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An Unbiased Family of Sampling Strategies for Estimation of Population Variance

Shashi Bhushan[a] and Chandni Kumari[b]

[a]Department of Mathematics & Statistics, Dr. Shakuntala Mishra National Rehabilitaion University, Lucknow, 226017, India.
[b]Department of Applied Statistics, Bahasaheb Bhimrao Ambedkar University, Lucknow, 226025, India.

Abstract

In this paper, we present the family of estimators $\hat{S}^2_{AM}$ for estimating the population variance under Midzuno-Lahiri-Sen (MLS) type sampling design P(s). The expressions for the unbiasedness and mean squared error for the proposed sampling strategy $\left(\hat{S}^2_{AM}, P(s)\right)$ are derived up to the first order of approximation. Further, some concluding remarks are also given. The comparison of proposed sampling strategy with respect to the some important estimators made. Finally, numerical illustration is also given in support of the present study.

Keywords Simple random sampling, Ratio type estimator, Midzuno-Lahiri-Sen type sampling design.

1 INTRODUCTION

In many surveys, information on an auxiliary variable that is highly correlated with the variable under study is readily available which can be used for improving the sampling strategy. Isaki (1983), Das and Tripathi (1978) proposed a generalized class of biased estimators for estimating population variance. Also, a sampling design was proposed by Midzuno (1952), Lahiri (1951) and independently by Sen (1952), under which the traditional ratio estimator is unbiased. An attempt has been made by the authors in the direction of Bhushan (2012, 2013, 2016), Bhushan et al. (2009), Bhushan and Pandey (2010), Bhushan and Katara (2010), Bhushan et al. (2011), to improve the existing sampling strategies by using auxiliary information which modifies both the estimator as well as the sampling scheme. These works were restricted to unbiased estimation of population mean. It is also noteworthy the very few works have been done in this direction.

Let the population consists of N units. Yi and Xi denote the i^th characteristics of the population. The population mean of the study variable is denoted by $\bar{Y}$ and population mean of the auxiliary variable which is known is denoted by $\bar{X}$. The population variance of the study variable and the auxiliary variable is denoted by S_y^2 and S_x^2. Let $\mu_{pq} = N^{-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^p\left(Y_i - \bar{Y}\right)^q$ be the population product moment between x and y. C_x and C_y be the coefficient of variation of auxiliary and study variable respectively. Thus, ρ be the correlation coefficient between the variable under study and auxiliary variable which measures the degree of linear relationship between two variables and it is given as $\rho = Cov(Y,X)/\sigma_Y\sigma_X$. Now, let us take a random sample of size n drawn without replacement. y_i be the i^th characteristics of the study variable of the sample and x_i be the i^th characteristics of the auxiliary variable of the sample. The sample mean of study variable for estimating the population mean is denoted by $\bar{y}\left(= n^{-1}\sum_{i=1}^{n} y_i\right)$. The sample mean of auxiliary variable is denoted by $\bar{x}\left(= n^{-1}\sum_{i=1}^{n} x_i\right)$. When the random sample s is selected by simple random sampling without replacement, the generalized Walsh type ratio estimator was considered by Bhushan (2016) for estimating population variance is reproduced below

$$\hat{S}_A^2 = \frac{s_y^2\bar{X}}{\bar{X} + A\left(\bar{x} - \bar{X}\right)}$$

where A is the characterizing scalar to be chosen suitably. Taking que from Bhushan (2016) the generalized ratio estimator for estimating population variance S_y^2 is modified to a more general class of estimators using prior information is proposed to be

$$\hat{S}_{AM}^2 = \frac{s_y^2\left(\alpha\bar{X} + \beta\right)}{\alpha\bar{X} + \beta + A\alpha\left(\bar{x} - \bar{X}\right)}$$

where A is the characterizing scalar to be chosen suitably; α and β represents the prior information in the form of the parameters based on auxiliary characters discussed later. We now consider this generalized ratio estimator $\boldsymbol{S}_{\text{AM}}^2$ under the proposed modified Midzuno-Lahiri-Sen type sampling design. The proposed MLS type sampling design for selecting a sample s of size n deals with selecting the first unit of the sample by PPS type sampling scheme described below and selecting the remaining $(n-1)$ units in a sample from $(N-1)$ units in the population by simple random sampling without replacement.

Therefore,

$P(s) = \sum_{i=1}^{n} P$ {selecting ith sample unit at first draw} X

P {selecting (n-1) units out of (N-1) units by SRSWOR}

$$= \sum_{i=1}^{n} \frac{(\alpha\bar{X}+\beta)+A\alpha(x_i - \bar{X})}{{}^{N-1}C_{n-1}(\alpha\bar{X}+\beta)}$$

$$= \frac{(\alpha\bar{X}+\beta)+A\alpha(\bar{x} - \bar{X})}{{}^{N}C_{n}(\alpha\bar{X}+\beta)}$$

2 SOME CLASSES OF PROPOSED FAMILY OF STRATEGIES

We explicitly propose some classes of strategies $(t_i, P(s_i))$ when the auxiliary information is available in the terms of known coefficient of variation, correlation coefficient, coefficient of skewness, coefficient of kurtosis, standard deviation, quartiles etc., for illustration and possible application. Now, the proposed estimators t_i under the respective Midzuno - Lahiri - Sen type sampling designs $P(s_i)$ are given below.

Estimator	**Sampling Design**
$t_1 = \frac{s_y^2(\bar{X}+C_x)}{\bar{X}+C_x+A\alpha(\bar{x}-\bar{X})}$	$P(t_1) = \frac{\bar{X}+C_x+A\alpha(\bar{x}-\bar{X})}{(\bar{X}+C_x)\ {}^{N}C_n}$
$t_2 = \frac{s_y^2(\bar{X}+\rho)}{\bar{X}+\rho+A\alpha(\bar{x}-\bar{X})}$	$P(t_2) = \frac{\bar{X}+\rho+A\alpha(\bar{x}-\bar{X})}{(\bar{X}+\rho)\ {}^{N}C_n}$
$t_3 = \frac{s_y^2(\bar{X}+\beta_2(x))}{\bar{X}+\beta_2(x)+A\alpha(\bar{x}-\bar{X})}$	$P(t_3) = \frac{\bar{X}+\beta_2(x)+A\alpha(\bar{x}-\bar{X})}{(\bar{X}+\beta_2(x))\ {}^{N}C_n}$
$t_4 = \frac{s_y^2(\beta_2(x)\bar{X}+C_x)}{\beta_2(x)\bar{X}+C_x+A\alpha(\bar{x}-\bar{X})}$	$P(t_4) = \frac{\beta_2(x)\bar{X}+C_x+A\alpha(\bar{x}-\bar{X})}{(\beta_2(x)\bar{X}+C_x)\ {}^{N}C_n}$
$t_5 = \frac{s_y^2(C_x\bar{X}+\beta_2(x))}{C_x\bar{X}+\beta_2(x)+A\alpha(\bar{x}-\bar{X})}$	$P(t_5) = \frac{C_x\bar{X}+\beta_2(x)+A\alpha(\bar{x}-\bar{X})}{(C_x\bar{X}+\beta_2(x))\ {}^{N}C_n}$

$t_6=\dfrac{s_y^2\left(\bar{X}+\sigma_x\right)}{\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}$	$P\left(t_6\right)=\dfrac{\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}{\left(\bar{X}+\sigma_x\right){}^N C_n}$
$t_7=\dfrac{s_y^2\left(\beta_2(x)\bar{X}+\sigma_x\right)}{\beta_2(x)\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}$	$P\left(t_7\right)=\dfrac{\beta_2(x)\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}{\left(\beta_2(x)\bar{X}+\sigma_x\right){}^N C_n}$
$t_8=\dfrac{s_y^2\left(\beta_1(x)\bar{X}+\sigma_x\right)}{\beta_1(x)\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}$	$P\left(t_8\right)=\dfrac{\beta_1(x)\bar{X}+\sigma_x+A\alpha\left(\bar{x}-\bar{X}\right)}{\left(\beta_1(x)\bar{X}+\sigma_x\right){}^N C_n}$
$t_9=\dfrac{s_y^2\left(\bar{X}+q_{1x}\right)}{\bar{X}+q_{1x}+A\alpha\left(\bar{x}-\bar{X}\right)}$	$P\left(t_9\right)=\dfrac{\left(\bar{X}+q_{1x}\right)\bar{X}+q_{1x}+A\alpha\left(\bar{x}-\bar{X}\right)}{\left(\bar{X}+q_{1x}\right){}^N C_n}$
$t_{10}=\dfrac{s_y^2\left(\bar{X}+q_{2x}\right)}{\bar{X}+q_{2x}+A\alpha\left(\bar{x}-\bar{X}\right)}$	$P\left(t_{10}\right)=\dfrac{\bar{X}+q_{2x}+A\alpha\left(\bar{x}-\bar{X}\right)}{\left(\bar{X}+q_{2x}\right){}^N C_n}$

3 PROPERTIES OF PROPOSED SAMPLING STRATEGY

Theorem 1 $\hat{S}_{AM}^2$ is an unbiased estimator of population variance under the proposed sampling design $P\left(s\right)$.

Proof. Consider, $\hat{S}_{AM}^2=\dfrac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}$

Taking expectation under the proposed sampling design, we get

$$E\left(\hat{S}_{AM}^2\right)=E_s\left\{\frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}\right\}$$

$$=\sum_{s=1}^{{}^N C_n}\left\{\frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}\right\}P\left(s\right)$$

$$=\sum_{s=1}^{{}^N C_n}\left[\frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}\right]\frac{\left\{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)\right\}}{{}^N C_n\left(\alpha\bar{X}+\beta\right)}$$

$$=\sum_{s=1}^{{}^N C_n}\left[\frac{s_y^2}{{}^N C_n}\right]$$

$=E\left(s_y^2\right)$ where E(.) denotes the expectation under SRSWOR

$$=S_y^2 \tag{3.1}$$

Thus $\hat{S}^2_{AM}$ is an unbiased estimator of population variance under the proposed sampling strategy.

Theorem 2 The variance of $\hat{S}^2_{AM}$ under the proposed sampling design is given by

$$\text{V}_\text{s}\left(\hat{S}^2_{AM}\right)_{opt} = \frac{\mu^2_{02}}{n}\left\{(\beta_{02}-1)-\left(\frac{\mu^2_{12}}{\mu^2_{02}\mu_{20}}\right)\right\}$$

Proof.

$$\text{V}_\text{s}\left(\hat{S}^2_{AM}\right) = \text{E}_\text{s}\left[\hat{S}^2_{AM}-S^2_y\right]^2$$

$$= E\left[\frac{s^2_y\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}-S^2_y\right]^2$$

$$= \sum_{s=1}^{{}^N C_n}\left[\frac{s^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y A\alpha\left(\bar{x}-\bar{X}\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}\right]^2 P(s)$$

$$= \sum_{s=1}^{{}^N C_n}\left[\frac{s^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y A\alpha\left(\bar{x}-\bar{X}\right)}{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)}\right]^2 \frac{\left\{\alpha\bar{X}+\beta+A\alpha\left(\bar{x}-\bar{X}\right)\right\}}{{}^N C_n\left(\alpha\bar{X}+\beta\right)}$$

$$= E\left[\frac{\left\{s^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y\left(\alpha\bar{X}+\beta\right)-S^2_y A\alpha\left(\bar{x}-\bar{X}\right)\right\}^2}{\left(\alpha\bar{X}+\beta\right)^2\left\{1+\frac{A\alpha\left(\bar{x}-\bar{X}\right)}{\alpha\bar{X}+\beta}\right\}}\right],\ \varepsilon_o = s^2_y - S^2_y,\ \varepsilon_1 = \bar{x}-\bar{X}$$

$$= \left\{E\left(\varepsilon_0\right)^2 - \frac{2S^2_y A\alpha E\left(\varepsilon_0\varepsilon_1\right)}{\alpha\bar{X}+\beta} + \frac{S^4_y A^2\alpha^2\, E\left(\varepsilon_1\right)^2}{\left(\alpha\bar{X}+\beta\right)^2}\right\}$$

Using the results of Sukhatme and Sukhatme, we have

$\text{E}\left(\varepsilon_o\right)=0$, $\text{E}\left(\varepsilon_1\right)=0$, $\text{V}(\varepsilon_o)=\text{E}(\varepsilon^2_0)=\frac{\mu_{04}-\mu^2_{02}}{n}$, $\text{V}(\varepsilon_1)=\text{E}(\varepsilon^2_1)=\frac{\mu_{20}}{n}$ and $\text{E}(\varepsilon_0\varepsilon_1)=\frac{\mu_{12}}{n}$

Then,

$$= \frac{\mu_{02}^2}{n}\left[(\beta_{02}-1)+\frac{A^2\alpha^2\mu_{20}}{(\alpha\bar{X}+\beta)^2}-\frac{2A\alpha}{(\alpha\bar{X}+\beta)}\left(\frac{\mu_{12}}{\mu_{02}}\right)\right]$$

$$= \frac{\mu_{02}^2}{n}\left[(\beta_{02}-1)+A^2\eta^2\mu_{20}-2A\eta\left(\frac{\mu_{12}}{\mu_{02}}\right)\right], \text{ where } \eta=\frac{\alpha}{\alpha\bar{X}+\beta}$$

which is the variance of $\hat{S}_{AM}^2$ under the proposed sampling strategy. The optimum value of A is given by

$$A_{opt}=\frac{\mu_{12}(\alpha\bar{X}+\beta)}{\alpha\mu_{02}\mu_{20}}=\frac{C}{\eta} \text{ (say), where } C=\frac{\mu_{12}}{\mu_{02}\mu_{20}}$$

Now, the minimum variance of $\hat{S}_{AM}^2$ under the proposed sampling strategy is given by

$$\mathrm{Vs}\left(\hat{S}_{AM}^2\right)_{opt}=\frac{\mu_{02}^2}{n}\left\{(\beta_{02}-1)-\left(\frac{\mu_{12}^2}{\mu_{02}^2\mu_{20}}\right)\right\} \quad (3.2)$$

which is the minimum variance of $\hat{S}_{AM}^2$ under Midzuno-Lahiri-Sen type sampling scheme.

4 EFFICIENCY COMPARISONS

The mean square error of the sample variance estimator $\hat{S}_y^2$, ratio estimator $\hat{S}_r^2=s_y^2\left(\bar{X}/\bar{x}\right)$, product estimator $\hat{S}_p^2=s_y^2\left(\bar{x}/\bar{X}\right)$, generalized ratio estimator (Tripathi) under SRSWOR $\hat{S}_A^2=s_y^2\left(\bar{X}/\bar{x}\right)^A$ and generalized ratio estimator (Walsh), reconsidered by Bhushan (2016) under Midzuno - Lahiri - Sen type sampling design $\hat{S}_{AM}^2=s_y^2\,\bar{X}\Big/\left\{\bar{X}+A\left(\bar{x}-\bar{X}\right)\right\}$ are given below.

$$\mathrm{MSE}\left(\hat{S}_y^2\right)=\frac{\mu_{02}^2}{n}(\beta_{02}-1) \quad (4.1)$$

$$\mathrm{MSE}\left(\hat{S}_r^2\right)=\frac{\mu_{02}^2}{n}\left[(\beta_{02}-1)+\frac{\mu_{20}}{\bar{X}^2}-2\frac{\mu_{12}}{\bar{X}\mu_{02}}\right] \quad (4.2)$$

$$\text{MSE}\left(\hat{S}_p^2\right)=\frac{\mu_{02}^2}{n}\left[\left(\beta_{02}-1\right)+\frac{\mu_{20}}{\bar{X}^2}+2\frac{\mu_{12}}{\bar{X}\mu_{02}}\right] \tag{4.3}$$

$$\text{MSE}\left(\hat{S}_A^2\right)==\frac{\mu_{02}^2}{n}\left[\left(\beta_{02}-1\right)+A^2\frac{\mu_{20}}{\bar{X}^2}-2A\frac{\mu_{12}}{\bar{X}\mu_{02}}\right] \tag{4.4}$$

Also the variance (MSE) of proposed sampling strategy is given in (3.2) which is as follows

$$\text{Vs}\left(\hat{S}_{AM}^2\right)_{opt}=\frac{\mu_{02}^2}{n}\left\{\left(\beta_{02}-1\right)-\left(\frac{\mu_{12}^2}{\mu_{02}^2\mu_{20}}\right)\right\} \tag{4.5}$$

4.1 Comparison with mean per unit estimator

$$\text{Vs}\left(\hat{S}_{AM}^2\right)_{opt}<\text{MSE}\left(\hat{S}_y^2\right) \tag{4.6}$$

4.2 Comparison with ratio estimator

$$\text{Vs}\left(\hat{S}_{AM}^2\right)_{opt}<\text{MSE}\left(\hat{S}_r^2\right) \tag{4.7}$$

4.3 Comparison with product estimator

$$\text{Vs}\left(\hat{S}_{AM}^2\right)_{opt}<\text{MSE}\left(\hat{S}_p^2\right) \tag{4.8}$$

4.4 Comparison with Tripathi estimator

$$\text{Vs}\left(\hat{S}_{AM}^2\right)_{opt}=\text{MSE}\left(\hat{S}_A^2\right) \tag{4.9}$$

The proposed sampling strategy under optimum conditions has lesser mean squared error than the estimators available in simple random sampling without replacement. The generalized ratio estimator by Bhushan(2016) for estimating population variance is equally efficient but it is a special case of the proposed sampling strategy.

5 EMPIRICAL STUDY

We have performed an empirical investigation over three different natural population to check the suitability of the proposed sampling strategy over conventional estimators under simple random sampling without replacement.

Population 1 (Choudhary F. S. and Singh D., Pg. no. 155).

The data concerns for studying milk yield, feeding and management practices of milch animals in the year 1977-78.

y : number of milch animals in surveys

x : number of milch animals in census.

Population 2 (Singh S., Pg. no. 1113). The data concerns the hypothetical situation of a small village having only 30 old persons.

y : duration of sleep (in minutes).

x : age (in years) of the persons.

Population 3 (Choudhary F. S. and Singh D., Pg. no. 132).

The data concerns the number of trees in the orchard.

y : yields (in 10 kg) of the 8 orchards

x : number of trees in the orchards.

The summary of the populations are given in the Table 1.

Table 1: Parameters of the data

Parameters	Population 1	Population 2	Population 3
N	17	30	8
N	8	12	3
μ_{20}	270.913	82.395	93.6
μ_{02}	431.585	3463.16	131.86
μ_{04}	508642.4	31996247	38610.93
μ_{12}	2422.297	5618.021	426.84

The bias of the estimator is obtained by

$$\text{Bias}\left(\hat{S}_A^2\right) = \frac{A}{n\bar{X}}\left\{\frac{(A+1)}{2}\frac{S_y^2\,\mu_{20}}{\bar{X}} - \mu_{12}\right\}$$

Particularly, the biases of $\hat{S}_r^2$ and $\hat{S}_p^2$ can be obtained by putting A = 1 and A = -1 respectively.

Table 2: Bias of the estimators

Estimator	Population 1	Population 2	Population 3
$\hat{S}_y^2$	0	0	0
$\hat{S}_r^2$	-0.254	-2.285	-0.084
$\hat{S}_p^2$	0.267	18.595	8.347
$\hat{S}_A^2$	-0.679	0	0
$\hat{S}_{AM}^2$	0	0	0

Table 3: MSE and PRE of the estimators

Estimator	Population 1		Population 2		Population 3	
	MSE	PRE	MSE	PRE	MSE	PRE
$\hat{S}_y^2$	10079.172	100.00	2500346	100.00	7074.677	100.00
$\hat{S}_r^2$	9938.315	101.417	2455336	84.71	6439.616	109.86
$\hat{S}_p^2$	10226.122	98.763	2599956	111.85	8663.115	81.66
$\hat{S}_A^2$	8379.760	120.28	2452464	115.95	6426.462	110.087
$\hat{S}_{AM}^2$	8370.760	120.28	2452464	139.92	6426.462	110.087

6 CONCLUDING REMARKS

Using suitable value of $C\left(=\mu_{12}/\mu_{02}\mu_{20}\right)$, we find some efficient estimators which have minimum mean square error as follows:

We know that sample variance in case of SRSWOR, $\hat{S}_{AM}^2$ is preferred to ratio and product estimator when 0<C<1/2 and -1/2<C<0 respectively. From efficiency condition (4.1), let the range information about C is known as $C < C_0$ be the prior information of C. Thus, choose A for satisfying efficiency condition (4.6) to get a class of estimators as

$$\hat{S}^2_{\frac{2c_0}{\eta}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2c_0}{\eta}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.1}$$

which are better than sample variance of without replacement and they are the most efficient estimator. Further it is known that C < C1 where -1/2 < C < 0. Thus we choose A for satisfying the efficiency condition (4.6) such that $A\eta = 2/3$. So that the class of estimators

$$\hat{S}^2_{\frac{2\eta}{3}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2\eta}{3}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.2}$$

are better than variance per unit in the sense of lesser mean square error. More specifically, for prior range information C<1/3. Choosing $A = 2/3\eta$, satisfying efficiency condition (4.6) we get

$$\hat{S}^2_{\frac{2}{3\eta}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2}{3\eta}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.3}$$

gives more efficient unbiased estimators and have smaller MSE than variance per unit i.e. $\hat{S}_y^2$.

Let the range information about C be known as C > C_0 (>1), then from the efficiency condition (4.7), we choose $C_0 = \frac{A\eta+1}{2}$ or $A\eta = 2C_0 - 1$ to get the class of estimators.

$$\hat{S}^2_{\frac{2C_0-1}{\eta}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2C_0-1}{\eta}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.4}$$

which are better than the ratio estimator since it has lesser mean sum of square. For example, if it is known that C > 3/2 we may choose $A = 2/\eta$ satisfying the efficiency condition (4.7) to obtain more efficient unbiased estimator.

$$\hat{S}^2_{\frac{2}{\eta}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2}{\eta}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.5}$$

than the ratio estimator $\hat{S}_r^2$ in the sense of having lesser mean square error.

If the range information about C be known as C > C_0 (>-1) then we choose A such that $C_0 = \frac{A\eta-1}{2}$ or $A\eta = 2C_0 + 1$satisfying the efficiency condition (4.8) to get the class of estimators.

$$\hat{S}^2_{\frac{2C_0+1}{\eta}} = \frac{s_y^2\left(\alpha\bar{X}+\beta\right)}{\alpha\bar{X}+\beta+\frac{2C_0+1}{\eta}\alpha\left(\bar{x}-\bar{X}\right)} \tag{6.6}$$

which are better than the product estimator in the sense of having lesser mean square error. Finally, from the comparative study, it is clear that $\hat{S}^2_{AM}$ is the most efficient estimator in the class of $\hat{S}^2_y$ and the ratio estimator $\hat{S}^2_r$. In case, the linear regression and the Walsh estimator they are equally efficient but the linear regression and Walsh estimator is unbiased only for the optimum value of the characterizing scalar whereas $\hat{S}^2_{AM}$ under the proposed Midzuno-Lahiri-Sen type sampling design is unbiased for all values of characterizing scalar A.

7 CONCLUSION

The main objective of current work is to develop family of unbiased sampling strategies for estimation of the population variance which are practical and provide substantial gain in efficiency. It is obvious from the numerical illustration; the proposed sampling strategies should be preferred over conventional estimators under SRSWOR. Hence, the proposed family of sampling strategies provides better estimation of population variance in terms of unbiasedness, efficiency and much more practical utility.

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