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Contra Harmonic Mean Labeling of Disconnected Graphs

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Abstract

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in$V with distinct element f(x) from 0,1…..................q in such a way that when each edge e = uv is labeled with

f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Keywords: Graph, Contra Harmonic mean labeling, Contra Harmonic mean graphs, Path, Cycle, Comb, etc.

1. INTRODUCTION

All graph in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harray [2].

S. Somasundaram and R. Ponraj introduced mean labeling for some standard graphs [3]. S.S. Sandhya and S. Somasundaram introduced Harmonic mean labeling of graph [4]. We have introduced Contra Harmonic mean labeling in [5]. In this paper we investigate the Contra Harmonic mean labeling behaviour of some disconnected graphs. The following definition are useful for our present study.

Definition 1.1

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x\in V$ with distinct elements f(x) from 0,1…..........q in such a way that when each edge e = uv is labeled with f(e=uv) = $\left\lceil\frac{f(u)^2+f(v)^2}{f(u)+f(v)}\right\rceil$ or $\left\lfloor\frac{f(u)^2+f(v)^2}{f(u)+f(v)}\right\rfloor$ with distinct edge labels. Then f is called Contra Harmonic mean labeling of G.

Definition 1.2: The union of two graphs $G_1 = (V_1,E_1)$ and $G_2= (V_2, E_2)$ is a graph $G = G_1\cup G_2$ with vertex set $V= V_1\cup V_2$ and edge set $E = E_1\cup E_2$

Definition 1.3: The corona of two graphs G_1 and G_2 is the graph $G = G_1\odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th}copy of G_2.

Theorem 1.4: Any Path is a Contra Harmonic mean graph.

Theorem 1.5: Any Cycle is a Contra Harmonic mean graph.

Theorem 1.6: Any Comb is a Contra Harmonic mean graph.

Theorem 1.7: Any Crown is a Contra Harmonic mean graph.

2. MAIN RESULTS

Theorem 2.1: $C_m\cup P_n$ is a Contra Harmonic mean graph ,for $m\geq 3$ and $n\geq 1$

Proof: Let C_m be the cycle $u_1……….u_m$ and P_n be the path $v_1……v_n$

Let $G = C_m\cup P_n$.

Define a function f: V(G) $\rightarrow$ {0,1,…..q} by

$f(u_i) = i-1,\ 1\leq i\leq m-1$, $f(u_m) = m$

$f(v_i) = m+i,\ 1\leq i\leq n$

Then the distinct edge labels are

$f(u_i\ u_{i+1}) = i,\ 1\leq i\leq m$

$f(v_i, v_{i+1}) = m+i,\ 1\leq i\leq n-1$

Clearly, $C_m\cup P_n$ is Contra Harmonic mean graph.

Example 2:2 The Contra Harmonic mean labeling of $C_5 \cup P_5$ is

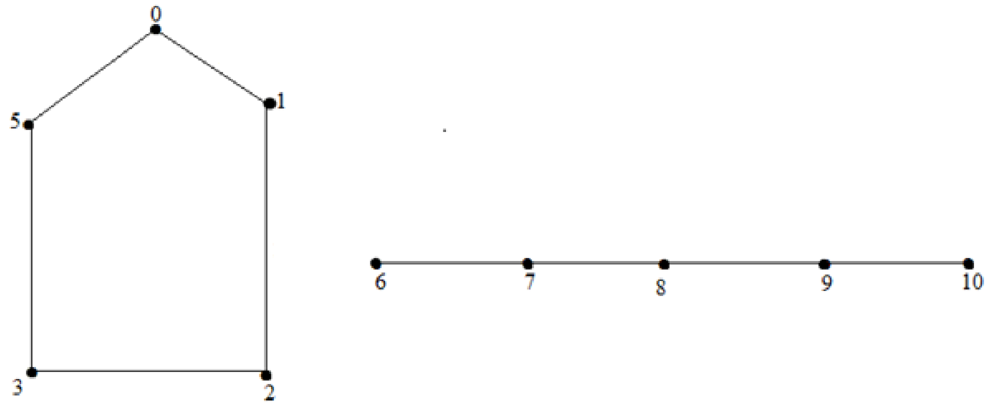

Figure 1

Theorem 2.3: $C_m \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof:

Let C_m be a cycle with vertices $u_1 \ldots\ldots u_m$ and Let $v_1 \ldots\ldots\ldots v_n$ be the path P_n. and let w_i be the vertices which is joined to the vertex v_i, $1 \le i \le n$ of the path P_n. The resultant graph is $P_n \odot K_1$. Let $G = C_m \cup (P_n \odot K_1)$

Define a function $f : V(G) \rightarrow \{0,1\ldots q\}$ by

$f(u_i) = i-1,\ 1 \le i \le m-1$

$f(u_m) = m$

$f(v_i) = m+2i-1,\ 1 \le i \le n$

$f(w_i) = m+2i,\ 1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = i,\ 1 \le i \le m$

$f(v_i, v_{i+1}) = m+2i,\ 1 \le i \le n-1$

$f(v_I, w_i) = m+2i-1,\ 1 \le i \le n$

Clearly, f is a Contra Harmonic mean graph of G.

Example 2.4: The Contra Harmonic mean labeling of $C_5 \cup (P_5 \odot K_1)$ is

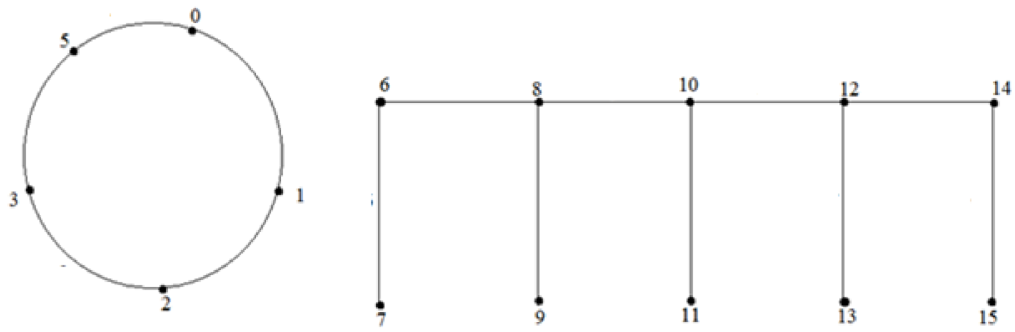

Figure 2

Theorem 2.5: $(C_m \odot K_1) \cup P_n$ is a Contra Harmonic mean graph.

Proof: Let $u_1, u_2 \ldots\ldots\ldots u_m$ be a cycle C_m and v_i be the vertex which is joined to the vertex u_i of the cycle C_m , $1 \leq i \leq m$.

The resultant graph is $C_m \odot K_1$. Let $w_1 w_2 \ldots . w_n$ be the path P_n

Let $G = (C_m \odot K_1) \cup P_n$.

Define a function $f: V(G) \to \{0,1,\ldots q\}$ by

$f(u_i) = 2i-2, 1 \leq i \leq m-1$

$f(u_n) = 2m$

$f(v_i) = 2i-1, 1 \leq i \leq m$

$f(w_i) = 2m+i, 1 \leq i \leq n$

Then the distinct edge labels are

$f(u_i\ u_{i+1}) = 2i, 1 \leq i \leq m$

$f(u_i\ v_i) = 2i-1, 1 \leq i \leq m$

$f(w_i\ w_{i+1}) = 2m+i, 1 \leq i \leq n-1$

Clearly, $(C_m \odot K_1) \cup P_n$ is a Contra Harmonic mean graph.

Example 2.6: The Contra Harmonic mean labeling of $(C_6 \odot K_1) \cup P_5$

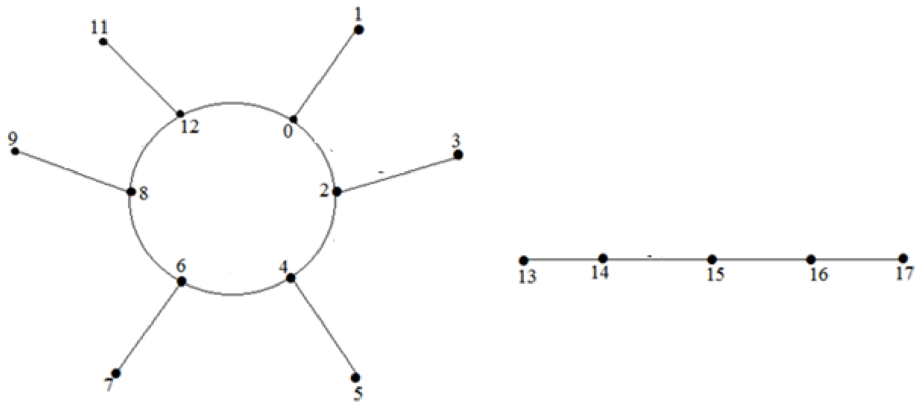

Figure 3

Theorem 2:7: $(C_m \odot K_1) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof: Let $u_1u_2\ldots..u_m$ be the cycle C_m and let v_i be the pendent vertex joined to the vertex u_i of $C_m 1 \le i \le m$.The resultant graph is $C_m \odot K_1$. Let $w_1\ldots\ldots w_n$ be the path P_n and t_i be the vertex which is joined to the vertex w_i, $1 \le i \le n$ of the path P_n. The resultant graph is $P_n \odot K_1$.

Let $G = (C_m \odot K_1) \cup (P_n \odot K_1)$.

Define a function $f: V(G) \rightarrow \{0,1\ldots..q\}$ by

$f(u_i) = 2i-2$, $1 \le i \le m-1$,$f(u_n) = 2m$,

$f(v_i) = 2i-1$, $1 \le i \le m$

$f(w_i) = 2m+2i-1$, $1 \le i \le n$

$f(t_i) = 2m+2i$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_iu_{i+1}) = 2i$, $1 \le i \le m$

$f(u_i\, v_i) = 2i-1$, $1 \le i \le m$

$f(w_iw_{i+1}) = 2m+2i$, $1 \le i \le n-1$

$f(w_it_i) = 2m+2i-1$, $1 \le i \le n$

Clearly, $(C_m \odot K_1) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Example: 2.8

The Contra Harmonic mean labeling of $(C_6 \odot K_1) \cup (P_5 \odot K_1)$

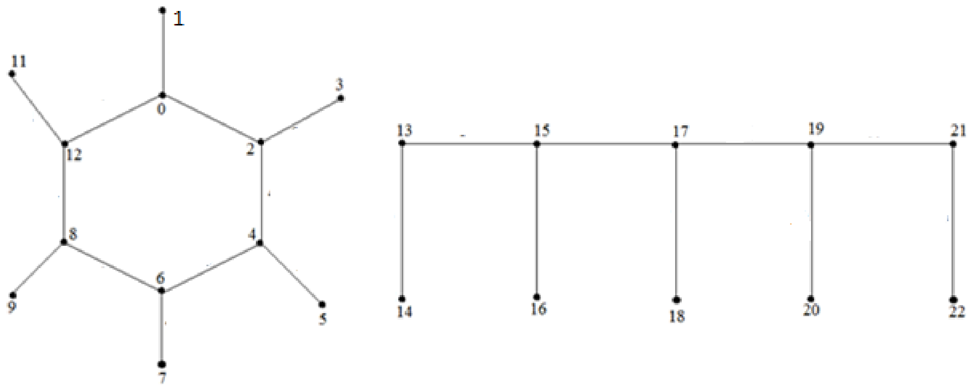

Figure 4

Theorem 2.9: $(C_m \odot K_1) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Proof: Let $u_1 u_2 \ldots u_m$ be the cycle C_m and let v_i be the vertex joined to the vertex u_i of C_m $1 \le i \le m$. The resultant graph is $C_m \odot K_1$. Let $w_1 \ldots w_n$ be the path P_n and let t_i and s_i be the vertices which are joined to the vertex w_i of path P_n ,$1 \le i \le n$. The resultant graph in $P_n \odot \overline{K_2}$.

Let $G = (C_m \odot K_1) \cup (P_n \odot \overline{K_2})$

Define $f: V(G) \to \{0,1,\ldots q\}$ by

$f(u_i) = 2i-2,\ 1 \le i \le m-1$,$f(u_m) = 2m$

$f(v_i) = 2i-1,\ 1 \le i \le m$

$f(w_i) = 2m+3i-2\ 1 \le i \le n$

$f(t_i) = 2m+3i-1,\ 1 \le i \le n$

$f(s_i) = 2m+3i,\ 1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 2i,\ 1 \le i \le m$

$f(u_i v_i) = 2i-1,\ 1 \le i \le m$

$f(w_i w_{i+1}) = 2m+3i,\ 1 \le i \le n-1$

$f(w_i t_i) = 2m+3i-2,\ 1 \le i \le n$

$f(w_i s_i) = 2m+3i-1,\ 1 \le i \le n$

Clearly, $(C_m \odot K_1) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Example 2.10 Contra Harmonic mean labeling of ($C_6 \odot K_1$) ∪($P_4 \odot \overline{K_2}$)

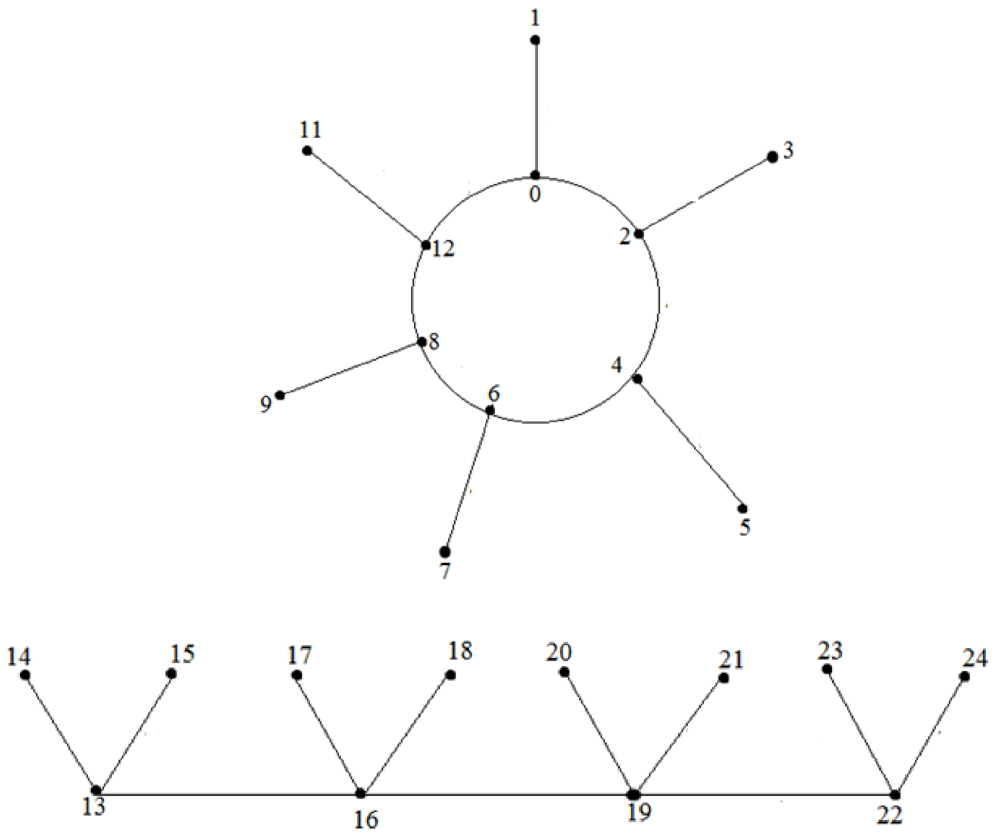

Figure 6

Theorem 2.11 : ($C_m \odot \overline{K_2}$)∪ P_n is a Contra Harmonic mean graph

Proof: $u_1u_2\ldots u_m$ be a cycle C_m and let v_i,w_i be the vertices that are joined to the vertex $u_i 1 \le i \le m$ of the cycle C_m.The resultant graph is ($C_m \odot \overline{K_2}$)

Let $s_1s_2\ldots s_n$ be the path P_n .

Let G = ($C_m \odot \overline{K_2}$) ∪ P_n

Define f: V(G) → {0,1….q} by

$f(u_i) = 3i-2, 1 \le i \le m-1$,$f(u_m) = 3m-1$

$f(v_i) = 3i-3, 1 \le i \le m$

$f(w_i) = 3i-1, 1 \le i \le m-1$,$f(w_m) = 3m$

$f(s_i) = 3m+i, 1 \le i \le n$

Then the distinct edge labels are

$f(u_i\, u_{i+1}) = 3i$, $1 \le i \le m-1$,$f(u_m, u_1) = 3m-1$

$f(u_i\, v_i) = 3i-2$, $1 \le i \le m$

$f(u_i w_i) = 3i-1$, $1 \le i \le m-1$, $f(u_m w_m) = 3m$

$f(s_i\, s_{i+1}) = 3m+i$, $1 \le i \le n-1$

Clearly, $(C_m \odot \overline{K_2}) \cup P_n$ is a Contra Harmonic mean graph.

Example : 2.12 The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup P_6$ is

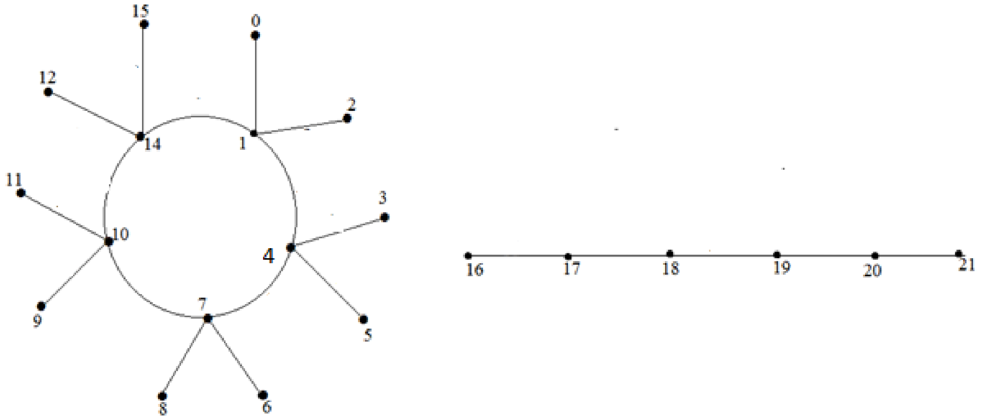

Figure 5

Theorem 2.13 :$(C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof: Let $u_1, u_2, \ldots. u_m$ be the cycle C_m. Let vi, w_i be the vertices that are joined to the vertex u_i, $1 \le i \le m$ of the cycle C_m.The resultant graph is $(C_m \odot \overline{K_2})$.

Let $s_1 s_2 \ldots. s_n$ be the path P_n and t_i be the vertex that are joined to the vertex s_i, $1 \le i \le n$ of P_n. .The resultant graph is $(P_n \odot K_1)$.

Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$.

Define $f: V(G) \to \{0,1,2,\ldots..q\}$ by

$f(u_i) = 3i-2$, $1 \le i \le m-1$, $f(u_m) = 3m-1$

$f(v_i) = 3i-3$, $1 \le i \le m$

$f(w_i) = 3i-1$, $1 \le i \le m-1$, $f(w_m) = 3m$

$f(s_i) = 3m+2i-1$, $1 \le i \le n$

$f(t_i) = 3m+2i,\ 1\le i\le n$

Then the distinct edge labels are

$f(u_iu_{i+1}) = 3i,\ 1\le i\le m-1$,$f(u_m, u_1) = 3m-1$

$f(u_i\, v_i) = 3i-2,\ 1\le i\le m$

$f(u_iw_i) = 3i-1,\ 1\le i\le m-1,\ f(u_mw_m) = 3m$

$f(s_i, s_{i+1}) = 3m+2i,\ 1\le i\le n-1$

$f(s_it_i) = 3m+2i-1,\ 1\le i\le n$

Clearly, $(C_m \odot \overline{K_2}) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph.

Example : 2.14

The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_6 \odot K_1)$ is

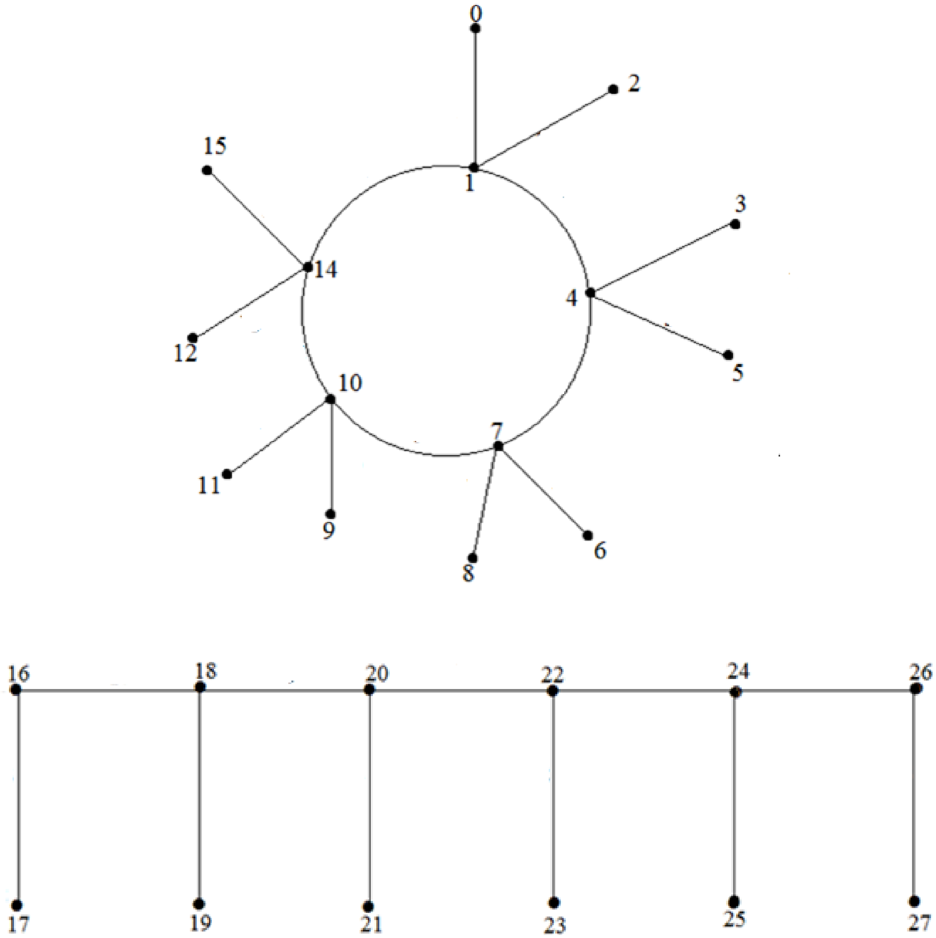

Figure: 7

Theorem : 2.15

Proof: $(C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$ is a Contra Harmonic mean graph.

Let $u_1 \ldots u_m$ be the cycle C_m and let v_i, w_i be the vertices that are joined to vertex u_i $1 \le i \le m$ of C_m. The resultant graph is $C_m \odot \overline{K_2}$. Let $z_1 \ldots z_n$ be the path P_n and let s_i, t_i be the vertices that are joined to the vertex z_i of the path P_n $1 \le i \le n$. The resultant graph is $P_n \odot \overline{K_2}$.

Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$

Define a function $f: V(G) \to \{0, 1 \ldots q\}$ by

$f(u_i) = 3i-2$, $1 \le i \le m-1$,$f(u_n) = 3m-1$

$f(v_i) = 3i-3$, $1 \le i \le m$

$f(w_i) = 3i-1$, $1 \le i \le m-1$, $f(u_m) = 3m$

$f(z_i) = 3m+3i-2$, $1 \le i \le n$

$f(s_i) = 3m+3i-1$, $1 \le i \le n$

$f(t_i) = 3m+3i$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_i\, u_{i+1}) = 3i$, $1 \le i \le m-1$,$f(u_m\, u_1) = 3m-1$

$f(u_i\, v_i) = 3i-2$, $1 \le i \le m$

$f(u_i w_i) = 3i-1$, $1 \le i \le m-1$, $f(u_i w_i) = 3m$

$f(z_i\, z_{i+1}) = 3m+3i-2$, $1 \le i \le n$

$f(z_i s_i) = 3m+3i-2$, $1 \le i \le n$

$f(z_i t_i) = 3m+3i-1$, $1 \le i \le n$

Clearly, $(C_m \odot \overline{K_2}) \cup (P_n \odot \overline{K_2})$is a Contra Harmonic mean graph.

Example 2.16:

The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_4 \odot \overline{K_2})$

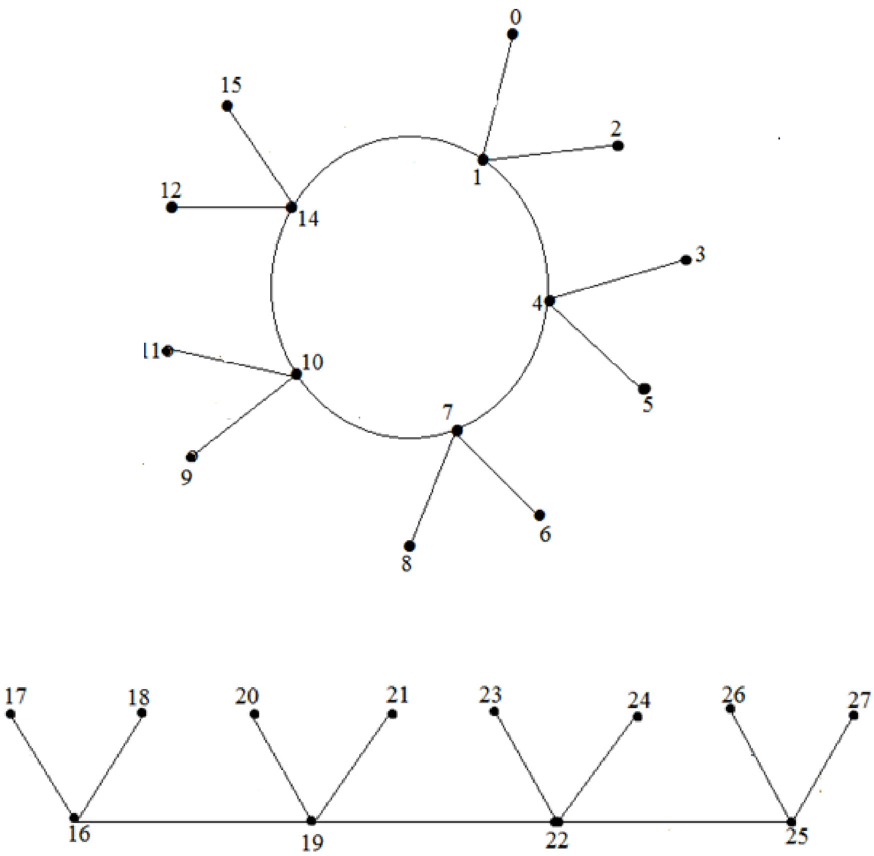

Figure 8

Theorem 2: 17: $(C_m \odot \overline{K_2}) \cup (P_n \odot K_3)$ is a Contra Harmonic mean graph.

Proof: Let $u_1 \ldots\ldots u_m$ be a cycle C_m and let v_i, w_i be the vertices joined to the vertex u_i $1 \le i \le m$. The resultant graph is $C_m \odot \overline{K_2}$.

Let $z_1 \ldots. z_n$ be the path P_n and let s_i, t_i be the vertex of K_3 that are joined to the vertex z_i of the path P_n $1 \le i \le n$, The resultant graph is $P_n \odot K_3$.

Let $G = (C_m \odot \overline{K_2}) \cup (P_n \odot K_3)$

Define $f: V(G) \to \{0,1 \ldots. q\}$ by

$f(u_i) = 3i-2$ $1 \le i \le m-1$, $f(u_m) = 3m-1$

$f(v_i) = 3i-3$, $1 \le i \le m$

$f(w_i) = 3i-1$, $1 \le i \le m-1$, $f(w_m) = 3m$

$f(z_i) = 3m+4i-3, 1 \le i \le n$

$f(s_i) = 3m+4i-2, 1 \le i \le n$

$f(t_i) = 3m+4i-1, 1 \le i \le n$

Then the distinct edge labels are

$f(u_i u_{i+1}) = 3i, 1 \le i \le m-1$, $f(u_m u_1) = 3m-1$

$f(u_i v_i) = 3i-2, 1 \le i \le m$

$f(u_i w_i) = 3i-1, 1 \le i \le m-1$, $f(u_m w_m) = 3m$

$f(z_i z_{i+1}) = 3m+4i, 1 \le i \le n-1$

$f(z_i s_i) = 3m+4i-3, 1 \le i \le n$

$f(z_i t_i) = 3m+4i-1, 1 \le i \le n$

$f(s_i t_i) = 3m+4i-2, 1 \le i \le n$

Clearly, $(C_m \odot \overline{K_2}) \cup (P_n \odot K_2)$ is a Contra Harmonic mean graph.

Example 2: 18: The Contra Harmonic mean labeling of $(C_5 \odot \overline{K_2}) \cup (P_4 \odot K_3)$ is

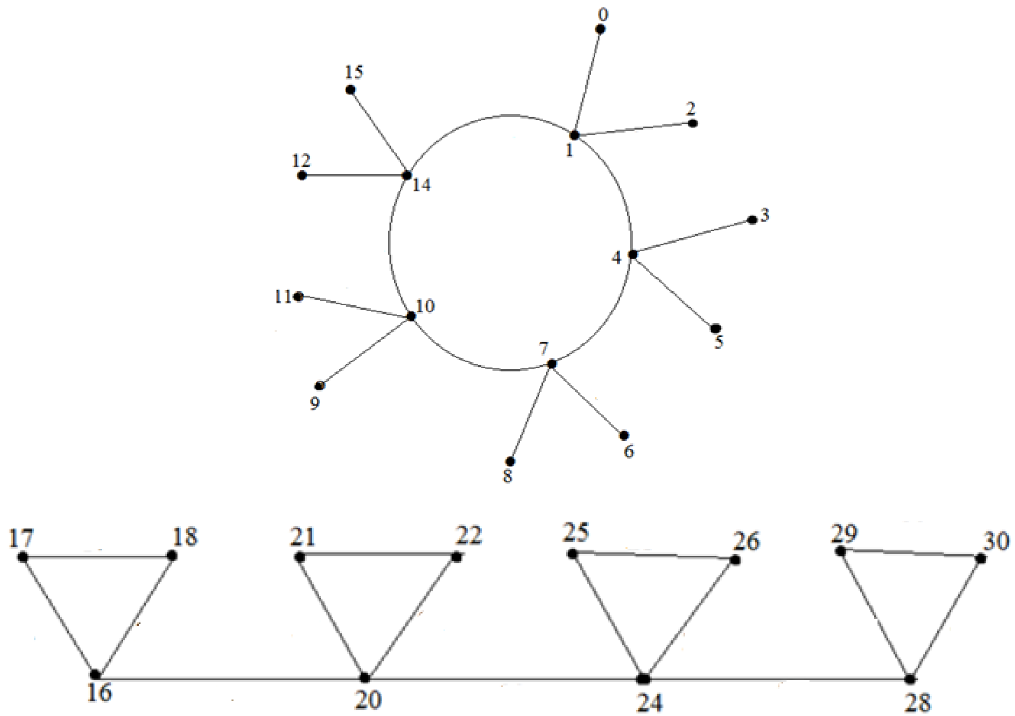

Figure 9

REFERENCES

[1] Gallian. 2012, A Dynamic Survey of graph labeling.The electronic Journal of Combination 17# DS6.

[2] Harary. F, 1988, Graph Theory, Narosa publishing House Reading, New Delhi.

[3] Somasundaram.S, Ponraj.R "Mean Labeling of Graphs', National Academy of Science Letter Volume 26, p210-213.

[4] Somasundaram.S and S.S. Sandhya S.S 'Harmonic Mean Labeling of Graphs', Journal of Combinatorial Mathematics and Combinatorial Computing.

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[5] Sandhya S.S, Somasundaram. S and Rajeshni Golda. J" 'Contra Harmonic mean labeling of graphs communicated to Journal of Mathematics Research .

[6] Sandhya S.S, Somasundaram. S and Rajeshni Golda. J" 'Contra Harmonic mean labeling of Some More graphs' communicated to International Journal of Contemporary Mathematical Science .