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Pairwise Fuzzy Globally Disconnected Spaces and Pairwise Fuzzy σ - Baire Spaces

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Abstract

In this paper, the concept of pairwise fuzzy globally disconnected spaces is introduced and studied. The condition under which pairwise fuzzy globally disconnected spaces become pairwise fuzzy σ-Baire spaces, is established. Several characterizations of pairwise fuzzy globally disconnected spaces, are obtained.

Keywords: Pairwise fuzzy open set, pairwise fuzzy F_σ -set, pairwise fuzzy G_δ -set, pairwise fuzzy σ-nowhere dense set, pairwise fuzzy first category set, pairwise fuzzy Baire space.

1. INTRODUCTION

In order to deal with uncertainties, the notions of fuzzy sets and fuzzy set operations were introduced by L.A.Zadeh [1] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. In 1968, C.L.Chang [2] defined fuzzy topological spaces by using fuzzy sets. The concept of σ-nowhere dense sets in classical topology was introduced and studied by Jiling Cao and Sina Greenwood in [3].

In 1989, A.Kandil [4] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.The concept of pairwise fuzzy σ-nowhere dense sets in fuzzy bitopological spaces is introduced and studied in [5]. By using pairwise fuzzy σ-nowhere dense sets, the concept of pairwise fuzzy σ - Baire spaces is defined and studied by the authors in [6], [7] and [8]. The purpose of this paper is to introduce the concept of pairwise fuzzy globally disconnected spaces and study several characterizations of pairwise fuzzy globally disconnected spaces and pairwise fuzzy σ-Baire spaces.

2. PRELIMINARIES

In order to make the exposition self-contained, some basic notations and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Lemma 2.1. [9] For a fuzzy set λ in a fuzzy topological space X,

(i) $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$, (ii) $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$.

Definition 2.1 [5] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1, 2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set in (X, T_1, T_2).

Definition 2.2 [5] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy.

G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)' s are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.3 [5] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X,T_1,T_2).

Definition 2.4 [10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy dense set if $\text{cl}_{T_1}\text{cl}_{T_2}(\lambda) = \text{cl}_{T_2}\text{cl}_{T_1}(\lambda) = 1$, in (X,T_1,T_2).

Definition 2.5 [11] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nowhere dense set if $\text{int}_{T_1}\text{cl}_{T_2}(\lambda) = \text{int}_{T_2}\text{cl}_{T_1}(\lambda) = 0$, in (X,T_1,T_2).

Definition 2.6 [5] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $\text{int}_{T_1}\text{int}_{T_2}(\lambda) = \text{int}_{T_2}\text{int}_{T_1}(\lambda) = 0$.

Definition 2.7 [6] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy σ-first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy.

σ-nowhere dense sets in (X,T_1,T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy σ-second category set in (X,T_1,T_2).

Definition 2.8 [6] If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy σ-residual set in (X, T_1, T_2).

Definition 2.9 [11] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $\text{int}_{T_i}(\vee_{k=1}^{\infty} (\lambda_k)) = 0$, (i= 1,2) where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2).

Definition 2.10 [5] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy σ-Baire space if $\text{int}_{T_i} (\vee_{k=1}^{\infty} (\lambda_k)) = 0$, (i = 1, 2) ,where (λ_k)'s are pairwise fuzzy σ - nowhere dense sets in (X, T_1, T_2).

Definition 2.11 A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a

(i) Pairwise fuzzy semi-open set if $\lambda \leq \text{cl}_{T_i}\text{int}_{T_j} (\lambda)$ ($i \neq j$, and i,j=1,2) [12].

(ii) Pairwise fuzzy semi-closed set if $\text{int}_{T_i}\text{cl}_{T_j} (\lambda) \leq \lambda$ ($i \neq j$, and i,j=1,2) [12].

(iii) Pairwise fuzzy pre-open set if $\lambda \leq \text{int}_{T_i}\, \text{cl}_{T_j} (\lambda)$ ($i \neq j$, and i,j=1,2) [12].

(iv) Pairwise fuzzy pre-closed set if $\text{cl}_{T_i}\, \text{int}_{T_j}(\lambda) \leq \lambda$ ($i \neq j$, and i,j=1,2) [12].

Definition 2.12 A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called

(i) Pairwise fuzzy regular open set if $\text{int}_{T_1}\text{cl}_{T_2} (\lambda) = \lambda = \text{int}_{T_2}\text{cl}_{T_1}(\lambda)$ [13].

(ii) Pairwise fuzzy regular closed set if $\text{cl}_{T_1}\text{int}_{T_2}(\lambda) = \lambda = \text{cl}_{T_2}\text{int}_{T_1}(\lambda)$ [13].

3. PAIRWISE FUZZY σ -BAIRE SPACES

Proposition 3.1. If the pairwise fuzzy nowhere dense set λ is a pairwise fuzzy F_σ -set in a fuzzy bitopological space (X,T_1,T_2) , then λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2) .

Proof. Let the pairwise fuzzy nowhere dense set λ be a pairwise fuzzy F_σ –set in (X,T_1,T_2) . Then $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, where (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). Since λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2) , $\text{int}_{T_i}\text{cl}_{T_j}(\lambda) = 0$ ($i \neq j$ and i,j=1,2) in (X,T_1,T_2). But $\text{int}_{T_i} (\lambda) \leq \text{int}_{T_i}\, \text{cl}_{T_j}(\lambda)$, implies that $\text{int}_{T_i} (\lambda) \leq 0$. That is, $\text{int}_{T_i}(\lambda) = 0$ in (X,T_1,T_2) and hence $\text{int}_{T_i}\, \text{int}_{T_j}(\lambda) = \text{int}_{T_i}(0) = 0$ ($i \neq j$ and i,j = 1,2). Thus λ is a pairwise fuzzy F_σ-set such that $\text{int}_{T_i}\, \text{int}_{T_j} (\lambda) = 0$. Therefore λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2).

Theorem 3.1 [11] Let (X,T_1,T_2) be a fuzzy bitopological space. Then the following are equivalent:

(1). (X,T_1,T_2) is a pairwise fuzzy Baire space.

(2). $int_{Ti}(\lambda) = 0$, $(i = 1,2)$,for every pairwise fuzzy first category set λ in (X,T_1,T_2).

(3). $cl_{Ti}(\mu) = 1$, $(i = 1,2)$,for every pairwise fuzzy residual set μ in (X,T_1,T_2).

Theorem 3.2 [6] Let (X,T_1,T_2) be a fuzzy bitopological space. Then the following are equivalent:

(1). (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

(2). $int_{Ti}(\lambda) = 0$, $(i = 1,2)$, for every pairwise fuzzy σ-first category set λ in (X,T_1,T_2).

(3) .$cl_{Ti}(\mu) = 1$, $(i = 1,2)$, for every pairwise fuzzy σ-residual set μ in (X,T_1,T_2).

Proposition 3.2. If each pairwise fuzzy σ-first category set is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

Proof. Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2) such that $int_{T_i} cl_{T_j}(\lambda) = 0$ ($i\neq j$ and $i,j =1,2$). Now $int_{T_i}(\lambda) \leq int_{T_i} cl_{T_j}(\lambda)$, implies that $int_{T_i}(\lambda) \leq 0$. That is, $int_{T_i}(\lambda) = 0$. Hence, for the pairwise fuzzy σ-first category set λ in (X,T_1,T_2), $int_{Ti}(\lambda) = 0$, implies by theorem 3.2, that (X,T_1,T_2) is a pairwise fuzzy σ -Baire space.

Proposition 3.3. If each pairwise fuzzy σ -first category set λ is a pairwise fuzzy closed set in a pairwise fuzzy σ-Baire space (X,T_1,T_2) , then λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2) .

Proof. Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2) such that $cl_{T_i}(\lambda) = \lambda$ ($i =1,2$). Since (X,T_1,T_2) is a pairwise fuzzy σ-Baire space**,** $int_{T_i}(\lambda) = 0$, in (X,T_1,T_2). Then , $int_{T_i} cl_{T_j}(\lambda) = int_{T_i}(\lambda) = 0$ in (X,T_1,T_2) and hence λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proposition 3.4. If $int_{T_i} (\vee_{k=1}^{\infty} (\lambda_k)) = 0$, where (λ_k)'s are pairwise fuzzy σ -first category sets such that $cl_{T_i}(\lambda_k) = \lambda_k$, in a pairwise fuzzy σ-Baire space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof. Let (λ_k)'s be pairwise fuzzy σ -first category sets such that $cl_{T_i}(\lambda_k) = \lambda_k$, in the pairwise fuzzy σ-Baire space (X,T_1,T_2). Then, by proposition 3.3, (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2).Thus $int_{T_i} (\vee_{k=1}^{\infty} (\lambda_k)) = 0$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2), implies that (X,T_1,T_2) is a pairwise fuzzy Baire space.

4. PAIRWISE FUZZY GLOBALLY DISCONNECTED SPACES AND PAIRWISE FUZZY σ-BAIRE SPACES

Definition 4.1. A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy globally disconnected space if each pairwise fuzzy semi-open set is a pairwise fuzzy open set in (X,T_1,T_2). That is, if $\lambda \leq cl_{T_i}\, int_{T_j}(\lambda)$ ($i \neq j$ and $i,j = 1,2$) , for a fuzzy set λ defined on X in a fuzzy bitopological space (X,T_1,T_2), then $\lambda \in T_i$ (i = 1, 2).

Proposition 4.1. If $cl_{T_i}\, int_{T_j}(\lambda) = cl_{T_i}(\lambda)$ for a fuzzy set λ defined on X in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then $int_{T_j}(\lambda) = \lambda$ in (X,T_1,T_2).

Proof. Let λ be a fuzzy set defined on X such that $cl_{T_i}\, int_{T_j}(\lambda) = cl_{T_i}(\lambda)$ ($i \neq j$ and $i,j = 1,2$) . Now, $\lambda \leq cl_{T_i}(\lambda)$ implies that $\lambda \leq cl_{T_i}\, int_{T_j}(\lambda)$ and hence λ is a pairwise fuzzy semi-open set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, the pairwise fuzzy semi-open set λ is a pairwise fuzzy open set in (X,T_1,T_2) and hence $int_{T_j}(\lambda) = \lambda$ in (X,T_1,T_2).

Proposition 4.2. If $cl_{T_i}(\lambda)$ (i =1,2), is a pairwise fuzzy regular closed set, for a fuzzy set λ defined on X , in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is a pairwise fuzzy pre-open set in (X,T_1,T_2).

Proof. Let $cl_{T_i}(\lambda)$ (i = 1,2) be a pairwise fuzzy regular closed set in (X,T_1,T_2). Then $cl_{T_i}\, int_{T_j}(cl_{T_i}(\lambda)) = cl_{T_i}(\lambda)$. Then, $cl_{T_i}\, int_{T_j}(cl_{T_i}(\lambda)) = cl_{T_i}[cl_{T_i}(\lambda)]$ in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, by proposition 4.1, $int_{T_j}(cl_{T_i}(\lambda)) = cl_{T_i}(\lambda)$ and then $\lambda \leq cl_{T_i}(\lambda)$, implies that $\lambda \leq int_{T_j}(cl_{T_i}(\lambda))$. Thus, λ is a pairwise fuzzy pre-open set in (X,T_1,T_2).

Proposition 4.3. If λ is a pairwise fuzzy nowhere dense set in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X,T_1,T_2). Then $int_{T_j}cl_{T_i}(\lambda)=0$ ($i\neq j$ and $i,j=1,2$) in (X,T_1,T_2). Then, $int_{T_j}(cl_{T_i}(\lambda)) \leq \lambda$ in (X,T_1,T_2) and hence λ is a pairwise fuzzy semi-closed set in (X,T_1,T_2). Then $(1- \lambda)$ is a pairwise fuzzy semi-open set in (X,T_1,T_2). Since (X,T_1,T_2) is pairwise fuzzy globally disconnected space, the pairwise fuzzy semi-open set $(1-\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2). Thus λ is a pairwise fuzzy closed set in (X,T_1,T_2).

Remark 4.1. In view of the above proposition, one will have the following result: ***"The pairwise fuzzy nowhere dense sets are pairwise fuzzy closed sets in pairwise fuzzy globally disconnected spaces ".***

Proposition 4.4 If λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy first category set in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, the pairwise fuzzy nowhere dense sets (λ_k)'s are pairwise fuzzy closed sets in (X, T_1, T_2) and hence $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, implies that λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

Proposition 4.5 If (X,T_1,T_2) is a pairwise fuzzy Baire and pairwise fuzzy globally disconnected space and if λ is a pairwise fuzzy first category set in (X,T_1,T_2), then λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy first category set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy Baire space, By theorem 3.1, $\text{int}_{T_i}(\lambda) = 0$, $(i = 1,2)$ in (X,T_1,T_2).Then $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = \text{int}_{T_i}(0) = 0$. Also since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, by proposition 4.4, the pairwise first category set λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2). Thus , λ is a pairwise fuzzy F_σ-set with $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0$ in (X,T_1,T_2) . Hence λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2).

Proposition 4.6. If λ is a pairwise fuzzy residual set in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is a pairwise fuzzy G_δ-set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy residual set in (X,T_1,T_2). Then$(1 - \lambda)$ is a pairwise fuzzy first category set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, by proposition 4.4, $(1 - \lambda)$ is a pairwise fuzzy F_σ-set in (X,T_1,T_2) and hence λ is a pairwise fuzzy G_δ-set in (X,T_1,T_2).

Proposition 4.7. If (X,T_1,T_2) is a pairwise fuzzy Baire and pairwise fuzzy globally disconnected space and if λ is a pairwise fuzzy residual set in (X,T_1,T_2), then λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy residual set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy Baire space, by theorem 3.1, $\text{cl}_{T_i}(\lambda) = 1$, $(i = 1,2)$ in (X,T_1,T_2). Also since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, by proposition 4.6, λ is a pairwise fuzzy G_δ-set in (X,T_1,T_2). Hence the pairwise fuzzy residual set λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X,T_1,T_2).

Definition 4.2 [5]. A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Volterra space if $\text{cl}_{T_i}(\wedge_{k=1}^{N}(\lambda_k)) = 1$, $(i = 1,2)$, where (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2).

Proposition 4.8. If $\text{cl}_{T_i}(\wedge_{k=1}^{N}(\lambda_k)) = 1$, $(i = 1,2)$,where (λ_k)'s are pairwise fuzzy residual sets in a pairwise fuzzy Baire and pairwise fuzzy globally disconnected space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy Volterra space.

Proof. Let (λ_k)'s be pairwise fuzzy residual sets in (X,T_1,T_2).Since (X,T_1,T_2) is a pairwise fuzzy Baire and pairwise fuzzy globally disconnected space, by proposition 4.7, the pairwise fuzzy residual sets (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2).Thus, $cl_{T_i}(\wedge_{k=1}^{N}(\lambda_k)) = 1$, where the fuzzy sets ($\lambda_k$)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2), implies that (X,T_1,T_2) is a pairwise fuzzy Volterra space.

Definition 4.3 [14] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nodec space if every non-zero pairwise fuzzy nowhere dense set in (X,T_1,T_2), is a pairwise fuzzy closed set in (X,T_1,T_2). That is , if λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2), then $1-\lambda \in T_i$ (i = 1, 2).

Proposition 4.9 If (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, then (X,T_1,T_2) is a pairwise fuzzy nodec space.

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy globally disconnected space, by proposition 4.3, λ is a pairwise fuzzy closed set in (X,T_1,T_2) and hence each pairwise fuzzy nowhere dense set is a pairwise fuzzy closed set in (X,T_1,T_2), implies that (X,T_1,T_2) is a pairwise fuzzy nodec space.

Definition 4.4 [15] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy extremally disconnected space if T_1 closure of each T_2 fuzzy open set is T_2 fuzzy open and T_2 closure of each T_1 fuzzy open set is T_1 fuzzy open. That is, (X,T_1,T_2) is a pairwise fuzzy extremally disconnected space if $cl_{T_i}(\lambda) \in T_i$,for each $\lambda \in T_i$ (i =1,2).

Proposition 4.10 If λ is a pairwise fuzzy nowhere dense set in a pairwise fuzzy extremally disconnected and pairwise fuzzy nodec space (X,T_1,T_2), then λ is a pairwise fuzzy pre-closed set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy nodec space , the pairwise fuzzy nowhere dense set λ is a pairwise fuzzy closed set in (X,T_1,T_2) and then $(1-\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2). Again since (X,T_1,T_2) is a pairwise fuzzy extremally disconnected space, $cl_{T_i}(1-\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2). Then by Lemma 2.1, $1-int_{T_i}(\lambda)$ is a pairwise fuzzy open set in (X,T_1,T_2) and hence $int_{T_i}(\lambda)$ is a pairwise fuzzy closed set in (X,T_1,T_2). Then, $cl_{T_j}\, int_{T_i}(\lambda) = int_{T_i}(\lambda)$ and hence $cl_{T_j}\, int_{T_i}(\lambda) \leq \lambda$ in (X,T_1,T_2). Hence λ is a pairwise fuzzy pre-closed set in (X,T_1,T_2).

Proposition 4.11. If the pairwise fuzzy globally disconnected space (X,T_1,T_2) is a pairwise fuzzy Baire space (X,T_1,T_2) and if (μ_k)'s are pairwise fuzzy first category sets in (X,T_1,T_2), then $\vee_{k=1}^{\infty}(\mu_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proof. Let the pairwise fuzzy globally disconnected space (X,T_1,T_2) be a pairwise fuzzy Baire space. Suppose that (μ_k)'s are pairwise fuzzy first category sets in (X,T_1,T_2). Then, by proposition 4.5, (μ_k)'s are pairwise fuzzy σ-nowhere dense sets in

(X,T_1,T_2) and hence $\vee_{k=1}^{\infty} (\mu_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proposition 4.12. If the pairwise fuzzy globally disconnected space (X,T_1,T_2) is a pairwise fuzzy Baire space and if $\text{int}_{T_i} (\vee_{k=1}^{\infty} (\mu_k)) = 0$, $(i = 1, 2)$, where (μ_k)'s are pairwise fuzzy first category sets in (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

Proof. Let the pairwise fuzzy globally disconnected space (X,T_1,T_2) be a pairwise fuzzy Baire space. Suppose that (μ_k)'s are pairwise fuzzy first category sets in (X,T_1,T_2) such that $\text{int}_{T_i}(\vee_{k=1}^{\infty} (\mu_k)) = 0$. By proposition 4.11, $(\vee_{k=1}^{\infty} (\mu_k))$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2) and hence $\text{int}_{T_i} (\vee_{k=1}^{\infty} (\mu_k)) = 0$ implies, by theorem 3.2, that (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

Definition 4.5 A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy pre F_σ-set if $\lambda= \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy pre-closed sets in (X, T_1, T_2).

Proposition 4.13 If **λ** is a pairwise fuzzy first category set in a pairwise fuzzy extremally disconnected and pairwise fuzzy nodec space (X,T_1,T_2), then λ is a pairwise fuzzy pre F_σ-set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy first category set in (X,T_1,T_2). Then $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, $(i = 1, 2)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy extremally disconnected and pairwise fuzzy nodec space,the pairwise fuzzy nowhere dense sets (λ_k)'s in (X,T_1,T_2) are pairwise fuzzy pre-closed sets in (X,T_1,T_2) and hence $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, where (λ_k)'s are pairwise fuzzy pre-closed sets, implies that λ is a pairwise fuzzy pre F_σ-set in (X,T_1,T_2).

5. CONCLUSION

In this paper, the concept of pairwise fuzzy globally disconnected spaces is introduced and several characterizations of pairwise fuzzy globally disconnected spaces and pairwise fuzzy σ-Baire spaces, are studied. A condition under which pairwise fuzzy nowhere dense sets in fuzzy bitopological spaces become pairwise fuzzy σ -nowhere dense sets , is obtained. The conditions for fuzzy bitopological spaces to become pairwise fuzzy σ-Baire spaces and pairwise fuzzy Volterra space, are also established . It is established that the pairwise fuzzy nowhere dense sets are pairwise fuzzy closed sets in pairwise fuzzy globally disconnected spaces.

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