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Elliptical Magnetic Binary Problem when the Primaries are Oblate Spheroids

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Abstract

In this paper we have discussed the existence and linear stability of equilibrium points in the elliptical magnetic binary problem when the primaries are oblate spheroids. It is observed that there exists three collinear and two non-collinear equilibrium points we have observed that all the collinear and non-collinear equilibrium points are unstable. We have also observed that the oblatness of the primaries affects the position of the equilibrium points when we change the eccentricity e and mass parameter μ.

Keywords: equilibrium points, elliptical magnetic binary problem, stability.

1. INTRODUCTION

The elliptical restricted three-body problem describes the dynamical system more accurately on account that the realistic assumptions of the, motion of the primaries are subjected to move along the elliptical orbit. Several mathematician [1–6] have been discussed the different aspects of the elliptical restricted three-body problem.

Stormer [7] has studied the motion of a charged particle which is moving in the field of a magnetic dipole as a two body problem. A. Mavragnais [8-11] and Mohd. Arif [12-13] have studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles. Mohd. Arif [14] have discussed the motion of a charged particle when the dipoles move on elliptic orbits. In this paper we have discussed the existence and linear stability of equilibrium points in the elliptical magnetic binary problem when the primaries are oblate spheroids

2. EQUATION OF MOTION

The equation of motion for the planar elliptical magnetic binary problem with the effects of oblateness of primaries in a dimensionless, pulsating rotating, co-ordinate system are as follows,

$$\xi'' - \eta' \text{f} = U_\xi \tag{2.1}$$

$$\eta'' + \xi' \text{f} = U_\eta \tag{2.2}$$

Where

$$\text{f} = 2 - 2\zeta \left(\frac{1}{r_1^3} + \frac{I_1}{2(1-\mu)\, r_1^5} + \frac{1}{r_2^3} + \frac{I_2\, \lambda}{2\, \mu\, r_2^5} \right), \quad U_\xi = \frac{\partial U}{\partial \xi} \text{ and } U_\eta = \frac{\partial U}{\partial \eta}$$

$$U = (\xi^2 + \eta^2) \left\{ \frac{m}{2(1+e\, cos\gamma)} + \zeta \left(\frac{1}{r_1^3} + \frac{I_1}{2(1-\mu)\, r_1^5} + \frac{1}{r_2^3} + \frac{I_2\, \lambda}{2\, \mu\, r_2^5} \right) \right\} - \xi\, \zeta \left\{ \frac{\mu}{r_1^3} - \frac{\mu\, I_1}{2(1-\mu)\, r_1^5} - \frac{\lambda(1-\mu)}{r_2^3} - \frac{I_2\, \lambda(1-\mu)}{2\, \mu\, r_2^5} \right\} - \frac{(1-\mu)}{(1+e\, cos\gamma)} \left\{ \frac{1}{r_1} + \frac{I_1}{2(1-\mu)\, r_1^3} + \frac{1}{r_2} + \frac{I_1}{2\mu\, r_2^3} \right\} + \frac{2\, q\, \xi\, \eta\, e \sin\gamma}{m\, c\, a^{3/2}\, (1-e^2)^{3/2}} \tag{2.3}$$

$\zeta = \dfrac{q\sqrt{a(1-e^2)}}{m\, c\, (1+e\, cos\gamma)}$, $\quad r_1^2 = (\xi - \mu)^2 + \eta^2$, $\; r_2^2 = (\xi + 1 - \mu)^2 + \eta^2$, $\; \lambda = \dfrac{M_2}{M_1}$ $\quad$ (M_1, M_2 are the

magnetic moments of the primaries which lies perpendicular to the plane of the motion)

$I_1 = (1-\mu)\dfrac{(R_{1e}^2 - R_{1p}^2)}{5}$, $\quad I_2 = \mu \dfrac{(R_{2e}^2 - R_{2p}^2)}{5}$, $\; R_{ie}$ = equatorial radii of the primaries, R_{ie} = Polar radii of the primaries $(i = 1,2)$, $\; c$ = velocity of light.

Now introduce the averaged potential function of the problem with respect to true anomaly as:

$$U^* = \frac{1}{2\pi} \int_0^{2\pi} U\, d\gamma, \tag{2.4}$$

Then

$$U^* = (\xi^2 + \eta^2) \left\{ \frac{m}{2\sqrt{(1-e^2)}} + a^{\frac{1}{2}} \left(\frac{1}{r_1^3} + \frac{I_1}{2(1-\mu)\, r_1^5} + \frac{1}{r_2^3} + \frac{I_2\, \lambda}{2\, \mu\, r_2^5} \right) \right\} - \xi\, a^{\frac{1}{2}} \left\{ \frac{\mu}{r_1^3} - \frac{\mu\, I_1}{2(1-\mu)\, r_1^5} - \frac{\lambda(1-\mu)}{r_2^3} - \frac{I_2\, \lambda(1-\mu)}{2\, \mu\, r_2^5} \right\} - \frac{(1-\mu)}{\sqrt{(1-e^2)}} \left\{ \frac{1}{r_1} + \frac{I_1}{2(1-\mu)\, r_1^3} + \frac{1}{r_2} + \frac{I_1}{2\mu\, r_2^3} \right\}$$

is the modified potential function where e is the eccentricity and a is the semi-major axis of the orbit, μ is the mass parameter, for numerical calculation we have taken a particular case $q = mc$.

3. EQUILIBRIUM POINTS

To find the locations of the equilibrium points, we must solve the following equations

$$\frac{\partial U^*}{\partial \xi} = 0 \text{ and } U_\eta = \frac{\partial U^*}{\partial \eta} = 0 \qquad (3.1)$$

We group the solution of equation (3.1) into two kinds; those $\eta = 0$ the collinear equilibrium points and those with $\eta \neq 0$ non-collinear equilibrium points.

We investigate the existence and location of the collinear equilibrium points into the following three intervals.

$$C_1 = \{\xi : \xi > \mu\},\ C_2 = \{\xi : -(\mu - 1) < \xi \leq \mu\} \text{ and } C_3 = \{\xi : \xi \leq -(\mu - 1)\}.$$

If $\xi \in C_1$ the substitution $r_1 = \xi - \mu = \tau$ and $r_2 = \xi + 1 - \mu = \tau + 1$ in (3.1) we have 15th degree equation

$$2\,\mu(1-\mu)[(\tau+1)^7\tau^7(\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(\tau+1)^7\tau^4\,(\tau+\mu) + 3(\tau+1)^4\tau^7\,(\tau+\mu)\lambda - (\tau+1)^7\tau^4(\tau+\mu) - (\tau+1)^4\tau^7(\tau+\mu)\,\lambda - (\tau+1)^7\tau^5 + \lambda\,(\tau+1)^5\tau^7\} - (\tau+1)^7\tau^5(1-\mu) - (\tau+1)^5\tau^7\mu] + a^{\frac{1}{2}}\sqrt{1-e^2}\{2(\tau+\mu)(\tau+1)^7\tau^2\mu\,I_1 + 2\lambda(\tau+\mu)(1-\mu)(\tau+1)^2\tau^7 I_2 - 5(\tau+\mu)^2\,(\tau+1)^7\tau\mu\,I_1 + \lambda(1-\mu)\tau^7(\tau+1)I_2 + \mu^2(\tau+1)^7\tau^2 I_1 + \lambda(1-\mu)^2\tau^7(\tau+1)^2 I_2 - 5\tau(\tau+\mu)\mu^2(\tau+1)^7 I_1 - 5\lambda(1-\mu)^2\tau^7(\tau+\mu)(\tau+1)I_2\} + 3I_1\mu(\tau+1)^7\tau^3 + 3\,I_2\,\tau^7(\tau+1)^3(1-\mu) = 0 \qquad (3.2)$$

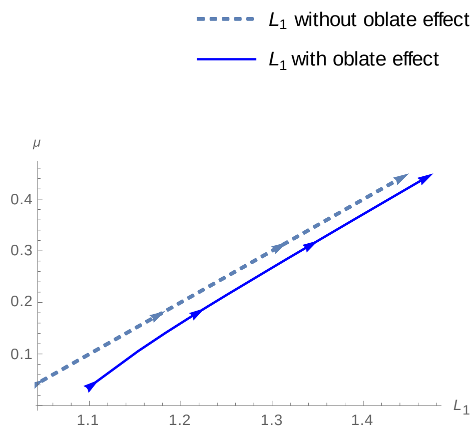
If $\xi \in C_2$ the substitution $r_1 = \xi - \mu = 1 - \tau$ and $r_2 = \xi + 1 - \mu = \tau$ in (3.1) again given 15th degree equation

$$2\,\mu(1-\mu)[(1-\tau)^7\tau^7(1-\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(1-\tau)^4\tau^7(1-\tau+\mu) + (1-\tau)^7\tau^4\,(1-\tau+\mu)\lambda - (1-\tau)^4\tau^7(1-\tau+\mu) - (1-\tau)^7\tau^4(1-\tau+\mu)\,\lambda - (1-\tau)^5\tau^7 + \lambda\,(1-\tau)^7\tau^5\} - (1-\tau)^5\tau^7(1-\mu) - (1-\tau)^7\tau^5\mu] + a^{\frac{1}{2}}\sqrt{1-e^2}\{2(1-\tau+\mu)\tau^7(1-\tau)^2\mu\,I_1 + 2\lambda(1-\tau+\mu)(1-\mu)\tau^2(1-\tau)^7 I_2 - 5(1-\tau+\mu)^2\,\tau^7(1-\tau)\mu\,I_1 + \lambda(1-\mu)(1-\tau)^7\tau I_2 + \mu^2\tau^7(1-\tau)^2 I_1 + \lambda(1-\mu)^2(1-\tau)^7\tau^2 I_2 - 5(1-\tau)(1-\tau+\mu)\mu^2\tau^7 I_1 - 5\lambda(1-\mu)^2(1-\tau)^7(1-\tau+\mu)\tau I_2\} + 3I_1\mu\tau^7(1-\tau)^3 + 3\,I_2\,(1-\tau)^7\tau^3(1-\mu) = 0 \qquad (3.3)$$

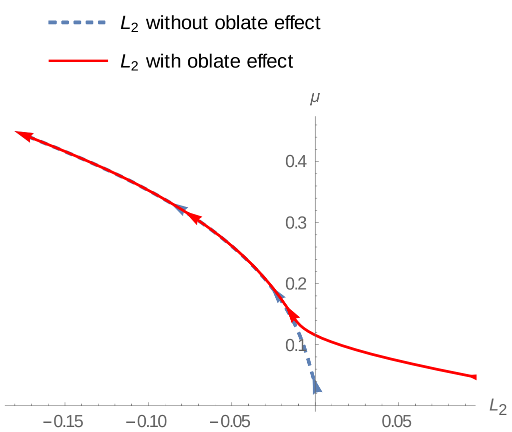
And when $\xi \in C_3$ the substitution $r_1 = \mu - \xi = 1 + \tau$ and $r_2 = -(\xi + 1 - \mu) = \tau$ in (3.1) we have again 15th degree equation

$$2\,\mu(1-\mu)[(1+\tau)^7\tau^7(-1-\tau+\mu)\,m - a^{\frac{1}{2}}\sqrt{1-e^2}\{3(1+\tau)^4\tau^7(-1-\tau+\mu) + (1+\tau)^7\tau^4\,(-1-\tau+\mu)\lambda - (1+\tau)^4\tau^7(-1-\tau+\mu) - (1+\tau)^7\tau^4(-1-\tau+\mu)\,\lambda - (1+\tau)^5\tau^7 + \lambda\,(1+\tau)^7\tau^5\} - (1+\tau)^5\tau^7(1-\mu) - (1+\tau)^7\tau^5\mu] + a^{\frac{1}{2}}\sqrt{1-e^2}\{2(-1-\tau+\mu)\tau^7(1+\tau)^2\mu\,I_1 + 2\lambda(-1-\tau+\mu)(1-\mu)\tau^2(1+\tau)^7 I_2 - 5(-1-\tau+\mu)^2\,\tau^7(1+\tau)\mu\,I_1 + \lambda(1-\mu)(1+\tau)^7\tau I_2 + \mu^2\tau^7(1+\tau)^2 I_1 + \lambda(1-\mu)^2(1+\tau)^7\tau^2 I_2 - 5(1+\tau)(-1-\tau+\mu)\mu^2\tau^7 I_1 - 5\lambda(1-\mu)^2(1+\tau)^7(-1-\tau+\mu)\tau I_2\} + 3I_1\mu\tau^7(1+\tau)^3 + 3\,I_2\,(1+\tau)^7\tau^3(1-\mu) = 0 \tag{3.4}$$

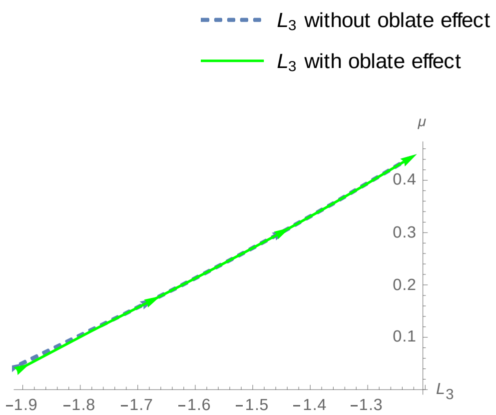
By use the Mathmatica-11 we solve the equations (3.2), (3.3) and (3.4) numerically and we have observed that the each equation have one real root for various values of mass parameter μ and eccentricity e and these roots are denoted by L_1, L_2 and L_3 respectively. The variation in the values of L_i $(i = 1,2,3)$ for various values of μ and e are shown in the figures (1…..6). We have seen that the oblateness of the primaries affects the position of the equilibrium points. The point L_1 shifted towards the primaries but this deviation decreases as μ increases fig(1).



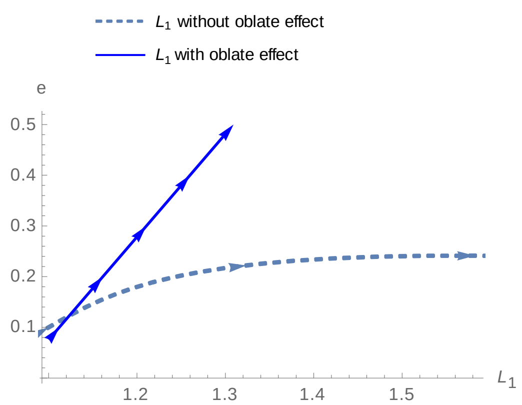
Fig(1)



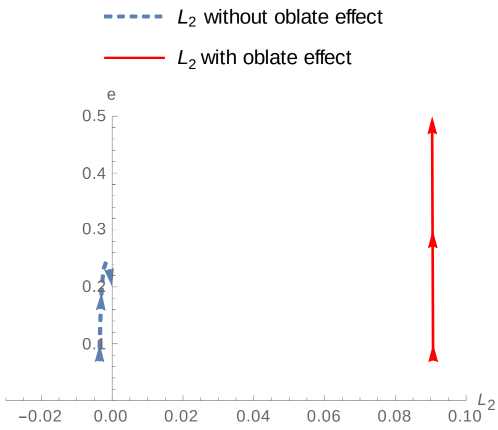
Fig(2)



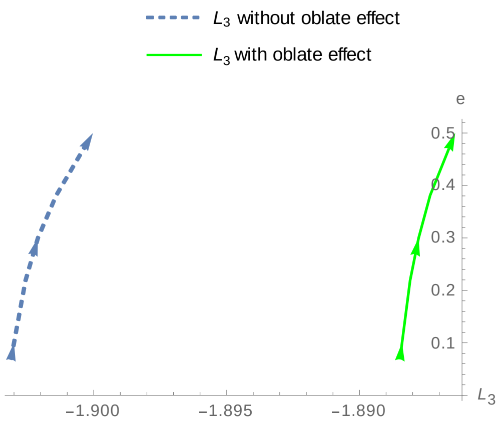
Fig(3)



Fig(4)



Fig(5)



Fig(6)

We have also observed that this effect is insignificant of the position of L_2 and L_3 for different values of μ. (fig(2) and (3)). Figures(4,5 and 6) shows that the oblateness of the primaries are more effective when we varying the value of eccentricity e. Here we have observed that the point L_1 and L_2 goes away from the centre of mass while the point L_3 move towards the centre of mass. Here we have seen that the deviation of L_1 increases as e increases.

The non-collinear equilibrium points denoted by L_4 and L_5 are the solution of the equation (3.1) when $\eta \neq 0$. In table (5) we give the position of the points L_4 and L_5 for various values of e. We observed that both L_4 and L_5 goes away from the centre of mass as e increases.

4. STABILITY OF EQUILIBRIUM POINTS

Let (ξ_0 , η_0) be the coordinate of any one of the equilibrium point and let α, β denote small displacement from the equilibrium point. Therefore we have

$$\alpha = \xi - \xi_0 ,$$

$$\beta = \eta - \eta_0 ,$$

Put this value of ξ and η in equation (2.1) and (2.2), we have the variation equation as:

$$\alpha'' - \beta' f_0 = \alpha\left(U^*{}_{\xi\xi}\right)^0 + \beta\left(U^*{}_{\xi\eta}\right)^0 \qquad (4.1)$$

$$\beta'' + \alpha' f_0 = \alpha\left(U^*{}_{\xi\eta}\right)^0 + \beta\left(U^*{}_{\eta\eta}\right)^0 \qquad (4.2)$$

Retaining only linear terms in α and β. Here superscript indicates that these partial derivative of U^* are to be evaluated at the equilibrium point $(\xi_0\ ,\eta_0)$. So the characteristic equation at the equilibrium points is

$$\lambda_1{}^4 + \lambda_1{}^2\left\{f_0{}^2 - \left(U^*{}_{\xi\xi}\right)^0 - \left(U^*{}_{\xi\xi}\right)^0\right\} + \left(U^*{}_{\xi\xi}\right)^0\left(U^*{}_{\eta\eta}\right)^0 - \left(U^*{}_{\xi\eta}\right)^{0^2} = 0 \qquad (4.3)$$

The equilibrium point $(\xi_0\ ,\eta_0)$ is said to be stable if all the four roots of equation (4.3) are either negative real numbers or pure imaginary.

From tables 1, 2, 3,4 and 5 it is clear that all the equilibrium point in these tables are unstable

Table (1) Table (2)

μ	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	1.11051	± 1.49929	$\pm 1.80231i$	.090506	± 7.5678	$\pm 32910.1i$
.1	1.14737	± 1.5227	$\pm 1.7574i$	$-.00256$	± 5.46578	$\pm 2054.7i$
.15	1.19086	± 1.5128	$\pm 1.7229i$	$-.01382$	± 4.3640	$\pm 514.9i$
.20	1.23668	± 1.4925	$\pm 1.6887i$	$-.026739$	± 3.7269	$\pm 201.5i$
.25	1.28366	± 1.4672	$\pm 1.6535i$	$-.04414$	± 3.2343	$\pm 98.378i$
.30	1.33132	± 1.4391	$\pm 1.6172i$	$-.06705$	± 2.7727	$\pm 56.457i$
.35	1.37944	± 1.4091	$\pm 1.5795i$	$-.09647$	± 2.1108	$\pm 37.906i$
.40	1.42789	± 1.3781	$\pm 1.5406i$	$-.13367$	± 1.6406	$\pm 31.8288i$
.45	1.4766	± 1.3462	$\pm 1.5003i$	$-.18032$	± 4.3074	$\pm 42.162i$
e	L_1	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$	L_2	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	1.11206	± 1.4503	$\pm 1.7093i$	.09065	± 7.7375	$\pm 32561.2i$
.2	1.16174	± 1.3137	$\pm 1.6978i$	.09062	± 7.7016	$\pm 32633.5i$
.3	1.21118	± 1.1911	$\pm 1.7198i$	.09057	± 7.6401	$\pm 32754.4i$
.4	1.26036	± 1.08711	$\pm 1.7693i$	.09493	± 7.5518	$\pm 32941.8i$
.5	1.30922	± 1.0048	$\pm 1.8421i$	.09393	± 7.4324	$\pm 33194.8i$
.6	1.35766	± 0.94701	$\pm 1.9362i$	.09023	± 7.2823	$\pm 33592.8i$

Table (3)

μ	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.05	-1.88741	± 0.15031	$\pm 6.6599i$
.1	-1.80329	± 0.22072	$\pm 7.13629i$
.15	-1.71202	± 0.23128	$\pm 7.8052i$
.20	-1.62199	± 0.22257	$\pm 8.56918i$
.25	-1.53458	± 0.20494	$\pm 9.41004i$
.30	-1.45007	± 0.18915	$\pm 10.3194i$
.35	-1.36851	± 0.19115	$\pm 11.2887i$
.40	-1.29007	± 0.22744	$\pm 12.298i$

Table (4)

e	L_3	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	-1.88843	± 0.07587	$\pm 6.6432i$
.2	-1.888823	± 0.096336	$\pm 6.6464i$
.3	-1.88788	± 0.12381	$\pm 6.6522i$
.4	-1.88731	± 0.15557	$\pm 6.6615i$
.5	-1.88642	± 0.19085	$\pm 6.6759i$
.6	-1.88497	± 0.22997	$\pm 6.6992i$

Table (5)

e	$(L_{4,5})_\xi$	$(L_{4,5})_\eta$	$(\lambda_1)_{1,2}$	$(\lambda_1)_{3,4}$
.1	-0.58901	± 0.62614	± 3.1659	$\pm 8.5936i$
.2	-0.59038	± 0.62515	± 3.1633	$\pm 8.6320i$
.3	-0.59278	± 0.62342	± 3.1588	$\pm 8.6998i$
.4	-0.5964	± 0.62076	± 3.1522	$\pm 8.8053i$
. 5	-0.6016	± 0.61691	± 3.14310	$\pm 8.9615i$
.6	-0.60900	± 0.61130	± 3.1310	$\pm 9.19617i$

CONCLUSION

In this paper we have observed that there exist five equilibrium points, three collinear and two non-collinear. It is fined that the oblateness of the primaries are more effective for different values of eccentricity e in the compression of mass parameter μ on the position of collinear-equilibrium points L_i $(i = 1,2,3)$. We also observed that the points L_4 and L_5 move away from the centre of mass as as e increases. We have also observed that all points given in tables 1,2,3 4 and 5 are unstable.

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