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Approximation of Zeros of Accretive Operators in a Compact Space

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Abstract

In this paper we consider the problem for finding of zeros of an accretive operator in a compact space and proving that the weak convergence results for resolvents of the accretive operator, and also we have been established some new results of accretive operators by introducing two iterative methods such as forword and backword related to Banach spaces [9].

Keywords: Banach operators, Topological spaces, m- accretive operators, iterative scheme.

1. INTRODUCTION

Let (X,T) be a metric space and k be a non-empty convex subset of (X,T) . Let $T : k \to k$ be an operator for any $x_0 \in k$, the sequence $\{x_n\}_{n=0}^{\infty} \subseteq k$ defined by

$$x_{n+1} = (1-a_n)x_n + a_n T y_n + v_n,$$

$$y_n = (1-b_n)x_n + b_n T x_n + u_n, n \geq 0$$

where $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are two summable sequence in X
i.e. $\sum_{n=1}^{\infty} u_n \leq \infty$ and $\sum_{n=1}^{\infty} u_n \leq \infty$

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therefor both above sequences with k, T and x_0 as in the sequence [2]$\{x_n\}_{n=0}^{\infty} \subseteq k$ defined by

$$x_{n+1} = (1 - a_n)x_n + a_n T x_n + u_n, n \geq 0$$

Hence it is clear that the ishikawa and Mann iterative operator processes are all special cases of the following distance i.e.

$$\ell X(T) = Sup\{\frac{1}{2}\|x + y\| + \|x - y\| - 1 : \|x\| = 1, \|y\| \leq r\}$$

satisfies that $\ell X(T) \to 0$ as $T \to 0$ for a real number $\ell > 1$. X is said to be ℓ- uniformly smooth $\exists$ a contant $c > 0$. Such that $\ell X(T) \leq CT^p$ for $0 < T < \infty$, it is know that L_p or ℓ_p and p- uniformly Smooth Banach Spaces are uniformly Smooth, when $p > 1$. So that we have been found the Zeros of an accretive operator in a Compact Space for weak convergent.

In this paper we have been established some new results of accretive operators by introducing two iterative methods such as forword and backword related to Banach spaces.

2. PRELIMINARIES

Let X and Y be a completed normed linear space (Banach Spaces), and

$$f : X \to Y$$

be a linear map,

(i) f is bounded in $\overline{\cup}(0, r)$ for some $r > 0$.

(ii) f is continuous at 0 with zero accretive operator.

(iii) f is continuous at X with accretive operator is defined in space $(X, 0)$.

(iv) f is uniformly continuous at origin in $(X, 0)$.

(v) $\|f(x)\| \leq \alpha\|X\| \; \forall x \in X$ where $\alpha > 0$.

(vi) The zero of space $Z(F)$ of F is closed in X, and the linear map $\overline{f} : X_{Z(F)} \to Y$ is defined by $\overline{f}(X + Z(F) = f(x)$ where $x \in X$ is continuous.

Proposition 2.1. *(see Cioranescu [8]). Let* $1 < p < \infty$.

(i) The Banach space X is smooth if and only if the duality mapping ∂_p is single valued.

(ii) The Banach space X is uniformly smooth if and only if the duality mapping $\jmath_p$ is single valued and norm-to norm uniformly continuous an bounded sets of X.

Lemma 2.2. *[8] Let real sequences $(\alpha_n), (\beta_n)$ satisfy the conditions:*

(i) $(\alpha_n) \subset [0,1], \sum_{n=0}^{\infty} \alpha_n = \infty$, or equivalenty, $\Pi_{n=0}^{\infty}(1-\alpha_n) = \lim_{n\to\infty} \Pi_{k=0}^{n}(1-\alpha_k) = 0$.

(ii) $\limsup_{n\to\infty} \beta_n \leq 0$. Assume a nonnegative sequence $\{s_n\}$ satisfies the inequality:

$$s_{n+1} \leq (1-\alpha_n)s_n + \alpha_n\beta_n, n \geq 0$$

Then $\lim_{n\to\infty} = 0$.

In [9] the two forward-backward methods previously studied, 3.29 and 3.41, find applications in other related problems such as variational inequalities, the convex feasibility problem, fixed point problems, and optimization problems.

Theorem 2.3. *[9] Let X be a uniformly convex and q-uniformly smooth Banach space. Let $A : X \to X$ be an α-isa of order q and $B : X \to 2^X$ an m-accretive operator. Assume that $S = (A+B)^{-1}(0) \neq \phi$. We define a sequence $\{x_n\}$ by the perturbed iterative scheme*

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n(Jr_n(x_n - r_n(Ax_n + a_n)) + b_n)$$

where $Jr_n = (I + r_nB)^{-1}, \{a_n\}, \{b_n\} \subset X, \{\alpha_n\} \subset (0,1]$, and $\{r_n\} \subset (0,+\infty)$.

3. MAIN RESULTS

Theorem 3.1. *Let $\|x,m\|_p$ define [1] Banach operator with index p and is finite with satisfying accretive operator, thus we have, for any $0 < p < \infty$. Let (X,T), where $T$is topological space with condition of T_0, T_1 ,.... $\|X,T,m\|_p < \infty$.*

Proof: We have the space $(X,m) < \infty$,Where,

$$0 < p < \infty \tag{3.1}$$

by Hann-Banach theorem for index p,

$$p \leq \|x,m\| < \infty \tag{3.2}$$

thus, from equation (3.1) and (3.2) we have, $0 \leq p \leq \infty$

$$\Rightarrow 0 \leq \|X, m\| \leq \infty \tag{3.3}$$

Now for topological space T_0, T_1 , we have

for any $x_1 \in X$, $x_2 \in X$,

$\Rightarrow (x_1T) \subseteq (XT)$ and $(x_2T) \subseteq (XT)$

Similarly for considering the finite number of points x_1, x_2, x_3, x_4,.......... , $x_n \in X$, we have

$(x_1T) \subseteq (x_2T) \subseteq (x_3T) \subseteq (x_4T) \subseteq$ .... $\subseteq (x_nT) \subseteq (XT)$

$\Rightarrow \sum x_1 \leq \sum x_2 \leq \sum x_3 \leq \sum x_4 \leq \sum$.......................... $\leq \sum x_n$

Which satisfying the accretive operator condition, therefore any finite number m we have with $0 \leq p \leq \infty$ (index).

We have $\sum(x_1, T, m) \sum[(x_2, T, m) \sum[(x_3, T, m)$................ $\sum[(x_p, T, m)$ With index p.

Therefore by Hann-Banach theorem we have, $\|X, T, m\|_p \leq \infty$. since $(0 < p < \infty)$.

Theorem 3.2. *: Let H be a Hilbert space, T be a monotone demicontinuous operator with domain $D(T) = H$, at range $R(T) \subseteq H$, then $B_\mu(0) = \delta B_\mu(0)$, where $\mu = (r - \|T(0)\|)/2$ and B is open ball.*

Proof: We have for fix $p \in B_\mu$ and let $x \in D(T)$, satisfies [4]

$Tx_n + (\frac{1}{n})x_n = p, n = 1, 2, ...$ we have,

$$Re\,\langle Tx_n - T(0), J(x_n)\rangle + Re\,\langle T(0), J_n(x)\rangle + (\frac{1}{n})\|X_n\|^2 = Re\,\langle p, J_n(n)\rangle \tag{3.4}$$

which may be accretiveness of T

$$(\frac{1}{n})\|X_n\| \leq \|T(0)\| + \|p\|, n = 1, 2.... \tag{3.5}$$

now we assume for some subsequence of

$\{x_n\}, n = 1, 2, ...\ \lim_{n\to\infty} \|X_n\| = \infty$ for given $\in > 0, \exists N(\in) > 0$, such that

$\|TX_n\| > r - \in, n > N(\in)$ it follows that

$$\|T(0)\| \geq r - 2\|p\| > r - 2\mu$$

$$\Rightarrow \lim_{n\to\infty} \{u(t)x_n - x_n\} = 0, \forall t \in R_+ \tag{3.6}$$

and since $\bigcap_m G_m = \{x_0\} \Rightarrow x_0$ is a common fixed point for $u(t), t > 0$
therefore from (3.4), (3.5) and (3.6) we have obtain that,

$$\begin{aligned} T(x_0) = \lim_{t\to 0^+} (1/t)(u(t) - I)x_0 = 0 \\ \Rightarrow Tx_0 = p \\ \Rightarrow B_\mu \subseteq R(T)\pi \end{aligned} \tag{3.7}$$

again for let $y \in X$; such that $\|y\| = \mu \Rightarrow y_m \to y$ as $m \to \infty$ and size the Ball B_μ contained in $R(T)$ by equation (3.7) we have $\{x_n\}$ is bounded.

Assume that $\{\overline{x}_m\}$ is a subsequence of it such that $\|\overline{x}_m\| \to \infty$ as $m \to \infty$
putting for $\overline{y}_m = T\overline{x}_m$ we obtain [3]

$$\|y\| = \lim_{n\to\infty} inf\|\overline{y}_m\| \geq \lim_{\{x\}\to\infty} inf\|T_x\| > r, x \in D(T). \tag{3.8}$$

$$\Rightarrow R(T) \subseteq B_\mu(0) \tag{3.9}$$

equating this distance at origin we have

$$B_\mu(0) = \delta B_\mu(0).$$

Theorem 3.3. *Let X be a Banach space, D an open bounded subset of X and let $T : D \to X$ an accretive mapping [5] i.e.,*

$$\{(\lambda - 1)(u - v) \leq \|(\lambda - 1)(u - v) + T(v) - T(u)\|\}$$

for all $u, v \in D$ and shown have that if T is discontinuous and accretive also if $\exists$ $x \in D$, and $\in > 0$ for which $\|T(X)\| \leq \|T(X)\| + \in$ for all $x \in \delta D$, then the open ball $B(0, X)$ is contained in the range of T for $Z = \{|T(x)| + \in\}$.

Proof: Let X denote a non-triviale Comlpex Banach Space by an accretive operator in X we mean a linear mapping

$$T : D(T) \to X$$

then we have

$$\|T\| = sup\{\|T_U\| : U \in D(T); \|u\| = 1\}$$

. Now we have given T is bounded if $\|T\| \leq \infty$ i.e. T is bounded on X, if it is bounded and $D(X) = T$. Now put the algebra of all bounded operators on X is denoted by $L(X)$.

We recall $T-$ densely defined [6] if $D(T)$ is dense in X and closed if it's Graph

$$G(T) = \{(u, T_u) : u \in D(T)\}$$

is a closed subspace of $X \times X$. These space of all closed operators is denoted as $C(X)$,

When we have the new operator are constructed from old, the domain are taken to be the largest for which the constructions make sense.

Example 3.4. *If S and T are linear operators then $S + T$ and ST are linearly defined by*

$$(S + T)u = Su + Tu : u \in D(S + T) = D(S) \cap D(T)$$

and

$$(ST)u = S(Tu) : u \in D(ST) = \{u \in D(T) : Tu \in D(S)\}$$

hence we find required result.

4. CONCLUSION

(4.1) In This paper we have been conclude that since the behavior of the Zero accretive operator in fourier series, so as for as convergence is concerned , for a particular value of x depends on the behavior of the function in the immediate neighborhood of this point only, hence the truth of the above all result is necessary.

(4.2) The result of Theorem 3.3 shows that iterative methods related with forword and backword operator.

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