

Numerical Analysis of Riccati equation using Differential Transform Method, He Laplace Method and Adomain Decomposition Method

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Abstract

We bring out a comparative study between Differential Transform Method (DTM), He Laplace Method (HLM), Adomain Decomposition Method (ADM) to solve non linear Riccati differential equation. It is shown that DTM has an advantage over the HLM and ADM which takes very less time to solve the non linear Riccati equation. Also DTM is more effective and powerful technique.

Keywords: Riccati equations, Differential Transform Method(DTM), Adomain Decomposition Method(ADM) and He Laplace Transform Method(HLM)

INTRODUCTION AND PRELIMINARIES

The French mathematician Liouville in 1841 provided Riccati equation was the one of the simplest non linear first order differential equation. This paper outlines a reliable comparison among the powerful methods that were recently developed. Differential Transform Method (DTM) was introduced by Zhou in 1986. The main advantage of this method is that it can be applied directly to nonlinear ordinary and partial

differential equations without requiring linearization, discretization or perturbation and also it is able to limit the size of computational work while still accurately providing the series solution with fast converge rate. It has been studied and applied during the last decades widely. He-Laplace method is an elegant combination of the Laplace transformation, the homotopy perturbation method and He's polynomials. The use of He's polynomials in the nonlinear term was first introduced by Ghorbani. The proposed algorithm provides the solution in a rapid convergent series which may lead to the solution in a closed form. The Adomian Decomposition Method (ADM) is a semi-analytical method for solving ordinary and partial nonlinear differential equations. The method was developed by George Adomian in the period 1970 - 1990. It was further extended to stochastic systems by using the Ito integral. The aim of this method is towards a unified theory for the solution of partial differential equations and has been superseded by the more general theory of the homotopy analysis method. The crucial aspect of the method is employment of the "Adomian polynomials" which allow for solution convergence of the nonlinear portion of the equation, without simply linearizing the system. These polynomials mathematically generalize to a Maclaurin series about an arbitrary external parameter, which gives the solution method more flexibility than direct Taylor series expansion. DTM is used to find the solution of various kinds of Riccati differential equation such solution of first order, second order and system of Riccati equations [1], solving linear and non linear system of ordinary differential equations [2], solving system of differential equations [3], solution of Non-Linear Differential equations [4], Solution of Riccati equation with variable co-efficient [5], Quadratic Riccati Differential Equation [6], on the solutions of Nonlinear Higher Order Boundary value problems [7]. ADM has been used to numerical analysis of Different second Order Systems [8], higher order and system of non-linear differential equations [9], solving second order nonlinear ordinary Differential Equations [10] and comparison study of Variational Iteration Method (VIM) and He -Laplace Method (HLM) and so on.

METHOD 1

Differential Transform Method

The transformation of the k^{th} derivative of a function $y(x)$ in one variable is defined as follows

$$Y(k) = \frac{1}{k!} \left[\frac{d^k(y(x))}{dx^k} \right]_{x=0} \quad (1)$$

and the inverse transform of $Y(k)$ is defined as

$$y(x) = \sum_{k=0}^{\infty} Y(k)x^k \quad (2)$$

The following are the important theorems of the one dimensional differential transform method

Theorem 1: If $y(x) = m(x) \pm n(x)$, then $Y(k) = M(k) \pm N(k)$

Theorem 2: If $y(x) = \alpha m(x)$, then $Y(k) = \alpha M(k)$

Theorem 3: If $y(x) = \frac{dm(x)}{dx}$, then $Y(k) = (K + 1)Y(k + 1)$

Theorem 4: If $y(x) = m(x)n(x)$, then $Y(k) = \sum_{r=0}^k M(r)N(k - r)$

Theorem 5: If $y(x) = x^l$, then $Y(k) = \delta(k - l) = \begin{cases} 1, & \text{if } k = l \\ 0, & \text{if } k \neq l \end{cases}$

Riccati Equation

A Differential equation of the form

$$y' + a(x)y + b(x)y^2 + c(x) = 0 \tag{3}$$

where $a(x)$, $b(x)$ and $c(x)$ are functions of x , is known as Riccati equation.

Method 2

He Laplace Method

Consider following nonlinear differential equation

$$y^{11} + p_1y^1 + p_2y + p_3f(y) = f(x) \tag{4}$$

$$y(0) = \alpha, y(0) = \beta \tag{5}$$

Where $p_1, p_2, p_3, \alpha, \beta$ are constants, $f(y)$ is a nonlinear function and $f(x)$ is the source term.

Taking Laplace Transform, we get

$$L[y''] + L[p_1y'] + L[p_2y] + L[p_3f(y)] = L[f(x)]$$

$$L[y''] + p_1L[y'] + p_2L[y] + p_3L[f(y)] = L[f(x)]$$

$$y(x) = F(x) - L^{-1}\left(\frac{p_2}{(s^2+p_1s)}L[y]\right) - L^{-1}\left(\frac{p_3}{(s^2+p_1s)}L[f(y)]\right) \tag{6}$$

where $F(x)$ represents the term arising from the source term and the prescribed initial conditions.

Applying Homotopy Perturbation method

$$y(x) = \sum_{n=0}^{\infty} p^n y_n(x) \tag{7}$$

where the term y_n are to be recursively calculated and the nonlinear term $f(y)$ can be decomposed as

$$f(y) = \sum_{n=0}^{\infty} p^n H_n(y) \quad (8)$$

for some He's polynomials H_n that are given by

$$H_n(y_0, y_1, y_2, \dots, y_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} [f(\sum_{i=0}^{\infty} p^i y_i)]_{p=0}, \quad n = 0, 1, 2, 3, \dots \quad (9)$$

Substituting (7) and (8) in (6), we get

$$\sum_{n=0}^{\infty} p^n y_n(x) = F(x) - L^{-1} \left(\frac{p_2}{(s^2 + p_1 s)} L[\sum_{n=0}^{\infty} p^n y_n(x)] \right) - L^{-1} \left(\frac{p_3}{(s^2 + p_1 s)} L[\sum_{n=0}^{\infty} p^n H_n(y)] \right) \quad (10)$$

This is the coupling of the Laplace transformation and the homotopy perturbation method using He's polynomials.

From equation (10)

$$p^0: y_0(x) = F(x)$$

$$p^1: y_1(x) = -L^{-1} \left(\frac{p_2}{(s^2 + p_1 s)} L[p^0 y_0(x)] \right) - L^{-1} \left(\frac{p_3}{(s^2 + p_1 s)} L[p^0 H_0(y)] \right)$$

$$p^2: y_2(x) = -L^{-1} \left(\frac{p_2}{(s^2 + p_1 s)} L[y_1(x)] \right) - L^{-1} \left(\frac{p_3}{(s^2 + p_1 s)} L[H_1(y)] \right)$$

$$p^3: y_3(x) = -L^{-1} \left(\frac{p_2}{(s^2 + p_1 s)} L[y_2(x)] \right) - L^{-1} \left(\frac{p_3}{(s^2 + p_1 s)} L[H_2(y)] \right)$$

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Method 3

Adomian Decomposition Method

The general form of a differential equation be

$$Fy = g \quad (11)$$

where F is the non-linear differential operator with linear and non-linear terms.

The linear term is decomposed as

$$F = L + R \quad (12)$$

where L is an invertible operator and R is the remainder of the linear operator.

For convenience, L is taken as the highest order derivative. Therefore the equation

may be written as

$$Ly + Ry + Ny = g \tag{13}$$

Where Ny corresponds to the non-linear terms. Solving Ly from (13) we have

$$Ly = g - Ry - Ny$$

As L is invertible,

$$L^{-1}[L(y)] = L^{-1}g - L^{-1}(Ry) - L^{-1}(Ny) \tag{14}$$

If L is a second order operator, then L^{-1} is a twofold integration operator

$$L^{-1} = \iint (\cdot) dt_1 dt_2$$

and

$$L^{-1}[L(y)] = y(t) - y(0) - ty'(0)$$

Then equation (14)

$$y(x) = y(0) + ty'(0) + L^{-1}(g) - L^{-1}(Ry) - L^{-1}(Ny) \tag{15}$$

Therefore, y can be presented as a series

$$y(x) = \sum_{n=0}^{\infty} y_n \tag{16}$$

with y_0 identified as $y(t_0) + ty'(0) + L^{-1}(g)$ and y_n ($n > 0$) is to be determined.

The non-linear term N(y) will be decomposed by the infinite series of Adomian polynomials

$$N(y) = \sum_{n=0}^{\infty} A_n \tag{17}$$

where A_n 's are obtained by writing

$$\mu(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \tag{18}$$

$$[\mu(\lambda)] = \sum_{n=0}^{\infty} \lambda^n A_n \tag{19}$$

Here λ is a parameter.

From (18) and (19), we have

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{n=0}^{\infty} \lambda^n y_n)] \Big|_{\lambda=0} \tag{20}$$

Now substituting (16) and (17) in (15), we have

$$\sum_{n=0}^{\infty} y_n = y_0 - L^{-1} \left[R \left(\sum_{n=0}^{\infty} y_n \right) \right] - L^{-1} \left(\sum_{n=0}^{\infty} A_n \right)$$

Consequently we can write

$$\begin{aligned}
 y_0 &= y(t_0) + ty'(t_0) + L^{-1}(g) \\
 y_1 &= -L^{-1}R(y_0) - L^{-1}(A_0) \\
 &\dots\dots\dots \\
 y_{n+1} &= -L^{-1}R(y_n) - L^{-1}(A_n)
 \end{aligned} \tag{21}$$

Consider the first order Riccati differential equation with constants coefficients where a, b, c are non negative

Example 1: $y' + y + 2y^2 + 3 = 0$ with $y(0) = 1$ (22)

i.e a = 1, b = 2, c = 3

Case (i)

Apply Differential Transform Method to equation (22),

$$(K + 1)Y(K + 1) + Y(K) + 2 \sum_{r=0}^k Y(r)Y(K - r) + 3\delta(k - 0) = 0$$

$$k = 0, Y(1) + Y(0) + 2Y(0)Y(0) + 3 = 0$$

$$Y(1) = -6$$

$$K = 1, 2Y(2) + Y(1) + 2.2Y(0)Y(1) = 0$$

$$Y(2) = 15$$

$$K = 2, Y(3) + Y(2) + 2(2Y(0)Y(2) + Y(1)Y(1)) = 0$$

$$Y(3) = -49$$

$$K = 3, 4Y(4) + Y(3) + 2 \sum_{r=0}^3 Y(r)Y(K - r) = 0$$

$$Y(4) = \frac{605}{4}$$

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Solution of (22) is given by $y(x) = Y(0)x^0 + Y(1)x^1 + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 \dots$

$$y(x) = 1 - 6x + 15x^2 - 49x^3 + \frac{605}{4}x^4 + \dots$$

Which converges to the exact solution by Taylor's series about $x = 0$ is

$$y(x) = 1 - 6x + 15x^2 - 49x^3 + \dots$$

Case (ii)

Applying He Laplace Method to equation (22), we get

$$y(x) = L^{-1} \left[\frac{1}{S} \right] - L^{-1} \left[\frac{3}{S^2} \right] - L^{-1} \left[\frac{1}{S} L(y) \right] - L^{-1} \left[\frac{2}{S} L(y^2) \right]$$

$$y(x) = 1 - 3x - L^{-1} \left[\frac{1}{S} L(y_n) \right] - L^{-1} \left[\frac{2}{S} L(H_n(y)) \right]$$

$$p^0: y_0(x) = 1 - 3x$$

$$p^1: y_1(x) = -L^{-1} \left[\frac{1}{S} L(y_0) \right] - L^{-1} \left[\frac{2}{S} L(H_0(x)) \right]$$

$$p^1: y_1(x) = -L^{-1} \left[\frac{1}{S} L(1 - 3x) \right] - L^{-1} \left[\frac{2}{S} L(1 - 3x)^2 \right]$$

$$p^1: y_1(x) = -3x + \frac{15}{2}x^2 - 6x^3$$

$$p^2: y_2(x) = -L^{-1} \left[\frac{1}{S} L(y_1) \right] - L^{-1} \left[\frac{2}{S} L(H_1(x)) \right]$$

$$p^2: y_2(x) = -L^{-1} \left[\frac{1}{S} L(-3x + \frac{15}{2}x^2 - 6x^3) \right] - L^{-1} \left[\frac{2}{S} L(2y_0y_1) \right]$$

$$p^2: y_2(x) = -L^{-1} \left[\frac{1}{S} L(-3x + \frac{15}{2}x^2 - 6x^3) \right] - L^{-1} \left[\frac{2}{S} L(2y_0y_1) \right]$$

$$p^2: y_2(x) = \frac{15}{2}x^2 + \frac{39}{2}x^3 - 27x^4 + \frac{72}{2}x^5$$

⋮
⋮
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Solution is $y(x) = y_0 + y_1 + y_2 + \dots$

$$y(x) = 1 - 6x + 15x^2 - \frac{27}{2}x^3 + \dots$$

Case (iii)

Applying Adomain Decomposition Method to equation (22), we get

$$L^{-1}(y^1) = L^{-1}(-3) - L^{-1}(y) - L^{-1}(2y^2)$$

$$y(x) = y(0) - 3x - L^{-1}(R(y_n)) - 2L^{-1}(N(y_n))$$

$$y_0 = 1 - 3x$$

$$y_1 = -L^{-1}(R(y_0)) - 2L^{-1}(N(y_0)) \text{ where } N(y_0) = y_0^2$$

$$y_1 = -L^{-1}(1 - 3x) - 2L^{-1}(1 - 6x + 9x^2)$$

$$y_1 = -[x - 3\frac{x^2}{2}] - 2[x - 6\frac{x^2}{2} + 9\frac{x^3}{3}]$$

$$y_1 = -3x + 15\frac{x^2}{2} - 6x^3$$

$$y_2 = -L^{-1}(R(y_1)) - 2L^{-1}(N(y_1)) \text{ where } N(y_1) = 2y_0y_1$$

$$y_2 = -L^{-1}(-3x + 15\frac{x^2}{2} - 6x^3) - 2L^{-1}(-6x + 33x^2 - 57x^3 + 36x^4)$$

$$y_2 = 3\frac{x^2}{2} - 5\frac{x^3}{2} + \frac{3}{2}x^4 + 6x^2 - 22x^3 + \frac{57}{2}x^4 - \frac{36}{5}x^5$$

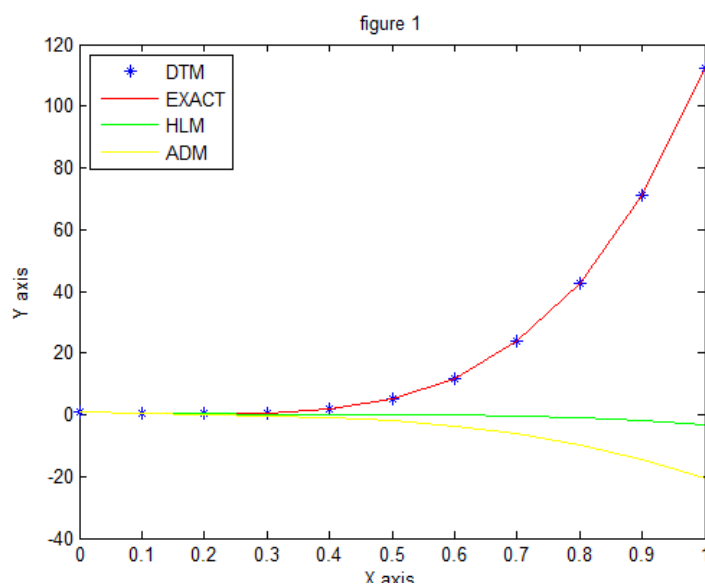
Solution is $y(x) = \sum_{n=0}^{\infty} y_n$

$$y(x) = y_0 + y_1 + y_2 + \dots$$

$$y(x) = 1 - 3x + (-3x + 15\frac{x^2}{2} - 6x^3) + (3\frac{x^2}{2} - 5\frac{x^3}{2} + \frac{3}{2}x^4 + 6x^2 - 22x^3 + \frac{57}{2}x^4 - \frac{36}{5}x^5) + \dots$$

$$y(x) = 1 - 6x + 15x^2 - \frac{61}{2}x^3 + \dots$$

Table-1							
x	DTM	EXACT	HLM	ADM	Error _{DTM}	Error _{HLM}	Error _{ADM}
0	1.00000	1.00000	1.00000	1.00000	0.000000	0.000000	0.000000
0.1	0.51613	0.51613	0.53650	0.51950	0.000000	0.020375	0.003375
0.2	0.25000	0.25000	0.29200	0.15600	0.000000	0.042000	0.094000
0.3	0.45213	0.45213	0.18550	-0.27350	0.000000	0.266625	0.725625
0.4	1.73600	1.73600	0.13600	-0.95200	0.000000	1.600000	2.688000
0.5	5.07813	5.07813	0.06250	-2.06250	0.000000	5.015625	7.140625
0.6	11.81800	11.81800	-0.11600	-3.78800	0.000000	11.934000	15.606000
0.7	23.65813	23.65813	-0.48050	-6.31150	0.000000	24.138625	29.969625
0.8	42.66400	42.66400	-1.11200	-9.81600	0.000000	43.776000	52.480000
0.9	71.26413	71.26413	-2.09150	-14.48450	0.000000	73.355625	85.748625
1	112.25000	112.25000	-3.50000	-20.50000	0.000000	115.750000	132.750000



Consider the first order Riccati differential equation with constants coefficients where a, b, c are negative

Example 2: $y' - y - y^2 - 2 = 0$ with $y(0) = 1$ (23)

i. e $a = -1, b = -1$ and $c = -2$

Case (i)

Applying Differential Transform Method to equation (23), we have

$$(K + 1)Y(K + 1) - Y(K) - \sum_{r=0}^k Y(r)Y(K - r) - 2\delta(k - 0) = 0$$

$$K = 0, Y(1) - Y(0) - Y(0)Y(0) - 2 = 0$$

$$Y(1) = 4$$

$$K = 1, 2Y(2) - Y(1) - 2Y(0)Y(1) = 0$$

$$Y(2) = 6$$

$$K = 2, 3Y(3) - Y(2) - 2Y(0)Y(2) - Y(1) - Y(1) = 0$$

$$Y(3) = \frac{34}{3}$$

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The solution of (23) is $y(x) = Y(0)x^0 + Y(1)x^1 + Y(2)x^2 + Y(3)x^3 + \dots$

$$y(x) = 1 + 4x + 6x^2 + \frac{34}{3}x^3 + \dots$$

which converges to the exact solution by Taylor's series about $x = 0$ is

$$y(x) = 1 + 4x + 6x^2 + \frac{34}{3}x^3 + \dots$$

Case (ii)

Applying He Laplace Method to the equation (23), we get

$$L[y'(x)] - L[y(x)] - L[y^2] - L[2] = 0$$

$$sL[y] - y(0) - L[y] - L[y^2] - \frac{2}{s} = 0$$

$$y(x) = L^{-1}\left[\frac{1}{s} + \frac{2}{s^2} + \frac{1}{s}L[y] + \frac{1}{s}L[y^2]\right]$$

$$y(x) = 1 + 2x + L^{-1}\left(\frac{1}{s}L[y]\right) + L^{-1}\left(\frac{1}{s}L[y^2]\right)$$

$$y_0 = 1 + 2x$$

$$y_1 = L^{-1}\left(\frac{1}{s}L[y_0]\right) + L^{-1}\left(\frac{1}{s}L[H_0(y)]\right)$$

$$y_1 = L^{-1} \left(\frac{1}{s} L[1 + 2x] \right) + L^{-1} \left(\frac{1}{s} L[1 + 4x^2 + 4x] \right)$$

$$y_1 = L^{-1} \left(\frac{1}{s} \left(\frac{1}{s} + \frac{2}{s^2} \right) \right) + L^{-1} \left(\frac{1}{s} \left(\frac{1}{s} + 4 \cdot \frac{2}{s^3} + 4 \cdot \frac{1}{s^2} \right) \right)$$

$$y_1 = 2x + 3x^2 + \frac{4}{3}x^3$$

$$y_2 = L^{-1} \left(\frac{1}{s} L[y_1] \right) + L^{-1} \left(\frac{1}{s} L[H_1(y)] \right)$$

$$y_2 = L^{-1} \left(\frac{1}{s} L \left[2x + 3x^2 + \frac{4}{3}x^3 \right] \right) + L^{-1} \left(\frac{1}{s} L \left[4x + 14x^2 + \frac{44}{3}x^3 + \frac{16}{3}x^4 \right] \right)$$

$$y_2 = 3x^2 + \frac{17}{3}x^3 + 4x^4 + \frac{16}{15}x^5$$

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The solution is given by $y(x) = 1 + 4x + 6x^2 + 7x^3 + 4x^4 + \frac{16}{15}x^5 + \dots$

Case (iii)

Apply Adomain Decomposition Method to equation (23), we have

$$L^{-1}(y^1) = L^{-1}(2) + L^{-1}(y) + L^{-1}(y^2)$$

$$y(x) = y(0) + 2x + L^{-1}(R(y_0)) + L^{-1}(N(y_0))$$

$$y_0 = 1 + 2x$$

$$y_1 = L^{-1}[R(y_0)] + L^{-1}[N(y_0)]$$

$$y_1 = L^{-1}[1 + 2x] + L^{-1}[1 + 4x^2 + 4x]$$

$$y_1 = 2x + 3x^2 + \frac{4x^3}{3}$$

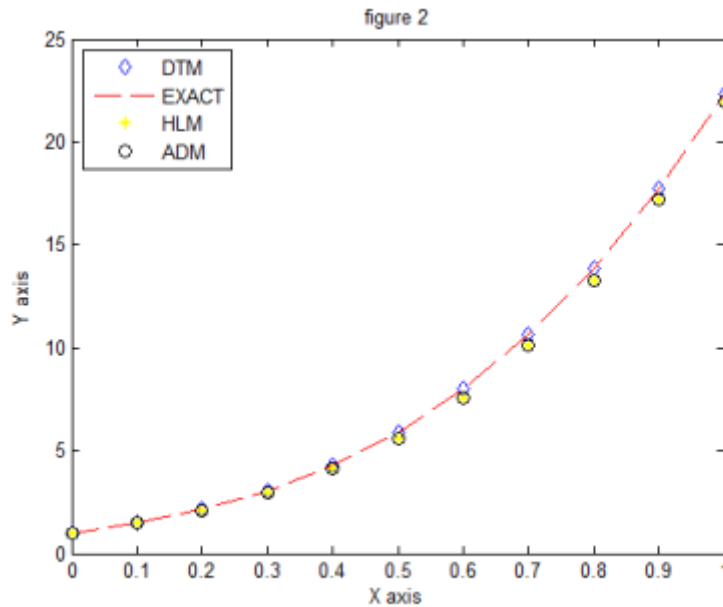
$$y_2 = L^{-1} \left[2x + 3x^2 + \frac{4x^3}{3} \right] + L^{-1} \left[4x + 14x^2 + \frac{44}{3}x^3 + \frac{16}{3}x^4 \right]$$

$$y_2 = 3x^2 + \frac{17}{3}x^3 + 4x^4 + \frac{16}{15}x^5$$

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The solution is $y(x) = 1 + 4x + 6x^2 + 7x^3 + 4x^4 + \frac{16}{15}x^5 + \dots$

Table-II							
x	EXACT	DTM	HLM	ADM	Error _{DTM}	Error _{HLM}	Error _{ADM}
0	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000
0.1	1.47133	1.47133	1.46740	1.46740	0.00000	0.00393	0.00393
0.2	2.13067	2.13067	2.10240	2.10240	0.00000	0.02827	0.02827
0.3	3.04600	3.04600	2.96140	2.96140	0.00000	0.08460	0.08460
0.4	4.28533	4.28533	4.11040	4.11040	0.00000	0.17493	0.17493
0.5	5.91667	5.91667	5.62500	5.62500	0.00000	0.29167	0.29167
0.6	8.00800	8.00800	7.59040	7.59040	0.00000	0.41760	0.41760
0.7	10.62733	10.62733	10.10140	10.10140	0.00000	0.52593	0.52593
0.8	13.84267	13.84267	13.26240	13.26240	0.00000	0.58027	0.58027
0.9	17.72200	17.72200	17.18740	17.18740	0.00000	0.53460	0.53460
1	22.33333	22.33333	22.00000	22.00000	0.00000	0.33333	0.33333



Consider the first order Riccati equation with constant coefficients where $a < 0$ and $b > 0, c > 0$

Example 3: $y^1 - y + 2y^2 + 1 = 0$ with $y(0) = 1$ (24)

i.e $a = -1, b = 2, c = 1$

Case (i)

Applying Differential Transform Method to equation (24)

$$(k + 1)Y(k + 1) - Y(k) + 2 \sum_{r=0}^k Y(r)Y(k - r) - \delta(k - 0) = 0$$

$$k = 0, Y(1) - Y(0) + 2Y(0)Y(0) + 1 = 0$$

$$Y(1) = -2$$

$$k = 1, 2Y(2) - Y(1) + 2 \sum_{r=0}^1 Y(r)Y(k - r)$$

$$Y(2) = 3$$

$$k = 2, 3Y(3) - Y(2) + 2 \sum_{r=0}^2 Y(r)Y(k - r) = 0$$

$$Y(3) = \frac{17}{3}$$

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The solution of (23) is $y(x) = Y(0)x^0 + Y(1)x^1 + Y(2)x^2 + Y(3)x^3 + \dots$

$$y(x) = 1 - 2x + 3x^2 + \frac{17}{3}x^3 + \dots$$

Which converges to the exact solution given by Taylor's about $x = 0$ is given by

$$y(x) = 1 - 2x + 3x^2 + \frac{17}{3}x^3 + \dots$$

Case (ii)

Applying He Laplace Method to equation (24), we have

$$L[y^1] - L[y] + 2L[y^2] + L[1] = 0$$

$$sL[y(x)] - y(0) - L[y] + L[y^2] + \frac{1}{s} = 0$$

$$y(x) = L^{-1} \left[\frac{1}{s} \right] - L^{-1} \left[\frac{1}{s^2} \right] + L^{-1} \left[\frac{1}{s} L(y) \right] - L^{-1} \left[\frac{2}{s} L(y^2) \right]$$

$$y(x) = 1 - x + L^{-1}\left[\frac{1}{s}L(y_n)\right] - L^{-1}\left[\frac{2}{s}L(H_n(y))\right]$$

$$p^0 : y_0 = 1 - x$$

$$p^1 : y_1 = L^{-1}\left[\frac{1}{s}L(y_0)\right] - L^{-1}\left[\frac{2}{s}L(H_0(y))\right]$$

$$y_1 = L^{-1}\left[\frac{1}{s}L(1 - x)\right] - L^{-1}\left[\frac{2}{s}L(1 + x^2 - 2x)\right]$$

$$y_1 = L^{-1}\left[\frac{1}{s^2} - \frac{1}{s^3}\right] - L^{-1}\left[\frac{2}{s^2} + \frac{4}{s^4} - \frac{4}{s^3}\right]$$

$$y_1 = -x + \frac{3x^2}{2} - \frac{2x^3}{3}$$

$$p^2 : y_2 = L^{-1}\left[\frac{1}{s}L(y_1)\right] - L^{-1}\left[\frac{2}{s}L(H_1(y))\right]$$

$$y_2 = L^{-1}\left[\frac{1}{s}L\left(-x + \frac{3x^2}{2} - \frac{2x^3}{3}\right)\right] - L^{-1}\left[\frac{2}{s}L\left(-2x + 5x^2 - \frac{13}{3}x^3 + \frac{4}{3}x^4\right)\right]$$

$$y_2 = \frac{3x^2}{2} - \frac{17x^3}{6} + 2x^4 + \frac{8}{15}x^5$$

and so on.

The solution is given by $y(x) = y_0 + y_1 + y_2 + \dots$

$$y(x) = 1 - 2x + 3x^2 - \frac{7}{2}x^3 + \dots$$

Case (iii)

Applying Adomian Decomposition Method to equation (24), we have

$$L^{-1}[y^1] = L^{-1}[-1] + L^{-1}[y] - L^{-1}[2y^2]$$

$$y(x) = y(0) - x + L^{-1}[R(y_n)] - 2L^{-1}[N(y_n)]$$

$$y_0 = 1 - x$$

$$y_1 = L^{-1}[R(y_0)] - 2L^{-1}[N(y_0)]$$

$$y_1 = L^{-1}[1 - x] - 2L^{-1}[1 + x^2 - 2x]$$

$$y_1 = -x + x^2 - \frac{2}{3}x^3$$

$$y_2 = L^{-1}[R(y_1)] - 2L^{-1}[N(y_1)]$$

$$y_2 = L^{-1}\left[-x + x^2 - \frac{2}{3}x^3\right] - 2L^{-1}\left[-2x + 3x^2 - \frac{x^3}{3} + \frac{4x^4}{3}\right]$$

$$y_2 = \left[-\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{6} + 2x^2 - 2x^3 + \frac{x^4}{6} - \frac{8x^5}{15}\right]$$

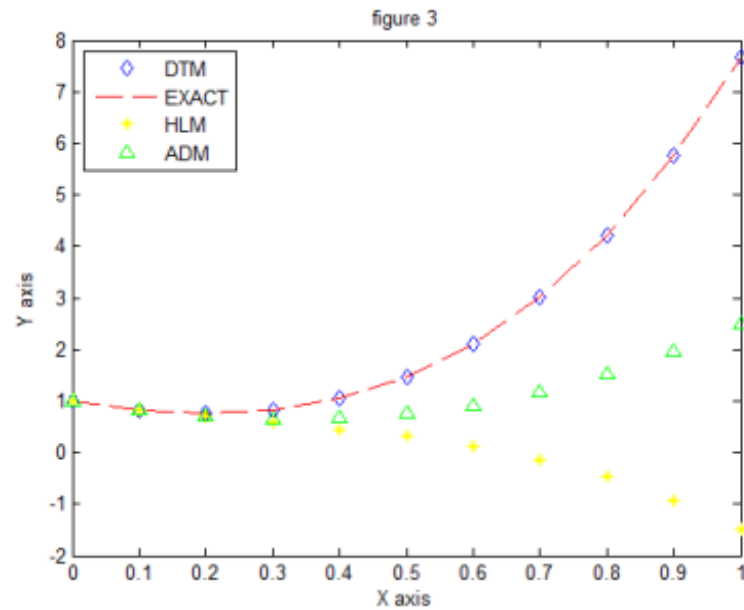
$$y_2 = \frac{3x^2}{2} + \frac{5x^3}{3}$$

and so on.

The solution is given by $y(x) = 1 - 2x + \frac{5x^2}{2} + x^3 + \dots$

Table-III

x	EXACT	DTM	HLM	ADM	Error _{DTM}	Error _{HLM}	Error _{ADM}
0	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000
0.1	0.83567	0.83567	0.82650	0.82600	0.00000	0.00917	0.00967
0.2	0.76533	0.76533	0.69200	0.70800	0.00000	0.07333	0.05733
0.3	0.82300	0.82300	0.57550	0.65200	0.00000	0.24750	0.17100
0.4	1.04267	1.04267	0.45600	0.66400	0.00000	0.58667	0.37867
0.5	1.45833	1.45833	0.31250	0.75000	0.00000	1.14583	0.70833
0.6	2.10400	2.10400	0.12400	0.91600	0.00000	1.98000	1.18800
0.7	3.01367	3.01367	-0.13050	1.16800	0.00000	3.14417	1.84567
0.8	4.22133	4.22133	-0.47200	1.51200	0.00000	4.69333	2.70933
0.9	5.76100	5.76100	-0.92150	1.95400	0.00000	6.68250	3.80700
1	7.66667	7.66667	-1.50000	2.50000	0.00000	9.16667	5.16667



Consider the first order Riccati equation with constant coefficients where $a > 0$, $b < 0$, $c = 0$

Example 4: $y' + y - y^2 = 0$ with $y(0) = 2$ (25)

i.e $a = 1, b = -1, c = 0$

Case (i)

Applying Differential Transform Method to equation (25), we get

$$(k + 1)Y(k + 1) + Y(k) - \sum_{r=0}^k Y(r)Y(k - r) = 0$$

$$k = 0, Y(1) + Y(0) - Y(0)Y(0) = 0$$

$$Y(1) = 2$$

$$k = 1, \quad 2Y(2) + Y(1) - 2Y(0)Y(1) = 0$$

$$Y(2) = 3$$

$$k = 2, \quad 3Y(3) + Y(2) - 2Y(0)Y(2) - Y(1)Y(1) = 0$$

$$Y(3) = \frac{13}{3}$$

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The solution is given by $y(x) = Y(0)x^0 + Y(1)x^1 + Y(2)x^2 + Y(3)x^3 + \dots$

$$y(x) = 2 + 2x + 3x^2 + \frac{13}{3}x^3 + \dots$$

Which converges to the exact solution by Taylor's series about $x = 0$ is

$$y(x) = 2 + 2x + 3x^2 + \frac{13}{3}x^3 + \dots$$

Case (ii)

Now apply He Laplace Method to equation (25), we get

$$L[y'] + L[y] - L[y^2] = 0$$

$$sL[y(x)] - y(0) = -L[y] + L[y^2]$$

$$L[y(x)] = \frac{2}{s} - \frac{1}{s} L[y] + \frac{1}{s} L[y^2]$$

$$y(x) = L^{-1}\left[\frac{2}{s} - \frac{1}{s} L[y] + \frac{1}{s} L[y^2]\right]$$

$$y_0 = L^{-1}\left[\frac{2}{s}\right]$$

$$y_0 = 2$$

$$y_1 = -L^{-1}\left[\frac{1}{s} L(y_0) + \frac{1}{s} L[H_0(y)]\right]$$

$$y_1 = -L^{-1}\left[\frac{1}{s} L(2) + \frac{1}{s} L(4)\right]$$

$$y_1 = -2x + 4x$$

$$y_1 = 2x$$

$$y_2 = -L^{-1}\left[\frac{1}{s} L(y_1) + \frac{1}{s} L(H_1(y))\right]$$

$$y_2 = -L^{-1}\left[\frac{1}{s} L(2x) + \frac{1}{s} L(8x)\right]$$

$$y_2 = 3x^2$$

$$y_3 = -L^{-1}\left[\frac{1}{s}L(y_2) + \frac{1}{s}L(H_1(y))\right]$$

$$y_3 = -L^{-1}\left[\frac{1}{s}L(3x^2) + \frac{1}{s}L(16x^2)\right]$$

$$y_3 = \frac{7}{3}x^3$$

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The solution is $y(x) = 2 + 2x + 3x^2 + \frac{7}{3}x^3 + \dots$

Case (iii)

Apply Adomian Decomposition Method to equation (25), we get

$$y_0 = 2$$

$$y_1 = -L^{-1}[R(y_0)] + L^{-1}[N(y_0)]$$

$$y_1 = -L^{-1}[2] + L^{-1}[4]$$

$$y_1 = -2x + 4x$$

$$y_1 = 2x$$

$$y_2 = -L^{-1}[R(y_1)] + L^{-1}[N(y_1)]$$

$$y_2 = -L^{-1}[2x] + L^{-1}[8x]$$

$$y_2 = -x^2 + 4x^2$$

$$y_2 = 3x^2$$

$$y_3 = -L^{-1}[R(y_2)] + L^{-1}[N(y_2)]$$

$$y_3 = -L^{-1}[3x^2] + L^{-1}[16x^2]$$

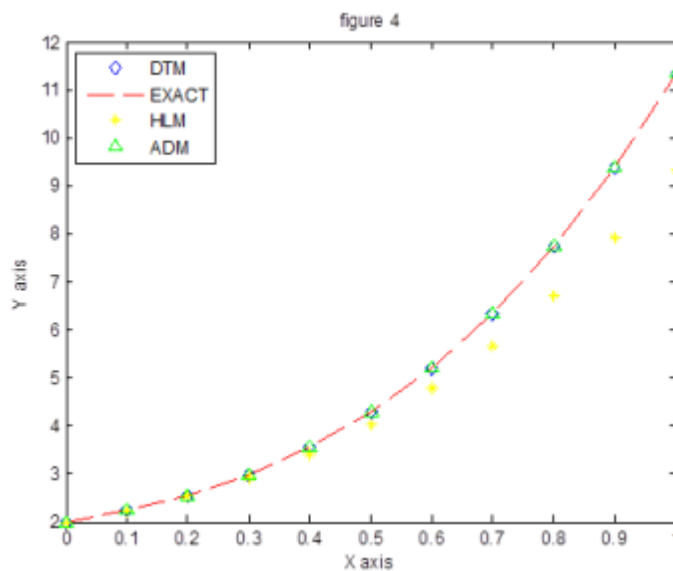
$$y_3 = -x^3 + \frac{16}{3}x^3$$

$$y_3 = \frac{13}{3}x^3$$

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The solution is $y(x) = 2 + 2x + 3x^2 + \frac{13}{3}x^3 + \dots$

Table-IV							
x	EXACT	DTM	HLM	ADM	Error _{DTM}	Error _{HLM}	Error _{ADM}
0	2.00000	2.00000	2.00000	2.00000	0.00000	0.00000	0.00000
0.1	2.23433	2.23433	2.23233	2.23433	0.00000	0.00200	0.00000
0.2	2.55467	2.55467	2.53867	2.55467	0.00000	0.01600	0.00000
0.3	2.98700	2.98700	2.93300	2.98700	0.00000	0.05400	0.00000
0.4	3.55733	3.55733	3.42933	3.55733	0.00000	0.12800	0.00000
0.5	4.29167	4.29167	4.04167	4.29167	0.00000	0.25000	0.00000
0.6	5.21600	5.21600	4.78400	5.21600	0.00000	0.43200	0.00000
0.7	6.35633	6.35633	5.67033	6.35633	0.00000	0.68600	0.00000
0.8	7.73867	7.73867	6.71467	7.73867	0.00000	1.02400	0.00000
0.9	9.38900	9.38900	7.93100	9.38900	0.00000	1.45800	0.00000
1	11.33333	11.33333	9.33333	11.33333	0.00000	2.00000	0.00000



CONCLUSION

In this paper, we compared the solution of Riccati equation by DTM, HLM, ADM. The result of DTM and Exact solution are in strong agreement with each other. DTM is reliable and powerful technique. We believe that the efficiency of DTM gives it much wider suitability which needs to be excavated further.

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