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Contra Harmonic Mean Labeling of Disconnected Graphs

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Abstract

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct element f(x) from 0,1…..................q in such a way that when each edge e = uv is labeled with

f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Keywords: Graph, Contra Harmonic mean labeling, Contra Harmonic mean graphs, Path, Cycle, Comb, etc

1. INTRODUCTION

All graph in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harray [2]. S. Somasundaram and R. Ponraj introduced mean labeling for some standard graphs in 2013. S.S. Sandhya and S. Somasundaram introduced Harmonic mean labeling of graph. We have introduced Contra Harmonic mean labeling in [5].In this paper we investigate the Contra

Harmonic mean labeling behaviour of some disconnected graphs. The following definition are useful for our present study.

Definition 1.1

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1…..........q in such a way that when each edge e = uv is labeled with f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels. Then f is called Contra Harmonic mean labeling of G.

Definition 1.2: The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$

Definition 1.3: The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Theorem 1.4: Any Path is a Contra Harmonic mean graph.

Theorem 1.5: Any Cycle is a Contra Harmonic mean graph.

Theorem 1.6: Any Comb is a Contra Harmonic mean graph.

Theorem 1.7: Any Crown is a Contra Harmonic mean graph.

2. MAIN RESULTS

Theorem 2.1: $C_m \cup P_n$ is a Contra Harmonic mean graph ,for $m \geq 3$ and $n \geq 1$

Proof: Let C_m be the cycle $u_1 \ldots u_m$ and P_n be the path $v_1 \ldots v_n$

Let $G = C_m \cup P_n$.

Define a function f: $V(G) \rightarrow \{0,1,\ldots,q\}$ by

$f(u_i) = i-1,\ 1 \leq i \leq m-1$, $f(u_m) = m$

$f(v_i) = m+i,\ 1 \leq i \leq n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = i,\ 1 \leq i \leq m$

$f(v_i, v_{i+1}) = m+i, 1 \le i \le n-1$

/ $C_m \cup P_n$ is Contra Harmonic mean graph

Example 2:2 The Contra Harmonic mean labeling of $C_5 \cup P_5$ is

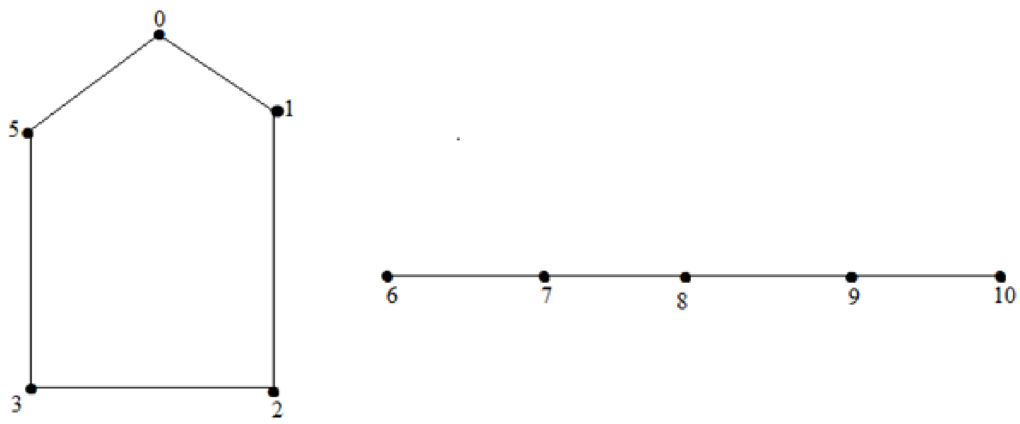

Figure 1

Theorem 2.3: $C_m \cup (P_n \odot K_1)$is a Contra Harmonic mean graph.

Proof:

Let C_m be a cycle with vertices $u_1 \ldots\ldots u_m$ and Let $v_1 \ldots\ldots\ldots v_n$ be the path P_n. and let w_i be the vertices which is joined to the vertex v_i, $1 \le i \le n$ of the path P_n. The resultant graph is $P_n \odot K_1$. Let $G = C_m \cup (P_n \odot K_1)$

Define a function $f : V(G) \rightarrow \{0,1\ldots q\}$ by

$f(u_i) = i-1, 1 \le i \le m-1$

$f(u_m) = m$

$f(v_i) = m+2i-1, 1 \le i \le n$

$f(w_i) = m+2i, 1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = i, 1 \le i \le m$

$f(v_i, v_{i+1}) = m+2i, 1 \le i \le n-1$

$f(v_I, w_i) = m+2i-1, 1 \le i \le n$

Then f is a Contra Harmonic mean graph of G.

Example 2.4: The Contra Harmonic mean labeling of $C_5 \cup (P_5 \odot K_1)$ is

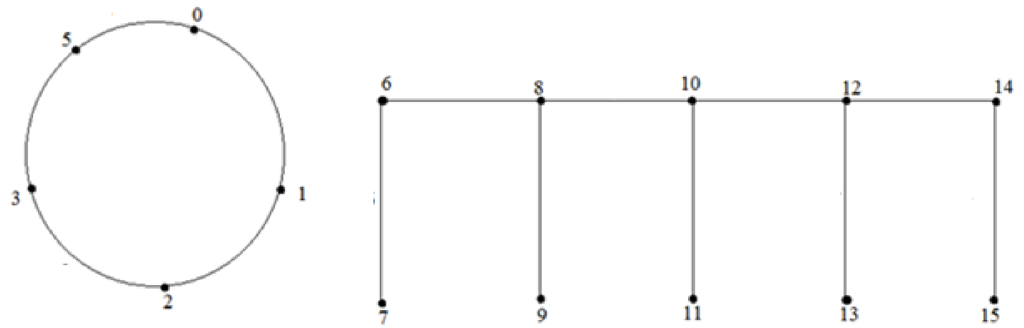

Figure: 2

Theorem 2.5: $(C_m \odot K_1) \cup P_n$ is a Contra Harmonic mean graph.

Proof: Let $u_1, u_2 \ldots\ldots\ldots\ldots u_m$ be a cycle C_m and v_i be the vertex which is joined to the vertex u_i of the cycle C_m, $1 \le i \le m$.

The resultant graph is $C_m \odot K_1$. Let $w_1 w_2 \ldots. w_n$ be the path P_n

Let $G = (C_m \odot K_1) \cup P_n$.

Define a function

f: $V(G) \to \{0,1,\ldots q\}$ by

$f(u_i) = 2i-2,\ 1 \le i \le m-1$

$f(u_n) = 2m$

$f(v_i) = 2i-1,\ 1 \le i \le m$

$f(w_i) = 2m+i,\ 1 \le i \le n$

Then the distinct edge labels are

$f(u_i\ u_{i+1}) = 2i,\ 1 \le i \le m$

$f(u_i\ v_i) = 2i-1,\ 1 \le i \le m$

$f(w_i\ w_{i+1}) = 2m+i,\ 1 \le i \le n-1$

/ $(C_m \odot K_1) \cup P_n$ is a Contra Harmonic mean graph

Example 2.6: The Contra Harmonic mean labeling of $(C_6 \odot K_1) \cup P_5$

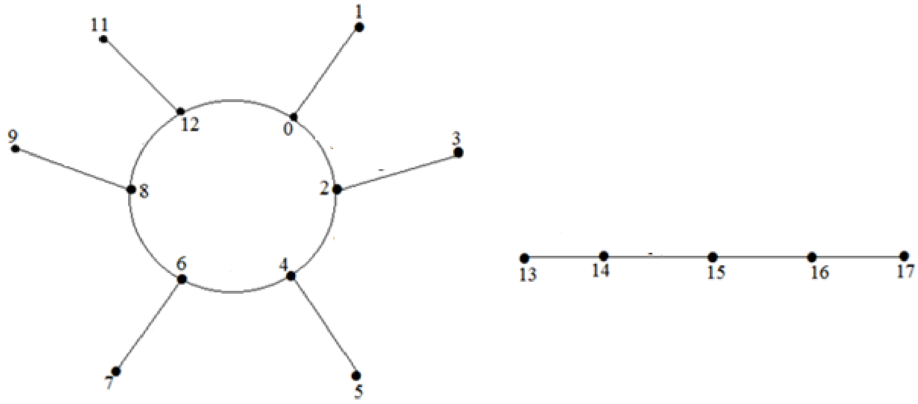

Figure: 3

Theorem 2:7

$(C_m \odot K_1) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof: Let $u_1u_2\ldots.u_m$ be the cycle C_m and let v_i be the pendent vertex joined to the vertex u_i of $C_m 1 \le i \le m$.The resultant graph is $C_m \odot K_1$. Let $w_1\ldots\ldots w_n$ be the path P_n and t_i be the vertex which is joined to the vertex w_i, $1 \le i \le n$ of the path P_n. The resultant graph is $P_n \odot K_1$.

Let $G = (C_m \odot K_1) \cup (P_n \odot K_1)$.

Define a function $f: V(G) \to \{0,1\ldots..q\}$ by

$f(u_i) = 2i-2$, $1 \le i \le m-1$,$f(u_n) = 2m$,

$f(v_i) = 2i-1$, $1 \le i \le m$

$f(w_i) = 2m+2i-1$, $1 \le i \le n$

$f(t_i) = 2m+2i$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_iu_{i+1}) = 2i$, $1 \le i \le m$

$f(u_i v_i) = 2i-1$, $1 \le i \le m$

$f(w_iw_{i+1}) = 2m+2i$, $1 \le i \le n-1$

$f(w_it_i) = 2m+2i-1$, $1 \le i \le n$

/ $(C_m \odot K_1) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Example: 2.8

The Contra Harmonic mean labeling of $(C_6 \odot K_1) \cup (P_5 \odot K_1)$

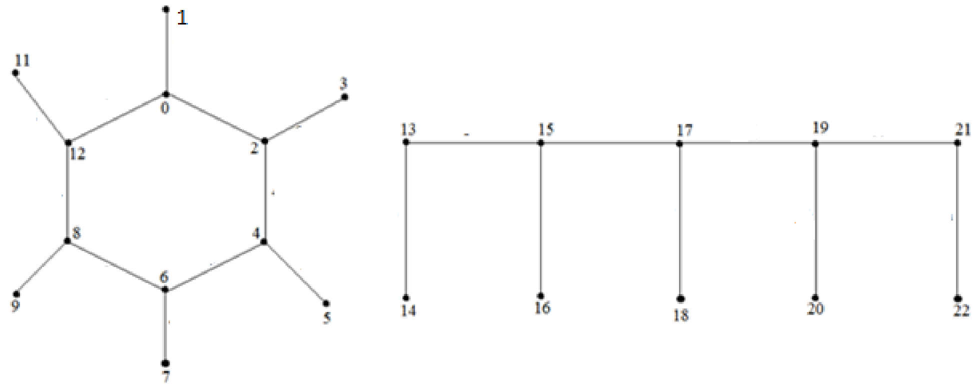

Figure: 4

Theorem 2.9: $(C_m \odot K_1) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Proof: Let $u_1 u_2 \ldots u_m$ be the cycle C_m and let v_i be the vertex joined to the vertex u_i of C_m $1 \le i \le m$. The resultant graph is $C_m \odot K_1$. Let $w_1 \ldots w_n$ be the path P_n and let t_i and s_i be the vertices which are joined to the vertex w_i of path P_n ,$1 \le i \le n$. The resultant graph in $P_n \odot \overline{K_2}$

Let $G = (C_m \odot K_1) \cup (P_n \odot \overline{K_2})$

Define $f: V(G) \to \{0,1,\ldots q\}$ by

$f(u_i) = 2i-2$, $1 \le i \le m-1$,$f(u_m) = 2m$

$f(v_i) = 2i-1$, $1 \le i \le m$

$f(w_i) = 2m+3i-2$ $1 \le i \le n$

$f(t_i) = 2m+3i-1$, $1 \le i \le n$

$f(s_i) = 2m+3i$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 2i$, $1 \le i \le m$

$f(u_i v_i) = 2i-1$, $1 \le i \le m$

$f(w_i w_{i+1}) = 2m+3i$, $1 \le i \le n-1$

$f(w_i t_i) = 2m+3i-2$, $1 \le i \le n$

$f(w_i s_i) = 2m+3i-1, 1 \leq i \leq n$

/ $(C_m \odot K_1) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph of G.

Example 2.10 Contra Harmonic mean labeling of $(C_6 \odot K_1) \cup (P_4 \odot \overline{K_2})$

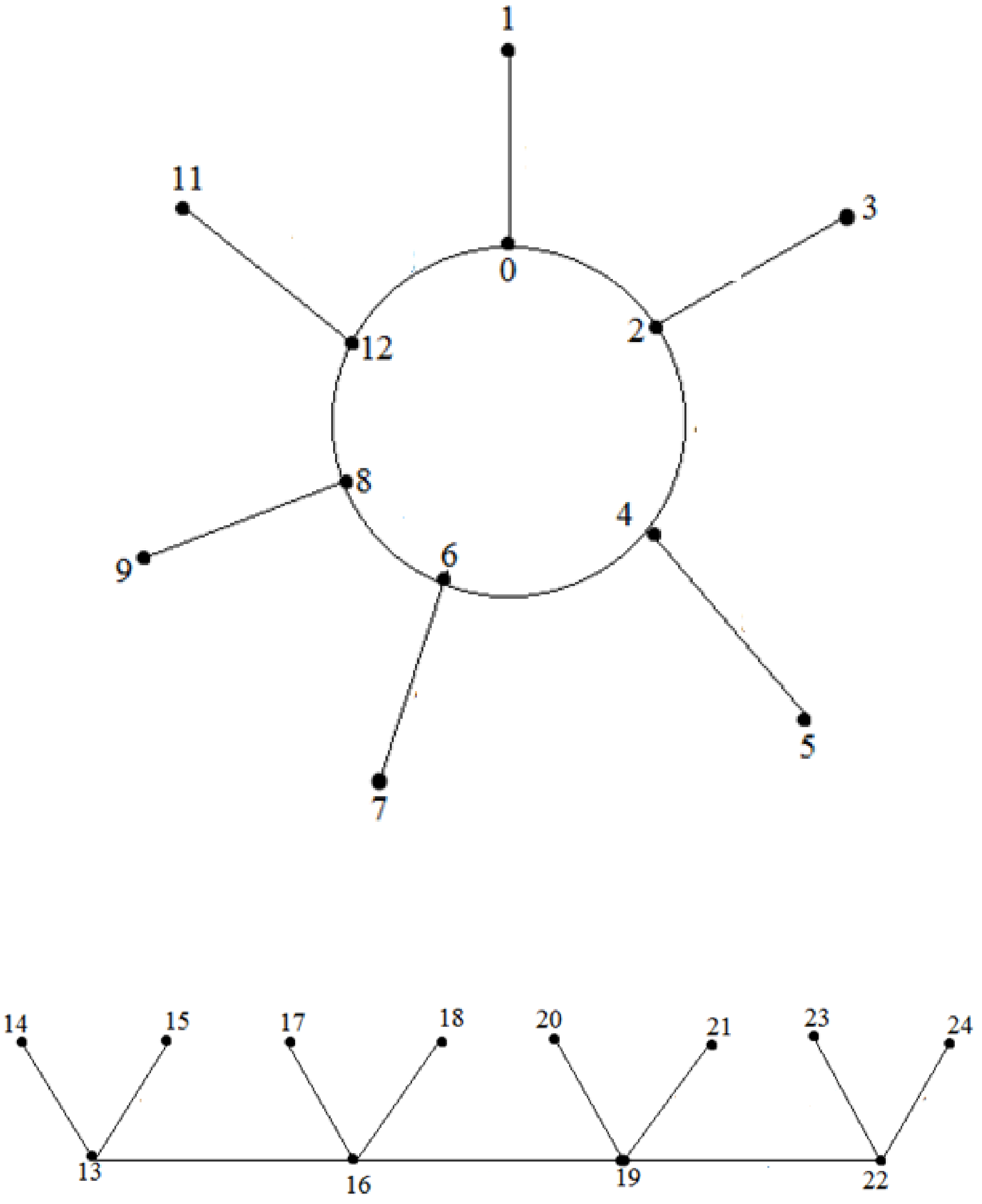

Figure: 6

Theorem 2.11 : $(C_m \odot \overline{K_2}) \cup P_n$ is a Contra Harmonic mean graph

Proof: $u_1u_2 \ldots u_mu_1$ be a cycle C_m and let v_i, w_i be the vertices that are joined to the vertex u_i $1 \le i \le m$ of the cycle C_m.

Let $G = (C_m \odot \overline{K_2}) \cup P_n$

Define $f: V(G) \rightarrow \{0, 1 \ldots .q\}$

by

$f(u_i) = 3i-2$, $1 \le i \le m-1$,$f(u_m) = 3m-1$

$f(v_i) = 3i-3$, $1 \le i \le m$

$f(w_i) = 3i-1$, $1 \le i \le m-1$,$f(w_m) = 3m$

$f(s_i) = 3m+i$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 3i$, $1 \le i \le m-1$,$f(u_m, u_1) = 3m-1$

$f(u_i v_i) = 3i-2$, $1 \le i \le m$

$f(u_iw_i) = 3i-1$, $1 \le i \le m-1$, $f(u_mw_m) = 3m$

$f(s_i s_{i+1}) = 3m+i$, $1 \le i \le n-1$

$(C_m \odot \overline{K_2}) \cup P_n$ is a Contra Harmonic mean graph.

Example : 2.12 The Contra Harmonic mean labeling of ($C_5 \odot \overline{K_2}$)$\cup$ P_6 is

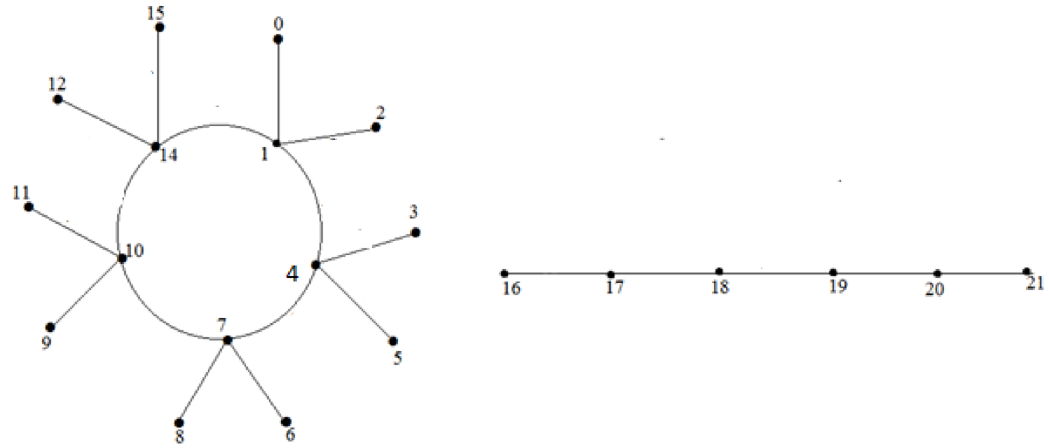

Figure 5

Theorem 2.13($C_m \odot \overline{K_2}$) $\cup$($P_n \odot K_1$) is a Contra Harmonic mean graph.

Proof: Let u_1, u_2,….u_m be the cycle C_m. Let vi, w_i be the vertices that are joined to the vertex u_i, $1 \leq i \leq m$ of the cycle C_m.

Let $s_1 s_2 \ldots s_n$ be the path P_n and t_i be the vertex that are joined to the vertex s_i, $1 \leq i \leq n$ of P_n.

Let G = ($C_m \odot \overline{K_2}$) $\cup$($P_n \odot K_1$)

Define f: V(G) $\rightarrow$ {0,1,2,…..q} by

$f(u_i) = 3i-2$, $1 \leq i \leq m-1$,$f(u_m) = 3m-1$

$f(v_i) = 3i-3$, $1 \leq i \leq m$

$f(w_i) = 3i-1$, $1 \leq i \leq m-1$, $f(w_m) = 3m$

$f(s_i) = 3m+2i-1$, $1 \leq i \leq n$

$f(t_i) = 3m+2i$, $1 \leq i \leq n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 3i$, $1 \leq i \leq m-1$,$f(u_m, u_1) = 3m-1$

$f(u_i\ v_i) = 3i-2$, $1 \leq i \leq m$

$f(u_i w_i) = 3i-1$, $1 \leq i \leq m-1$, $f(u_m w_m) =3m$

$f(s_i, s_{i+1}) = 3m+2i$, $1 \le i \le n-1$

$f(s_i t_i) = 3m+2i-1$, $1 \le i \le n$

/ $(C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Example : 2.14

The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_6 \odot K_1)$ is

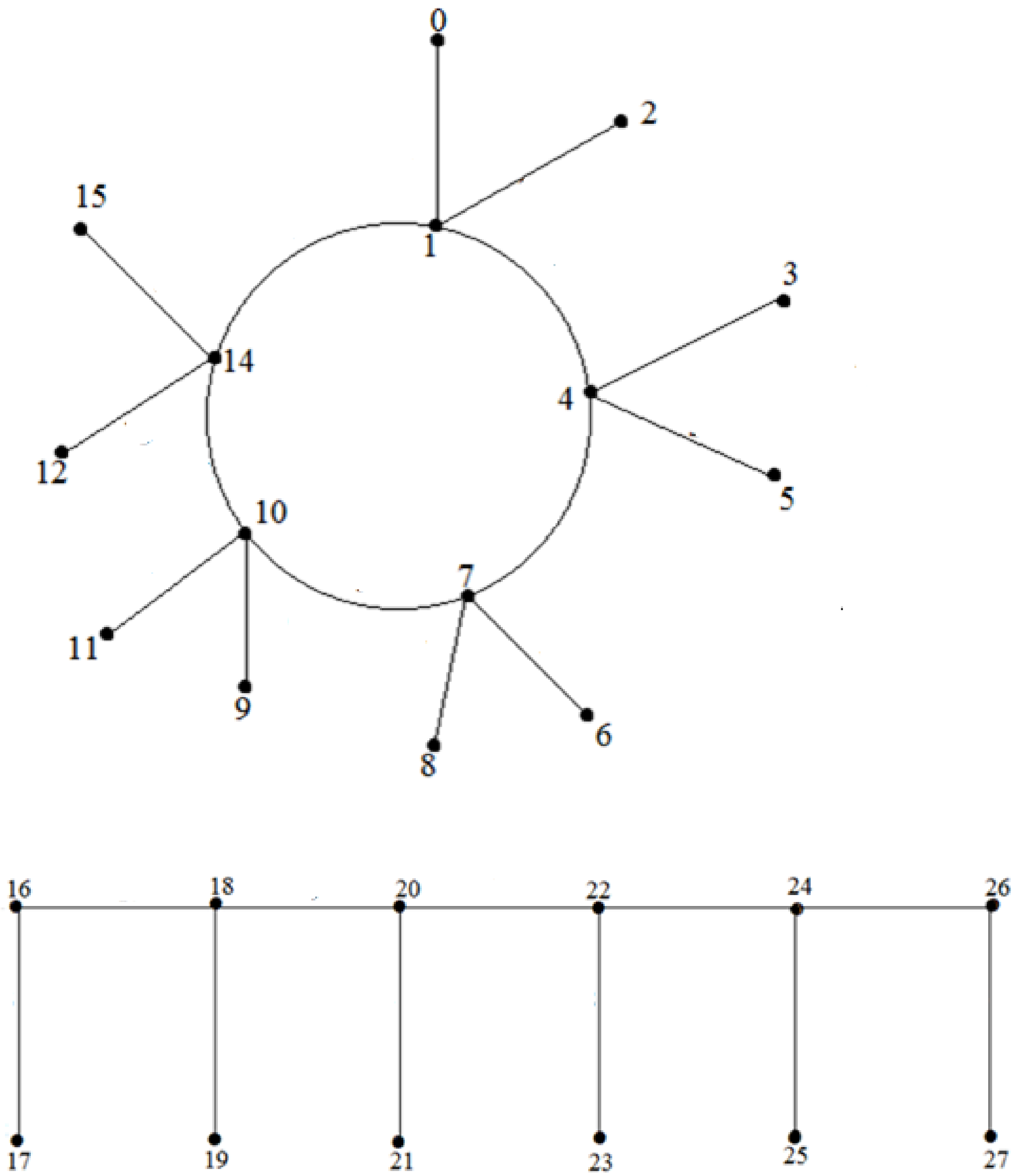

Figure: 7

Theorem : 2.15

Proof: $(C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Let $u_1 \ldots u_m$ be the cycle C_m and let v_i, w_i be the vertices that are joined to vertex u_i $1 \le i \le m$ of C_m. Let $z_1 \ldots z_n$ be the path P_n and let s_i, t_i be the vertices that are joined to the vertex z_i of the path P_n $1 \le i \le n$.

Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$

Define a function $f: V(G) \to \{0, 1 \ldots q\}$ by

$f(u_i) = 3i-2, 1 \le i \le m-1, f(u_n) = 3m-1$

$f(v_i) = 3i-3, 1 \le i \le m$

$f(w_i) = 3i-1, 1 \le i \le m-1, f(u_m) = 3m$

$f(z_i) = 3m+3i-2, 1 \le i \le n$

$f(s_i) = 3m+3i-1, 1 \le i \le n$

$f(t_i) = 3m+3i, 1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 3i, 1 \le i \le m-1, f(u_m u_1) = 3m-1$

$f(u_i v_i) = 3i-2, 1 \le i \le m$

$f(u_i w_i) = 3i-1, 1 \le i \le m-1, f(u_i w_i) = 3m$

$f(z_i z_{i+1}) = 3m+3i-2, 1 \le i \le n$

$f(z_i s_i) = 3m+3i-2, 1 \le i \le n$

$f(z_i t_i) = 3m+3i-1, 1 \le i \le n$

/ $(C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Example 2.16:

The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_4 \odot \overline{K_2})$

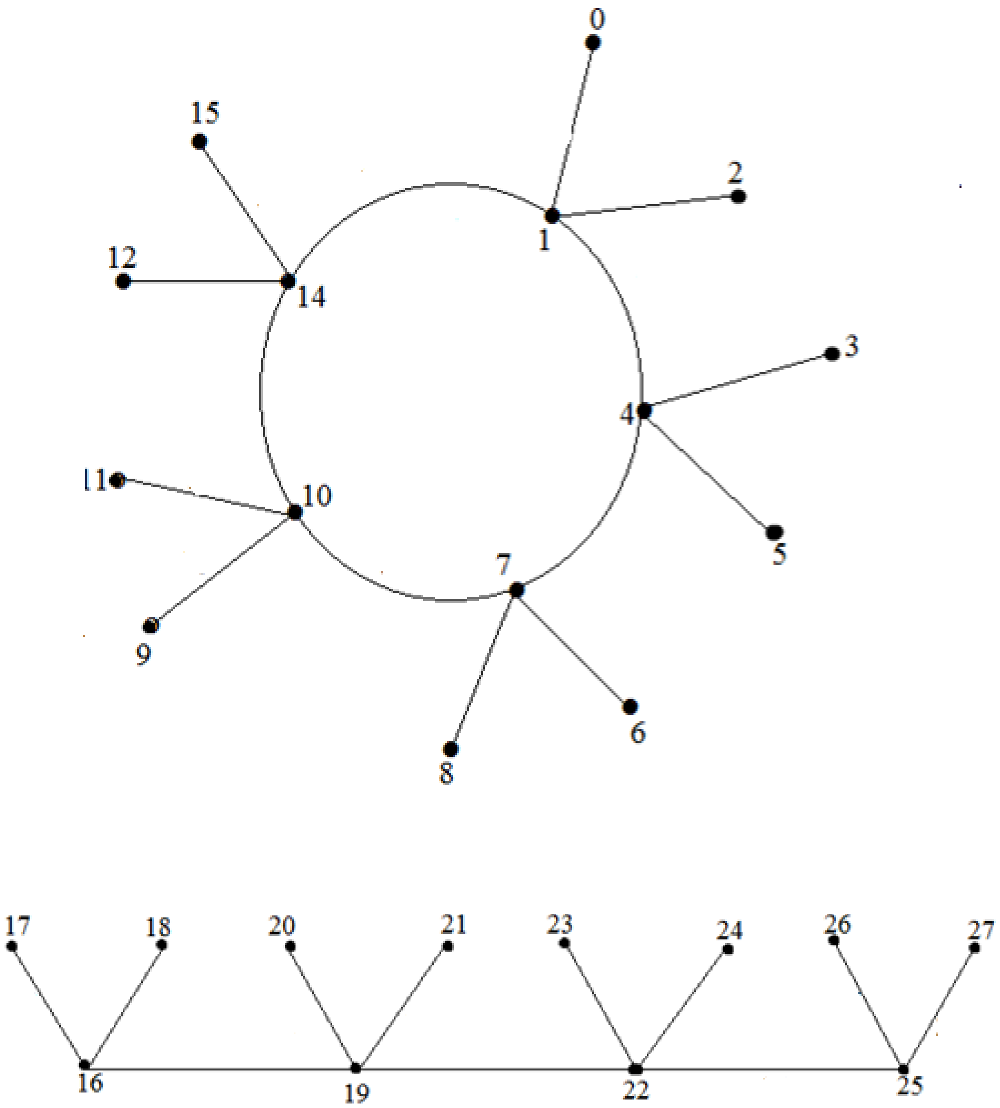

Figure: 8

Theorem 2: 17

$(C_m \odot \overline{K_2}) \cup (P_n \odot K_3)$ is a Contra Harmonic mean graph.

Proof: Let $u_1 \ldots\ldots u_m$ be a cycle C_m and let v_i, w_i be the vertices joined to the vertex u_i $1 \le i \le m$.

Let $z_1 \ldots . z_n$ be the path P_n and let s_i, t_i be the vertex of K_3 that are joined to the vertex z_i of the path P_n $1 \le i \le n$

Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot K_3)$

Define $f: V(G) \rightarrow \{0,1 \ldots . q\}$ by

$f(u_i) = 3i-2$ $1 \le i \le m-1$,$f(u_m) = 3m-1$

$f(v_i) = 3i-3$, $1 \le i \le m$

$f(w_i) = 3i-1$, $1 \le i \le m-1$, $f(w_m) = 3m$

$f(z_i) = 3m+4i-3$, $1 \le i \le n$

$f(s_i) = 3m+4i-2$, $1 \le i \le n$

$f(t_i) = 3m+4i-1$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 3i$, $1 \le i \le m-1$, $f(u_m u_1) = 3m-1$

$f(u_i, v_i) = 3i-2$, $1 \le i \le m$

$f(u_i w_i) = 3i-1$, $1 \le i \le m-1$, $f(u_m w_m) = 3m$

$f(z_i z_{i+1}) = 3m+4i$, $1 \le i \le n-1$

$f(z_i s_i) = 3m+4i-3$, $1 \le i \le n$

$f(z_i t_i) = 3m+4i-1$, $1 \le i \le n$

$f(s_i t_i) = 3m+4i-2$, $1 \le i \le n$

/ $(C_m \odot \overline{K_2}) \cup (P_n \odot K_2)$ is a Contra Harmonic mean graph.

Example 2: 18: The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_4 \odot K_3)$ is

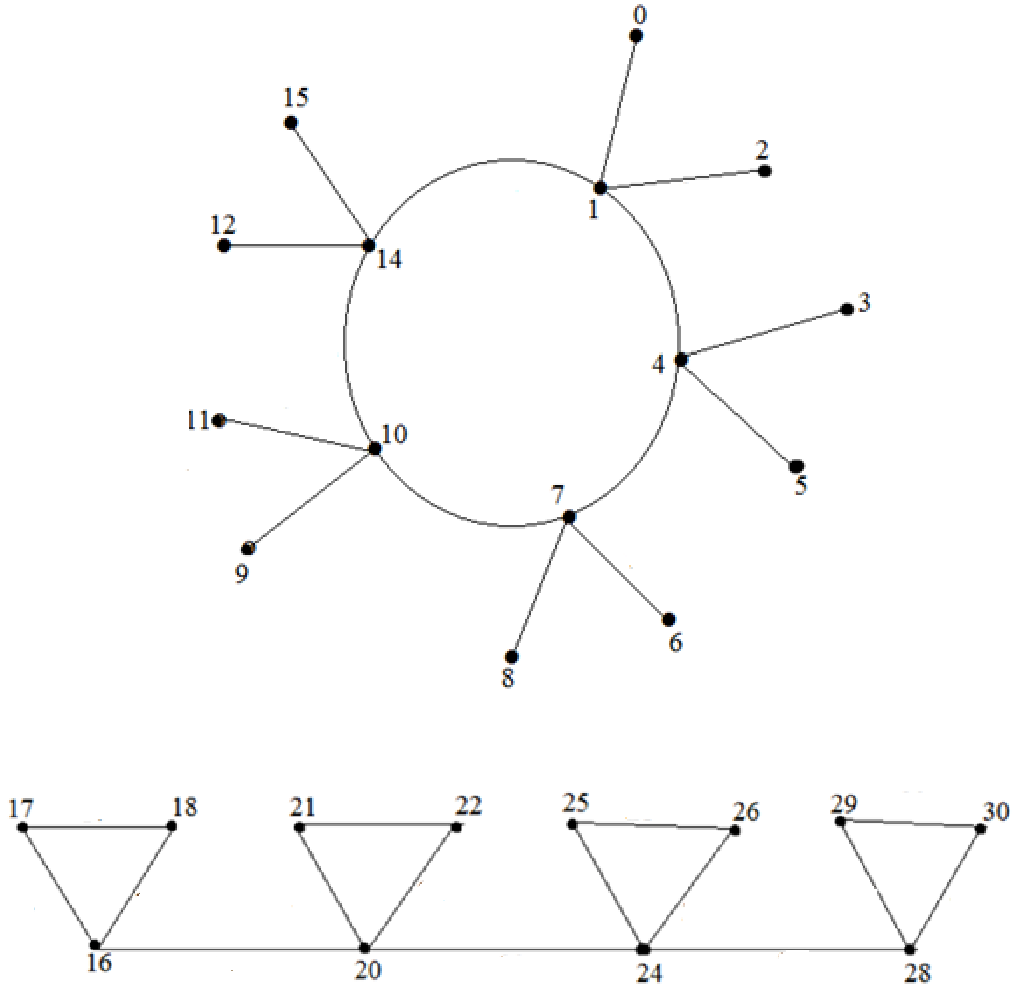

Figure: 9

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