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Regular and Irreguar m-polar Fuzzy Graphs

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Abstract

In this paper we define irregular m-polar fuzzy graphs and its various classifications.Size of regular m-polar fuzzy graphs are derived.The relation between highly and neighbourly irregular m-polar fuzzy are established.Some basic theorems related to the stated graphs have also been presented.

Keywords: m-polar fuzzy graphs , regular m-polar fuzzy graphs , irregular and degree of m-polar garphs.

INTRODUCTION:

The origin of graph theory started with Konigsberg bridge problem in 1735.This problem led to the concept of Eulerian graph.Euler studied the Konigsberg problem and constructed a structure that solves the problem that is referred to as an Eulerian graph.In this paper we introduced the notation of the m-polar fuzzy set as a generalization of m-polar fuzzy graph.

Currently,concept of graph theory are highly utilize by computer science applications,especially in area of research, including data mining image segmentation,clustering and net working. The first basic definitions of m-polar fuzzy graph was proposed by G.Ghorai and M.pal[1] from a study on m-polar fuzzy graphs.

In 1999 ,Molodtsov [9]introduced the concept of soft set theory to solve imprecise problems in the field of engineering, Social science, economics, medical science and environment. Molodtsov applied this theory to several directions such as smoothness of fuction, game theory, operation research, probability and measurement theory.In resent times, a number of research studies contributed into fuzzification of soft set theory. As a result many researchers were more active doing research on soft set.

However H.Rashmantou, S.Samanta, M.pal, R.A.Borzooel [4] introduced m-poar fuzzy graphs with categorical properties . Also S.Samanta, M.pal [5] defined basic definition of irregular m-polar fuzzy graphs. In 2015, G.Ghorai and M.pal [2,11,12] also introduced another group of explained definition of complement and isomorphism of m-polar fuzzy graph and some operations and density of m-polar fuzzy graphs. At this same time they elaborated faces and dual of m-polar fuzzy planar graphs. The complement of a fuzzy graphs, Indian journal of pure and Applied mathematics was introduced by M.S.Sunitah, A.Vijayakumar[9].

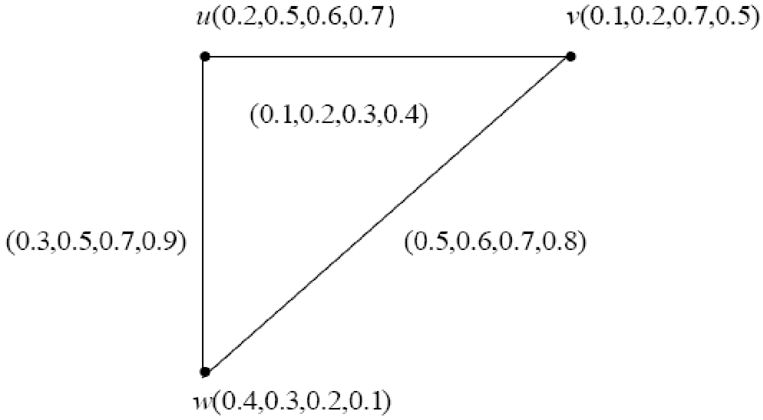
In 2011,Akram[13] introduced the concept of m-polar fuzzy graphs and defined different operations on it. Now m-polar fuzzy graph theory is growing and expanding its application. Also we refer basic definitions of fuzzy set theory and K.Kalaiarasi[10] defined optimization of fuzzy integrated Vendor-Buyer inventory models. Some basic Definitions and basis of regular and irregular theorems are discussed.

Definition 1.1

An m-polar fuzzy graph of a graph of a graph $G^* = (V, E)$ is a pair $G = (A, B)$ where $A : V \to [0,1]^m$ is an m-polar fuzzy set in V and $B : \overline{V^2} \to [0,1]^m$ is an m-polar fuzzy set in $\overline{V^2}$ such that for each $i = 1,2,....m.pi \circ [B(xy)] \leq \min \{ pi \circ A(x), pi \circ A(y)$ for all $xy \in \overline{V^2}$ and $B(xy) = 0$ for all $xy \in \overline{V^2} - E, \{0 = 0,0,0.....,0\}$ is the smallest element in $[0,1]^m$.

A is called the m-polar fuzzy vertex and B is called the m–polar fuzzy edge.

Example:1.1



Definition 1.2

The order of the m-polar fuzzy graph $G=(V,A,B)$ is denoted by $|V|$ or $(O(G))$ where

$$O(G)=|V|=\sum_{x\in v}\frac{1+\sum_{i=1}^{m} pi\circ A(x)}{2}$$

Definition 1.3

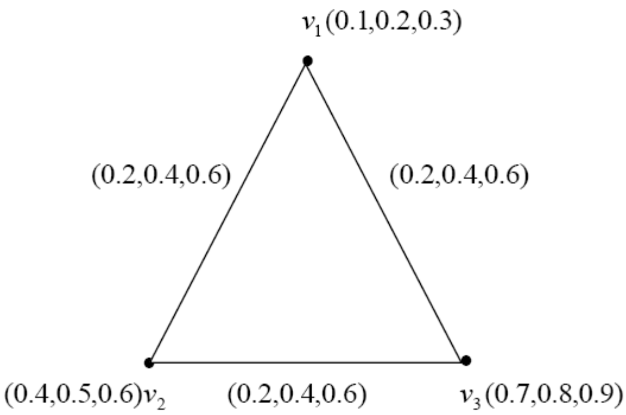
The size of $G=(A,B)$ is denoted by $|E|$ or $S(G)$ where

$$S(G)=|E|=\sum_{xy\in v}\frac{1+\sum_{i=1}^{m} pi\circ B(xy)}{2}$$

Definition 1.4

Let $G=(A,B)$ be a m-polar fuzzy graph where $A:V\to[0,1]^m$ is an m-polar fuzzy set in V and $B:\overline{V^2}\to[0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E\subseteq v\times v$ respectively.If $d(u)=k_1,d(v)=k_2$ for all $u,v\in U,V$. k_1,k_2 are two real numbers,then the graph is called k_1,k_2 regular m-polar fuzzy graph.If there exists a vertex which is adjacent to a vertex with same degree.

Exmaple:1.4



$d(v_1) = 0.4, 0.8, 1.2$ $d(v_2) = 0.4, 0.8, 1.2$ $d(v_3) = 0.4, 0.8, 1.2$

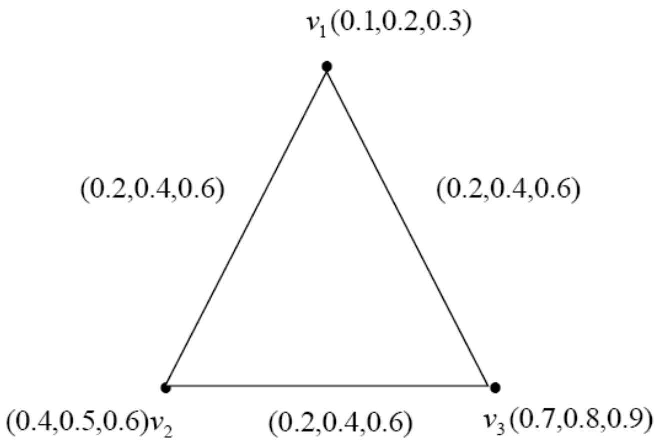
$$\therefore d(v_1) = d(v_2) = d(v_3)$$

$\therefore$ The graph is regular 3-polar fuzzy graph .he graph is said to regular .

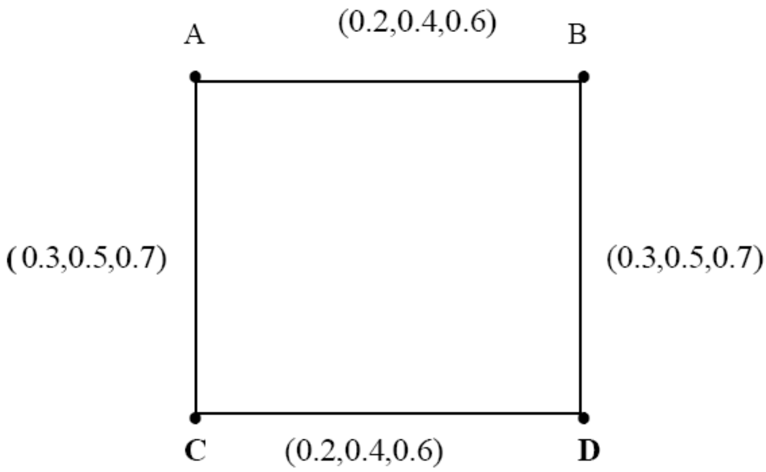
Remark:

If the graph is said to regular if the odd graph or even graph of degree are same. Example of odd graph and even graph are given below.

Example of odd reguar graph:



Example of Even Regular graph:



In even regular graph the alternative edges are equal.

Definition 1.5

Let $G=(A,B)$ be a m-polar fuzzy graph where $A:V\to[0,1]^m$ is an m-polar fuzzy set in V and $B:\overline{V^2}\to[0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E\subseteq v\times v$ respectively. G is said to be totally regular m-polar fuzzy graph. If there exists a vertex which is adjacent to a vertex with same total degree $td(u)$.

Definition 1.6

Let G be a connected m-polar fuzzy graph.Then G is called neighbourly totally regular

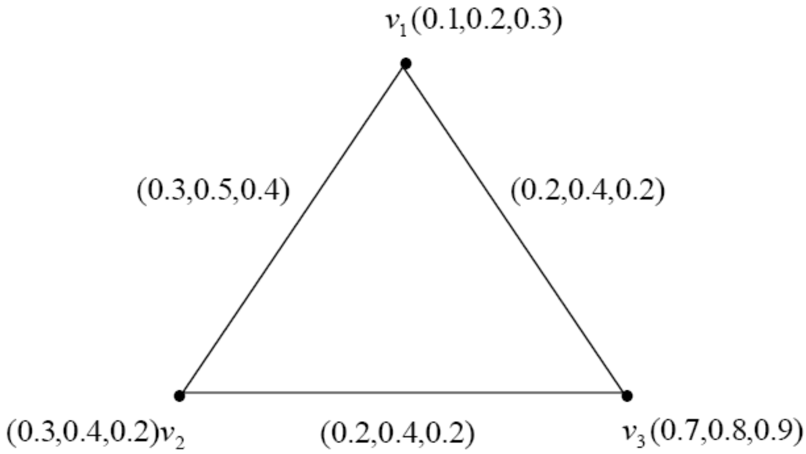
m-polar fuzzy graph if for every two adjacent vertices of G have same total degrees.

Definition 1.7

Let G be a connected m-polar fuzzy graph.Then G is called neighbourly regular

m-polar fuzzy graph if for every two adjacent vertices of G have same degrees.

Example:1.7



$d(v_1) = 0.5,0.9,0.6$ $\qquad$ $d(v_2) = 0.5,0.9,0.6$ $\qquad$ $d(v_3) = 0.4,0.8,0.4$

$$\therefore d(v_1) = d(v_2) \quad \text{and} \quad d(v_2) \neq d(v_3)$$

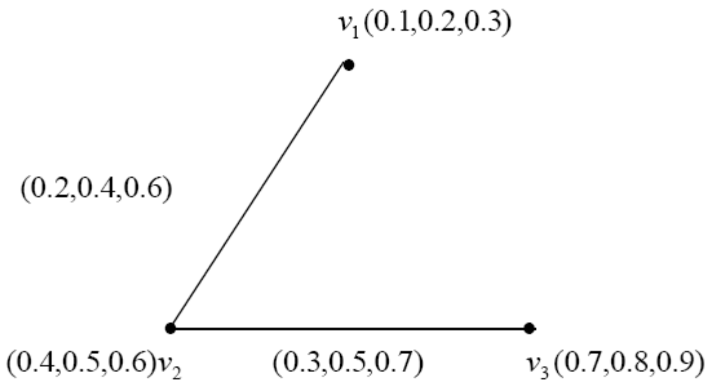
$\therefore$ So this graph is an example of an irregular 3-polar fuzzy graph.

Definition 1.8

Let G be a connected m-polar fuzzy graph.Then G is called neighbourly irregular m-polar fuzzy graph if for every two adjacent vertices of G have distinct degrees.

That is $d(v_1) \neq d(v_2) \neq d(v_3)$

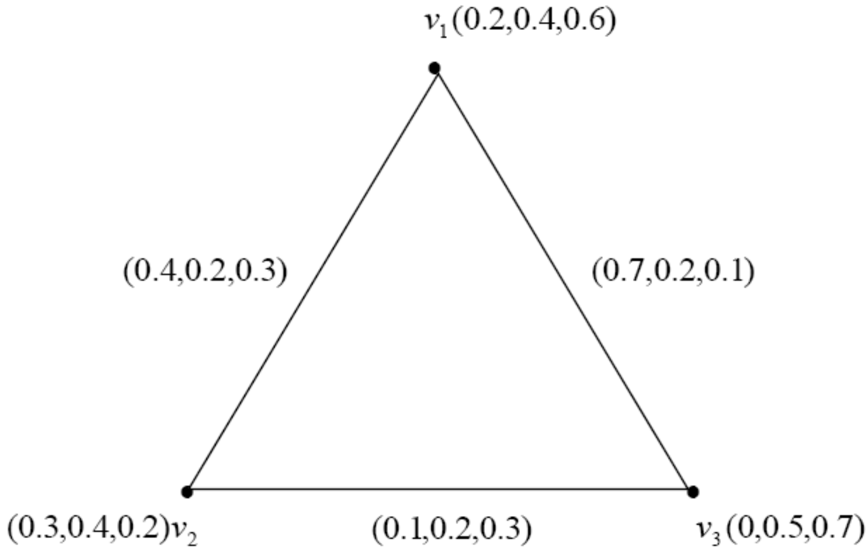
Example:1.8



Definition :1.9

Let $G=(A,B)$ be a m-polar fuzzy graph where $A:V\to[0,1]^m$ is an m-polar fuzzy set in V and $B:\overline{V^2}\to[0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E\subseteq v\times v$ respectively. G is said to be totally irregular m-polar fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct total degree $td(u)$.

Example:1.9



$d(v_1)=0.5,0.9,0.6$ $d(v_2)=0.5,0.9,0.6$ $d(v_3)=0.4,0.8,0.4$

$td(v_1)=1.3,0.8,1.0$ $td(v_2)=0.8,0.9,1.3$ and $td(v_3)=0.8,0.9,1.3$

$td(v_2)=td(v_3)$ and $td(v_1)\neq td(v_2)$.

There exist a vertex which is adjacent to a vertex with distinct total degrees.So this graph is an example of Totally irregular m-polar fuzzy graphs.

Definition 1.10

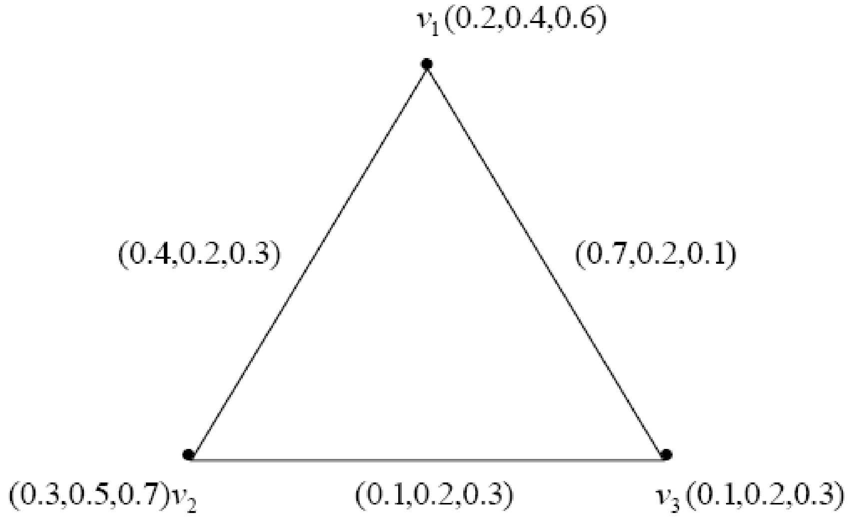
Let G be a connected m-polar fuzzy graph.Then G is called neighbourly totally irregular

m-polar fuzzy graph if for every two adjacent vertices of G have distinct total degrees.

Remark:

It is need not be highly irregular m-polar fuzzy graph.That is any adjacent vertices of the graph G have the same degree.Other than any adjacent vertices of the graph G have the distinct degree.

Example:1.10



$d(v_1) = 1.1,0.4,0.4$ $d(v_2) = 0.5,0.4,0.6$ $d(v_3) = 0.8,0.4,0.4$

$td(v_1) = 1.3,0.8,1.0$ $td(v_2) = 0.8,0.9,1.3$ and $td(v_3) = 0.9,0.6,0.7$

$$td(v_1) \neq td(v_2) \neq td(v_3)$$

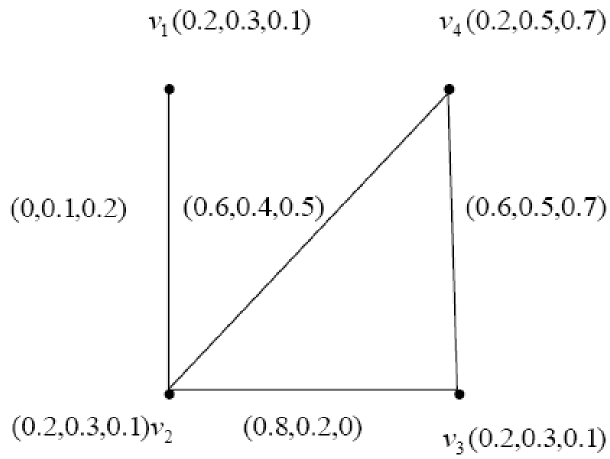
This graph is an example of neighbourly totally irregular m-polar fuzzy graphs.

Definition 1.11

Let G be a connected m-polar fuzzy graph.Then G is called highly irregular

m-polar fuzzy graph if every vertex of G is adjacent to vertices with distinctl degrees.

Example:1.11



$d(v_1) = 0.6,0.5,0.7$ $d(v_2) = 1.4,0.7,0.7$ $d(v_3) = 1.4,0.7,0.7$ $d(v_1) = 1.2,0.9,1.2$

$d(v_2) = d(v_3)$ but $(v_1) \neq d(v_2) \neq d(v_4)$

$\therefore$ This is highiy irregular m -polar fuzzy graph.It is need not be neighbourly irregular.

Theorem 2.1

Let $G = (A, B)$ be a m -polar fuzzy graph.Then G is highly irregular m -polar fuzzy graph and neighbourly irregular m -polar fuzzy graph iff the degrees of all vertices of G are distinct.

Proof:

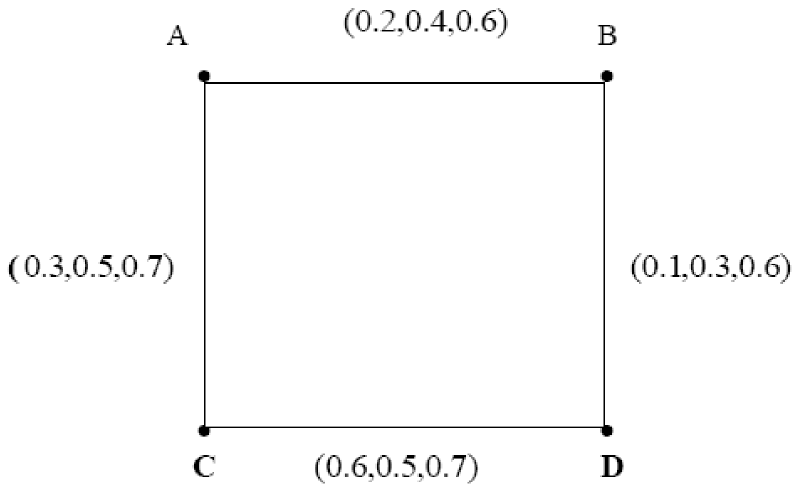
Let $G = (A, B)$ be a m -polar fuzzy graph where $A : V \to [0,1]^m$ is an m -polar fuzzy set in V and $B : \overline{V^2} \to [0,1]^m$ be two m -polar fuzzy sets on a non-empty finite set V and $E \subseteq v \times v$ respectively.

Let $V = \{v_1, v_2, ...v_n\}$.We assume that G is highly irregular and neighbourly irregular m -polar fuzzy graph.

To prove:

If the degrees of all vertices of G are distinct.

Let the adjacent vertices V_1 be $v_2, v_3, ..., v_n$ with degrees $d(v_1), d(v_2), ...d(v_n)$ respectively.



We know that G is highly irregular m-polar fuzzy graph and neighbourly irregular m-polar fuzzy graph so $d(v_1) \neq d(v_2) \neq .... \neq d(v_n)$.

So it is obivious that all vertices are of distinct degrees.

Conversely,

We assume that the degrees of all vertices of G are distinct.

This means that every two adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees.

Hence G is highly irregular and neighbourly irregular m-polar fuzzy graph.

Theorem 2.2

Let G be a m-polar fuzzy graph if G is neighbourly irregular then G is a neighbourly totally irregular m-polar fuzzy graph.

Proof:

Let $G = (A, B)$ be a m-polar fuzzy graph where $A : V \to [0,1]^m$ is an m-polar fuzzy set in V and $B : \overline{V^2} \to [0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E \subseteq v \times v$ respectively.

Assume that G is a neighbourly irregular m-polar fuzzy graph.

To prove:

G is a neighbourly totally irregular m-polar fuzzy graph.

If the graph G is a neighbourly irregular m-polar fuzzy graph.That is the degree of every two adjacent vertices are distinct.

The adjacent vertices are u_1 and u_2 with distinct degrees $d(u_1)$ and $d(u_2)$ respectively.Also let $m(u_1) = c_1, c_2, m(u_2) = c_3, c_4$ m-membership function. c_1, c_2, c_3, c_4 are constants.

$$\therefore td(u_1) = d(u_1) + c_1 + c_2$$

$$td(u_2) = d(u_2) + c_3 + c_4$$

Clearly,

$$td(u_1) \neq td(u_2).$$

Therefore for any two adjacent vertices v_1 and v_2 with distinct degrees.its total degrees are also distinct.

Hence G is a neighbourly totally irregular m-polar fuzzy graph.

Theorem:2.3

Let G be a m-polar fuzzy graph.If G is neighbourly totally irregular then G is neighbourly irregular m-polar fuzzy graph.

Proof:

Let $G = (A, B)$ be a m-polar fuzzy graph where $A : V \to [0,1]^m$ is an m-polar fuzzy set in V and $B : \overline{V^2} \to [0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E \subseteq v \times v$ respectively.

Assume that G is a neighbourly totally irregular fuzzy graph.(ie)Total degrees of every two adjacent vertices are distinct.

To prove:

G is neighbourly irregular m-polar fuzzygraph.

Consider two adjacent vertices v_1 and v_2 with degrees (x_1, x_1) and (x_2, x_2) respectively.

Also assume that $v_1 = c_1, c_2$ where c_1, c_2 are constant.

Then $td(v_1) \neq td(v_2)$

$$\Rightarrow d(v_1) \neq d(v_2)$$

$$\Rightarrow x_1 + c_1 \neq x_2 + c_2$$

$$\Rightarrow x_1 \neq x_2 \text{ and } c_1 \neq c_2.$$

Hence degree of adjacent vertices of G are distinct.This is true for every pair of adjacent vertices in G.

Hence the result.

Remark:

Also we have to prove G is neighbourly irregular m-polar fuzzy graph then G is neighbourly totally irregular fuzzy graph.

Theorem:2.4

The size of a (k_1, k_2) regular m-polar fuzzygraph is $\frac{pk_1}{2}, \frac{pk_2}{2}$ where $p = |v|$ .That is p-vertex.

Proof:

Let $G = (A, B)$ be a m-polar fuzzy graph where $A : V \to [0,1]^m$ is an m-polar fuzzy set in V and $B : \overline{V^2} \to [0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E \subseteq v \times v$ respectively.

The size of G is

$$S(G) = \sum \frac{1 + \sum_{i=1}^{m} pi \circ B(xy)}{2}$$

We've

$$\sum d(v) = 2S(G)$$

$$2S(G) = (\sum k_1, \sum k_2)$$

$$2S(G) = pk_1, pk_2$$

$$S(G) = \frac{pk_1}{2}, \frac{pk_2}{2}$$

Hence the theorem.

Theorem:2.5

Let $G = (A, B)$ be a m-polar fuzzy graph.Then G is highly regular m-polar fuzzy graph and neighbourly regular m-polar fuzzy graph iff the degrees of all vertices of G are same.

Proof:

Let $G = (A, B)$ be a m-polar fuzzy graph where $A : V \to [0,1]^m$ is an m-polar fuzzy set in V and $B:\overline{V^2} \to [0,1]^m$ be two m-polar fuzzy sets on a non-empty finite set V and $E \subseteq v \times v$ respectively.

Let $V = \{v_1, v_2, ...v_n\}$.We assume that G is highly regular and neighbourly regular m-polar fuzzy graph.

To prove:

If the degrees of all vertices of G are same.

Let the adjacent vertices V_1 be $v_2, v_3, ..., v_n$ with degrees $d(v_1), d(v_2), ...d(v_n)$ respectively.

We know that G is highly regular m-polar fuzzy graph and neighbourly regular m-polar fuzzy graph so $d(v_1) = d(v_2) = ... = d(v_n)$.

So it is obivious that all vertices are of same degrees.

Conversely,

We assume that the degrees of all vertices of G are same.

This means that every two adjacent vertices have same degrees and to every vertex the adjacent vertices have same degrees.

Hence G is highly regular and neighbourly regular m-polar fuzzy graph.

CONCLUSION:

Graph theory is an extremly useful tool for solving combinatorial problems in different areas, including algebra, number theory, Geomentry, topology,o perations research, optimisation and computer science. Because research on modelling of real world problems often involve muliti-agent, multi-attribute, multi-object, multi-index, multi-polar information, uncertainty and process limits. m-polar fuzzy graphs are very useful.

In this paper we have described order, size of a m-polar fuzzy graphs. The necessary and sufficient conditions for a m-polar fuzzy graph to be the regular m-polar and irregular m-polar fuzzy graphs have been presented .Size of a m-polar fuzzy graphs and relation between size and order of a m-polar fuzzy graphs have been calculated.We have define irregular,neighbourly irregular,totally and highly irregular m-polar fuzzy graphs.Some relations about the defined graphs have been proved.

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