratification conditions. The paper describes experimental results on the upper ocean model, with reliance on automatic buoy data, where the suggested scheme and other, more commonly applied calculation algorithms for turbulence factor were used. The paper demonstrates that the suggested model has certain advantages over its likes in the equatorial belt, namely it exhibits the structure of temperature profiles in the upper ocean in the way that is most close to available empirical data.

Keywords: upper ocean layer, turbulent mixing, convection, turbulent coefficient.

INTRODUCTION

One of the specific properties of vertical turbulent flow in ocean is that it most often occurs in stable environments. And whereas density gradients in the sub-surface quasi-homogeneous layer are relatively low, which allows us to use conventional turbulence description methods, turbulence in the thermocline is practically absent, if taken in its classic sense, as turbulent flows are suppressed by strong stratification. Still, even in the thermocline, turbulent mixing is significantly more intensive that molecular diffusion. This is the reason why heat and salt diffusion through the thermocline has a determinative impact on the evolution of the upper ocean. We can single out the key mechanisms of turbulence generation in the thermocline:

- internal wave collapse;
- local dynamic instability;
- double diffusion;
- secondary motion (e.g. Langmuir circulation).

Since the now common ocean models are not capable of explicitly providing solutions to the aforementioned processes, taking the latter into account in water mass circulation is carried out via parameterization. Turbulent mixing parameterization lies in calculating the turbulence factor on the basis of complexes of large-scale variables (flow velocity, temperature, salt content, density, etc.). We can distinguish several basic requirements for such parameterization:

- proceeding from reasonable physical relations, including those derived from similarity theory, dimension theory and differential equations;
- consistency with experimental data;
- calculation efficiency;
- easiness of testing and debugging.

The last two requirements are most urgent for the schemes used in ocean circulation models that are highly demanding on computer resources. Testing and debugging procedure implies carrying out experiments for defining optimal values of adjustment coefficients; it is the more complicated the more such factors are involved.

Up till now there have been developed several dozen parameterization versions for vertical turbulent exchange in the upper ocean, based on different principles (see reviews in [1 - 2]). This impressive amount of approaches testifies to the fact that the problem has not been convincingly solved yet. A good example of this can be given, as follows: the most common schemes (like in [3-4]) rely on atmospheric boundary layer theory and involves practically no alterations to it, despite the fact that stratification conditions in the two environments are significantly different. Also, experimental data on the upper ocean are scarce, insufficiently accurate and obtained under many aggravating factors, mostly that of the unknown advection, which brings forward the requirement of subjecting time sequences to extra processing and filtering. Such procedures were carried out for the development of turbulent mixing scheme based on a similarity theory modification [5]. Similarity theory [6-7] has been used for describing turbulent conditions in the ground air since 1960ies. In [8] similarity theory is applied also to the upper ocean layer without regard for differences between the two environments, which makes the KPP scheme suggested in [8] not quite valid. [5] puts forward a formulation of similarity theory where instead of heat flow, as an external parameter, the stratification function is used based on density profile,

$$St(z) \quad \frac{g}{\ _0 v_*^2} \int_0^z (\ (z) \quad \ (0))dz \tag{1}$$

where v_* – dynamic velocity, g – gravity factor, ρ – potential density. The theory itself appears as follows:

$$D(t,z) \quad D_* f(\frac{g}{\ _0} \quad (z,t), v_*, z) \tag{2}$$

where D(t,z) can be any dynamic characteristic of turbulence: kinetic energy, dissipation rate or exchange coefficient, D_* – a typical scale for *D(t,z)* (having the same dimension). This definition is not inconsistent with the basic statements of similarity theory [6-7]. Conversely, it is a generalization, in the event that the layer under scrutiny demonstrates some density lag. In [5] it was suggested to calculate the turbulence coefficient (k) on the basis of (1-2),

$$\begin{aligned} k \quad & v_*(z \quad C_s \frac{v_*^2}{g})(1 \quad St) \quad , St \quad 0 \\ k \quad & v_*(z \quad C_s \frac{v_*^2}{g})(1 \quad St)^{0.5}, St \quad 0 \end{aligned} \tag{3}$$

where α, β, γ, C_s - adjustment coefficients and κ – Karman constant.

This algorithm was further expanded and tested in the upper ocean models [1] and Baltic sea circulation model [9]. One of the drawbacks of the definition (1-3) was a steady rise in the turbulence coefficient under neutral stratification conditions. In [9], in order to avoid this, a flow velocity shear module was used instead of dynamic velocity on the surface.

$$_* \quad z \frac{V}{z} \tag{4}$$

This correction was a forced shift from similarity theory and possibly not the best remedy for the aforementioned defect. This study suggests a new formulation for calculating turbulence factor and provide testing thereof.

Formulation of the scheme

Under neutral stratification, turbulence conditions in the ocean can be successfully simulated by models based on the equations of turbulence kinetic energy balance. Such a model was used for a stationary case under horizontal homogeneity conditions, which is a commonly adopted simplification;

$$\begin{aligned} \frac{d^2}{dz^2} \quad & \overline{k} \quad 0 \\ \frac{d^2}{dz^2} \quad & \frac{}{k} \quad 0 \end{aligned} \tag{5}$$

$$^{2} \quad ^{2} \quad k\frac{d}{dz}k\frac{db}{dz} \quad 0 \quad , \qquad - \tag{6}$$

where λ – Coriolis parameter, $k\frac{du}{dz}$ $\quad k\frac{dv}{dz}$.

The equations were closed by Kolmogorov hypotheses:

$$b^2 \quad k \quad l\sqrt{b} \quad , \qquad , \tag{7}$$

where c_n and - are constants. The characteristic scale of turbulent mixing length *(l)* was defined as follows:

$$l \qquad \frac{}{d \;/\; dz}, \qquad \frac{b}{k} \;. \tag{8}$$

Using formulae (6 - 8) we obtained a universal profile of dimensionless turbulence factor (k_n) and the function of dimensionless depth (σ),

$$L \quad \frac{v_*}{}, \; z_n \quad \frac{z}{L}, \; k_n \quad \frac{k}{v_* L} \;. \tag{9}$$

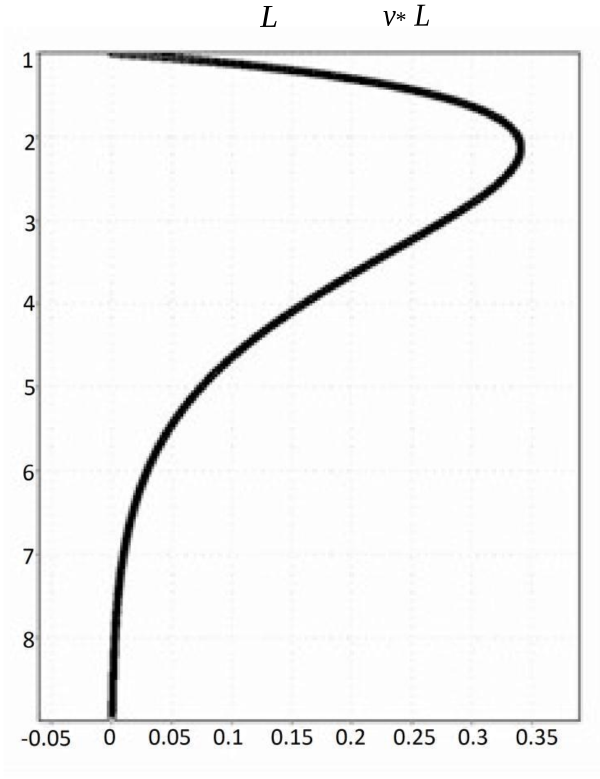

Figure 1. Dimensionless turbulence factor's vertical profile

This profile was approximated by the following function:

$$k_n \quad \frac{1}{(a \quad bz_n{}^2 \quad ce^{z_n})} \,, \tag{10}$$

using which we can plot the turbulence factor profile for neutral stratification by two parameters: dynamic velocity and Coriolis parameter.

Using (10) a new formulation for the coefficient parameterization was defined relying on similarity theory. The turbulence factor is presented as the sum of two components: $k=k_w+k_u$, where k_w— defines mixing generated by a flow of turbulent energy from the atmosphere, whereas k_u is a convective appliance.

$$k_w \quad \frac{v_*(z \quad z_0)}{(a \quad b \quad {}^2 \quad ce \quad)} (1 \quad St) \tag{11}$$

$$St \quad \frac{g}{{}_0 v_*^2} \int_{z(\min(\ ))}^{H} (\ (z) \quad \min(\ ))dz, z \quad z(\min(\ )) \,. \tag{12}$$

$$St \quad 0, z \quad z(\min(\ ))$$

To calculate k_u we find the minimal depth for which the following condition is fulfilled:

$$\frac{1}{z}\int_0^z (z)dz \quad (z \quad dz) \,. \tag{13}$$

This depth is regarded to be the depth of unstable layer z_{mix}, while the integral ρ_M

$${}_M \quad \frac{1}{z_{mix}} \int_0^{z_{mix}} (z)dz \,, \tag{14}$$

mixed layer density.

Further on we calculate the integral instability function Ut and k_u:

$$Ut \quad \frac{g}{{}_0}\int_0^z (\ (z) \quad {}_M)dz, z \quad z_{mix} \,, \tag{15}$$

$$Ut \quad 0, z \quad z_{mix}$$

$$k_u \quad z_{mix}\sqrt{Ut} \,, \tag{16}$$

where γ – a constant.

Under neutral stability (St=0), the k_w will produce a result consistent with the theory of boundary layers. Under stable stratification ($St>0$), the k_w will be approaching 0 faster, but adjustment factors α,β allow us to adjust the scheme in such a way that realistic turbulent exchange is preserved in highly stratified layers.

Several constraints were also added to the scheme.

$$k \quad \max(k, 10^{\;5} m^2/s)$$
$$\frac{dv}{dz} \quad 10^{\;3}, z_n \quad 6, k \quad \max(k, 10^{\;4} m^2 \;/ s) \quad . \tag{17}$$

The physics of the first constraint are such that the vertical mixing coefficient may not be smaller that the coefficient of molecular diffusion. The second constraint pertains to simulation of mixing processes generated by velocity shear in layers below the thermocline.

TESTING OF THE MODEL

Validation of algorithm (11)-(17) was done in a model for upper ocean relying on data from buoy-based stations within the project TAO-TOGA [10-12]. Data sequences comprise: velocity and direction of wind, air temperature and moisture, pressure, incoming short-wave/long-wave radiation, air pressure, ocean temperature and salt content at different layers. Proceeding from data provided by three buoys, flows of heat, moisture and momentum coming into the ocean were calculated using the COARE procedure [13]. With this weather impact, series of experiments were carried out using the model of upper ocean. Each series included experiments with model versions relying on different turbulence factor parameterizations. I used the following schemes: one based on similarity theory (11)-(16) (hereinafter ST), KPP; two schemes based on the equation of turbulence kinetic energy balance, the first one (hereinafter TKE) relying on a closure derived from the turbulent formations' characteristic scale from [4], the second one (K-E) resting on the second differential equation for dissipation rate [3], and also the so-called second order scheme [14] (hereinafter SO). The latter one has to be highlighted as it is based on six extremely complicated evolutionary equations and about 20 differential diagnostic relations. The model's resolution - 60 levels with vertical increment of 5 meters, time increment - 1 hour. Initial temperature / salinity profiles were based on Kitaigorodskii and Miropolskii self-similarity concept [15]:

$$(z), S(z) \quad \begin{cases} F_s, z \quad h_m \\ F_s \quad (F_s \quad F_b) M(\;), h_m \quad z \quad h_b \end{cases} ,$$

$$M(\;) \quad \frac{8}{3} \quad 2 \;^2 \quad \frac{1}{3} \;^4 \tag{17}$$

where F_s, F_b – values measured on the surface and at the lower boundary (h_b), determined according to empirical data; h_m – mixed layer thickness that was selected from available data taken at different ocean depths. Other variables were deemed to remain constant along the entire column: , U and V=0 m/s, $b=10^{-8}$ m^2/s^2, $\varepsilon=10^{-12}$ m^2/s^3. Adjustment coefficients for the ST scheme had the values as follows: h=2 Σ=2., γ=10. Similar experiments with other buoy station data in [1] demonstrated that the largest variance between the experiments is found in temperature profiles.

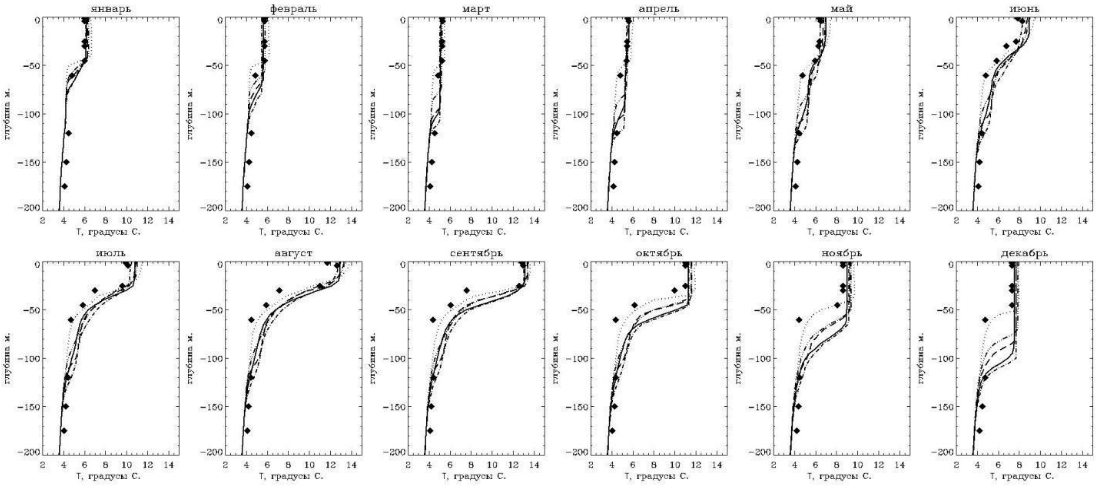

Figure 2. Monthly average patterns of calculated temperature, and empirical data, buoy 1 (50 north latitude, 145 western longitude), year 2011.

ST scheme, KPP scheme, TKE scheme,

— K-E scheme, SO scheme, observations

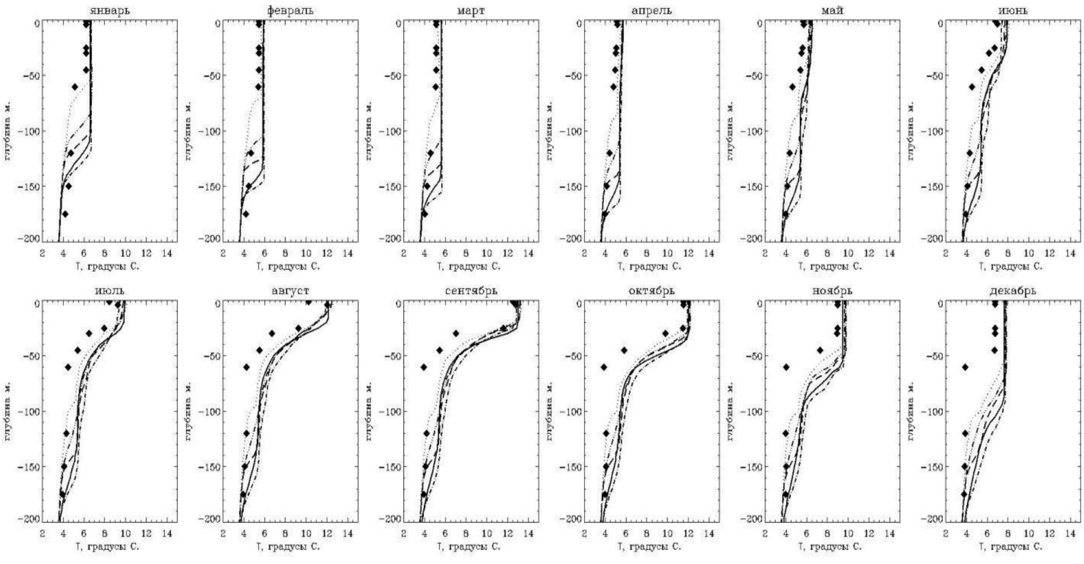

Figure 3. Monthly average patterns of calculated temperature, and empirical data, buoy 1 (50 north latitude, 145 western longitude), year 2012.

ST scheme, KPP scheme, TKE scheme,

K-E scheme, SO scheme, observations

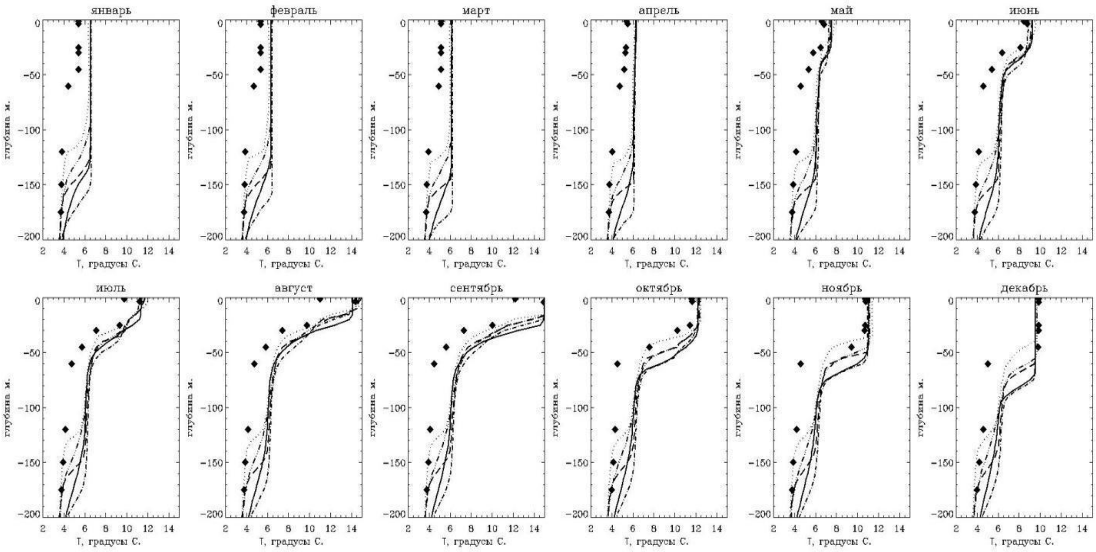

Figure 4. Monthly average patterns of calculated temperature, and empirical data, buoy 1 (50 north latitude, 145 western longitude), year 2013.

ST scheme, KPP scheme, TKE scheme,

— K-E scheme, SO scheme, observations

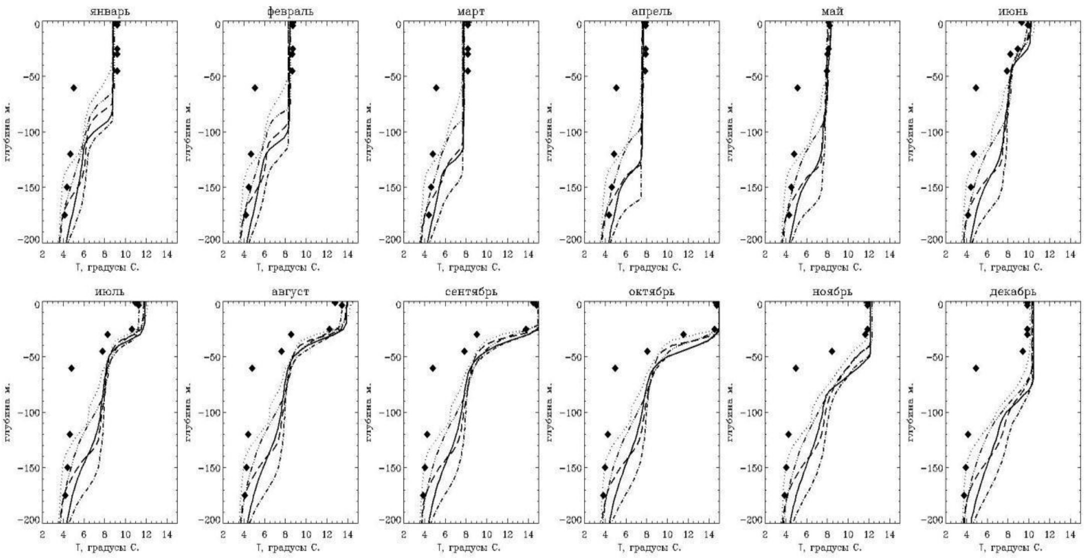

Figure 5. Monthly average patterns of calculated temperature, and empirical data, buoy 1 (50 north latitude, 145 western longitude), year 2014.

ST scheme, KPP scheme, TKE scheme,

— K-E scheme, SO scheme, observations Looking at the experimental results, we have to bear in mind that the upper ocean model can only simulate very local processes. Due to this reason, significant variance in the findings, particularly at the points of high advection, is inevitable.

Data from buoy 1 (Fig. 2 - 5) covered the longest span of uninterrupted observation - 4 years. It can be noted here that closer to the end of experimenting - regardless of the scheme used - the model results contained sizeable observation fallacies in modelling the upper ocean structure. The depth of the quasi-homogeneous layer was inflated nearly twofold. Such a result can be a testimony to the fact that this variance in observations is linked to non-local processes as all the schemes employed (structured along different principles) did eventually bring very similar experimental outcomes. The KPP scheme proved to be most efficient at this particular point. It exhibited the depth of the quasi-mixed layer very similar to the observed one. The poorest performance was shown by the TKE model that gave the most exaggerated depth value for the quasi-homogeneous layer and the lowest gradient in the thermocline. The ST scheme patterns were closest to the KE scheme, with the only exception that the latter - in some months - demonstrated the second layer of temperature discontinuity (e.g., since May till December, 2013) to be lower than the seasonal thermocline; this second layer of discontinuity was also shown by the KPP scheme.

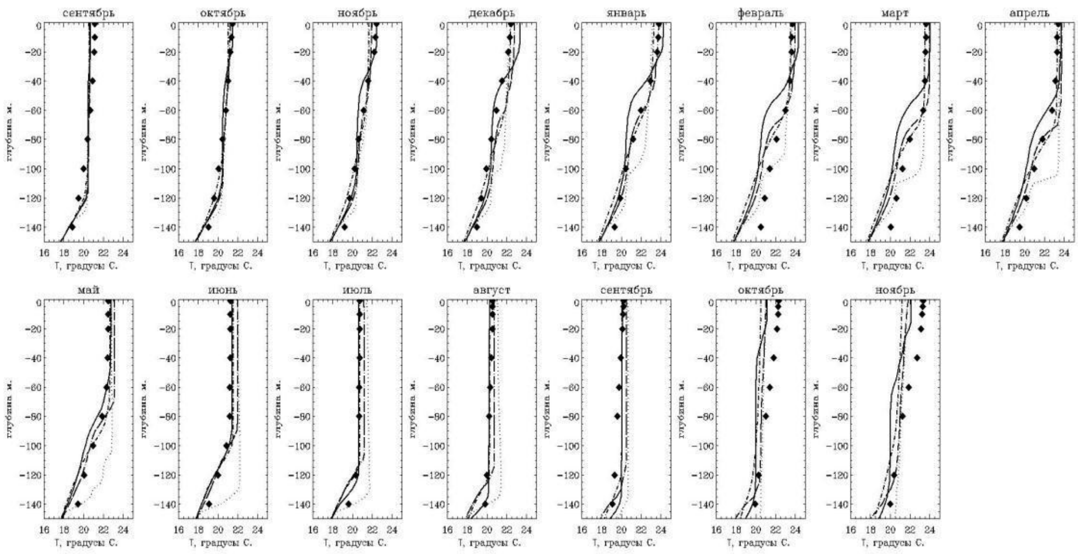

Figure 6. Monthly average patterns of calculated temperature, and empirical data, buoy 2 (25 north latitude, 100 western longitude), years 2012 -2013.

ST scheme, KPP scheme, TKE scheme,

К-E scheme, SO scheme, observations

Model results for buoy 2 (Fig. 6) are presented for the layer where the highest variance was observed (up to 150 m). The KPP scheme showing the best results for buoy 1, proved to be the worst at this point. This was particularly so in Spring and Summer months. The ST scheme demonstrated rather controversial results. On the one hand, ST profiles are largely different from empirical data in the period between February and April. On the other, in Autumn vertical distribution was closest to the observed data: a thin quasi-homogeneous layer up to 20 m and a low temperature gradient. The other schemes, for the same months, demonstrated a quasi-homogeneous layer that was too deep (beyond 100 m).

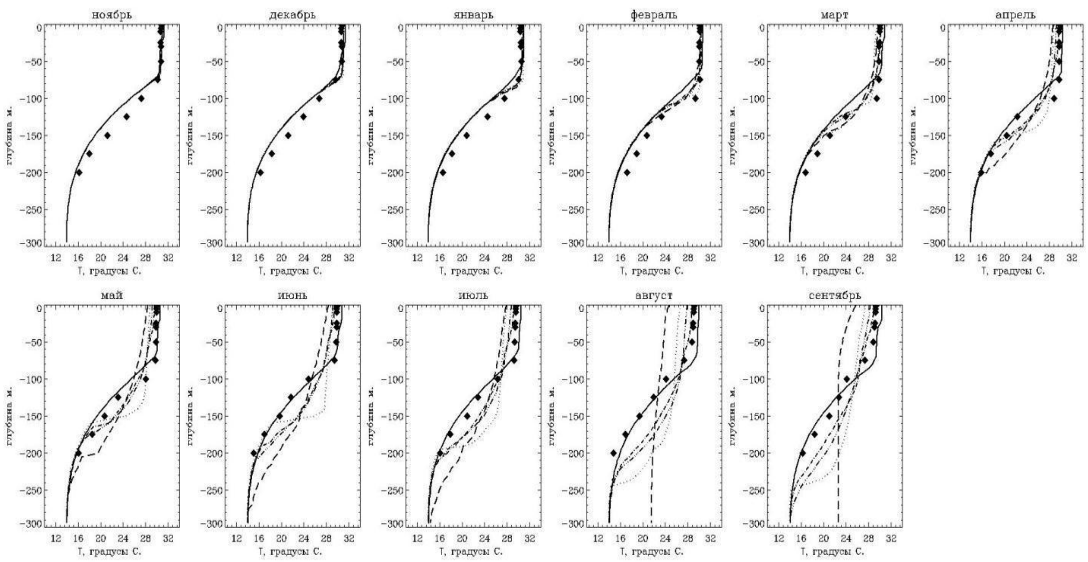

Figure 7. Monthly average patterns of calculated temperature, and empirical data, buoy 3 (0 north latitude, 165 western longitude), years 2014 -2015.

ST scheme, KPP scheme, TKE scheme,

K-E scheme, SO scheme, observations

The best result for buoy 3 was shown by the SK scheme. It was capable of modelling both the depth of the quasi-homogeneous layer and the thermocline gradient very close to the empirical data obtained. In the second half of the experiment, all the other schemes did not show a layer of the same temperature: they demonstrated a thermocline from the surface downwards with a low gradient that was increasing down to the 200 m depth. The TKE scheme, closer to the end of the experiment, entered the extreme mode of work virtually mixing up the whole of the integration range. This sort of results is most likely determined by the way the algorithms do work if they are based on the equation of turbulence kinetic energy balance in the equatorial belt where the depth of the Ekman layer tends to infinity. The K-E scheme proved stable in this area, most probably due to the use of a more complicated closure.

CONCLUSION

The upper layer of ocean is a most tricky object for investigation and modelling. The key obstacle to building an adequate theory is the presence of a highly stratified zone below the layer where direct interaction with the atmosphere is taking place. It is known that many a data on the ground air layer - a seemingly well studied one - are inconsistent, so there arose a wide range of functions describing wind and temperature patterns. Similar data on the ocean are several times more scarce; but, unlike the case of the atmosphere, dozens if not hundreds of models have been suggested to describe turbulence in the upper ocean.

Still, the goal of this study is to formulate and implement a new algorithm for vertical mixing in the ocean. Such a parameterization was built on the scheme deriving itself from the modified similarity theory in [5]. Certain changes were introduced to the original algorithm that were not inconsistent with the fundamentals of the method and allowed me to model some processes in a more correct way.

A series of experiments were carried out using a one-dimensional model of the upper ocean with reliance on the similarity theory-based scheme and on other more commonly used parameterizations. The study findings demonstrated that this scheme is capable of successfully simulating turbulent diffusion processes along with convective mixing. At the least, the quality of my calculations, proceeding from this scheme, is definitely comparable with that of model versions using other parameterizations. It has also been demonstrated that more complex formulations do not necessarily lead to better results. In this study, the cumbersome and awkward second order scheme did not bring any promising results.

Turbulent mixing algorithms are very sensitive to calculation settings the latter being expressed in the optimal choice of coefficients and factors. The suggested parameterization has a relatively small number of adjustment parameters, which makes it quite easy and simple in use. This scheme, in its mathematics, relies on a series of basic ocean environment parameters and does not require additional variables.

It is also worth noting that this method proceeds from relatively simple physics, but despite this fact, it is able to simulate vertical mixing processes not worse that complicated schemes relying on various, often unverified hypotheses.

Many a differential model schemes for the upper ocean share a common disadvantage: the vanishing of the diffusion factor under stable stratification. Such models, in their pure form, cannot describe turbulent exchange between the mixed layer and the upper thermocline. This drawback can be remedied by introduction of a threshold value for the diffusion coefficient. This coefficient can be modified and selected at ease, but it is exactly this factor that eventually works in the long-term dynamics; so, the various complications of equations used in the layer that lies above the discontinuity layer turn out to be, strictly speaking, redundant as the structure of the mixed layer is relatively simple.

Unsuitability of full turbulence conventional differential models is determined by the fact that the physics of mixing in a stable environment differ fundamentally from the physics of mixing under unstable or weak positive stratification. Mixing in the upper thermocline takes place, apparently, at internal wave collapse. The result of this are irreversible non-linear effects in the field of external waves and in Langmuir circulation Following from this, we can state that turbulence is sporadic in its nature, i.e. it characterized by high intermittence. This is the reason why flows of salt and momentum occur due to the working of a long-term integral mechanism. Obviously enough, this process cannot be described using conventional approaches schemeed for modelling turbulent flows.

There is a strong possibility that the scheme suggested here is not the best way of applying the modified similarity theory to mixing in ocean. Possible are other techniques of approximating the function f(St). But the method per se appears to be rather promising, particularly so if compared to schemes relying on the equation of turbulence kinetic energy, whose improvement potential is practically exhausted, in whole, at this stage of computing development.

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