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Pricing floating strike lookback put option under heston stochastic volatility

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Abstract

The pricing problems of the exotic options in the finance do not have the analytic solutions under stochastic volatility and so it is difficult to calculate the option prices or at least it requires much of time to compute them. This study provides the required theoretical framework to practitioners for the option price estimation. This paper focuses on pricing for floating strike lookback put option and testing option pricing formulas for the Heston stochastic volatility model, which defines the asset volatility as the stochastic process. Our pricing method is depending on the PDE approach on Heston stochastic volatility model and homotopy analysis method. Heston model has received the most attention then it can give a acceptable explanation of the underlying asset dynamics. The resulting formula is well connected to a Black–Scholes price that is the first term of the series expansion, which makes computing the option prices fairly efficient.

AMS subject classification:
Keywords: lookback put option, option pricing,stochastic volatility model, Heston model, homotopy analysis method.

1. Introduction

European Look back options are kind of the exotic option with path-dependent, introduced at first by Goldman, Sossin and Gatto (1997) having their settlement based on the minimum or the maximum value underlying asset registered during the life of the option time. At maturity, the holder can look back and select the most convenient price of the underlying that occurred during this period: therefore they offer investors the opportunity at a price of buying a stock at its lowest price and selling a stock at its highest price. Since this scheme guarantees the best possible result for the option holder, he or she will never regret the option payoff. As a consequence, a look back option is more expensive than any other option with similar payoff function stochastic volatility is a statistical model used for to evaluate derivative securities, such as options.stochastic volatility treats the volatility of the underlying asset as a random variable. This improves the accuracy of models and forecasts.Stochastic volatility models for the pricing derivative securities have been developed as the extension to the original, constant volatility model of the Black and Scholes (1973). Under the Black and Scholes model, closed-form pricing formulas for the continuously monitored lookback options were derived by Goldman and Conze and Viswanathan. Heynen and Kat(1979) derived the analytical formulas for discretely monitored lookback options under the Black-Scholes setting and the corresponding formulas were further generalized to the general LÃ©vy setting by Agliardi. In the Black-Scholes model, the underlying asset price process is assumed to follow the geometric Brownian process, in which the volatility of the asset price is a constant. This assumption used in option price with the Black-scholes formula is inconsistent with the phenomena of real data in some studies, for example see [8],[22].under the assumption constant volatility, option price can not affected by the change in the price of the underlying asset but in the stochastic volatility option price can affected by the change in the price of the underlying asset. So that by assuming that the volatility of the underlying price is stochastic process rather than a constant, it becomes possible to model derivatives more accurately.

The pricing of the lookback options under the stochastic volatility model poses interesting mathematical challenges.In this work, we use the homotopy analysis method to derive the analytic pricing formulas for lookback options under Heston's stochastic volatility model There are a number of methods one can use to model volatility stochastically. Hull and White (1987) model the variance using a geometric Brownian motion, as well as an Ornstein-Uhlenbeck process with mean-reversion related to the volatility. In the general case, mean-reversion is considered to be an essential feature of observed volatility, and thus all plausible models are of the Ornstein-Uhlenbeck type. Wiggins (1987) models the logarithm of the volatility with mean-reversion, whereas Scott (1987), Johnson and Shanno (1987), Heston (1993) and Stein and Stein (1991) model the variance using a square root process. Zhu (2000) also considers a double square root process, which is an extension of the basic square root process in which both the drift and diffusion coefficients involve the volatility. In this paper we focus on HestonâŁ™s square root model,under which Heston (1993) provides an analytic expression for European option prices.

This paper is concerned with a pricing of the floating strike lookback put option. The price of floating strike lookback put option depends on the maximum or minimum of the underlying asset price. In this work, we exercise the homotopy analysis method to drive the pricing formulas for the floating strike lookback put option under Heston stochastic volatility model.The homotopy analysis method has been used by Liao (1992, 1997, 2003) in mechanics, and Zhu (2006), Zhu et al. (2010), Park and Kim (2011) and Leung (2013) in the finance.Now we are going to explore the relationship between the lookback put option price with the floating strike and stock price. The paper is organized as follows. In Section 2, we introduce Heston stochastic volatility model. In section 3, we derive a governing partial differential equation (PDE) for the lookback options. In Section 4, By the homotopy analysis method we used to derive a pricing formula for a floating strike lookback put option. The simulation results are presented in section 5.The conclusions are presented in Section 6.

2. The heston stochastic model

The Heston SV model assumes the following stochastic process for the underlying asset price s_t under risk neutral measure at the time t given as:

$$dS_t = r_t S_t dt + S_t \sqrt{v_t} dW_t^1 \tag{1}$$

and the variance follows the process:

$$dv_t = k(\Theta - v_t)dt + \sigma\sqrt{v_t} dW_t^2 \tag{2}$$

$$dW_t^1 dW_t^2 = pdt$$

To take into account leverage effect, Wiener stochastic processes W^1, W^2 should be correlated $dW_t^1 dW_t^2 = pdt$. In which the correlation coefficient is p. The stochastic model (2) for the variance is related to the square-root process of Feller (1951) and Cox, Ingersoll and Ross (1985). For the square-root process (2) the variance is always positive and if $2k\Theta > \sigma^2$ then it cannot reach zero. Note that the deterministic part of process (2) is asymptotically stable if $k > 0$. Clearly, that equilibrium point is $V_t = 0$.

The parameters used in the model are as follows:

S_t is the price of the underlying asset at the time t

K is the rate of the mean reversion

r is the risk free interest rate

Θ is the long term mean variance

v_t is the variance at the time t

σ is the volatility of the variance process.

Therefore, under the Heston model, the underlying asset follows an evolution process which is similar to the Black-scholes model, but it also introduces a stochastic behavior for the volatility process. In particular, Heston makes the assumption that the asset variance V_t follows a mean reverting Cox-Ingersoll-Ross process.Consequently, the Heston model provides a versatile modelling framework that can accommodate many of the specific characteristics that are typically observed in the behavior of financial assets. In particular, the parameter σ controls the kurtosis of the underlying asset return distribution, while p sets its asymmetry. We plot an example of the stock price and volatility of the stochastic process in the Figure 1 and Figure 2 respectively.

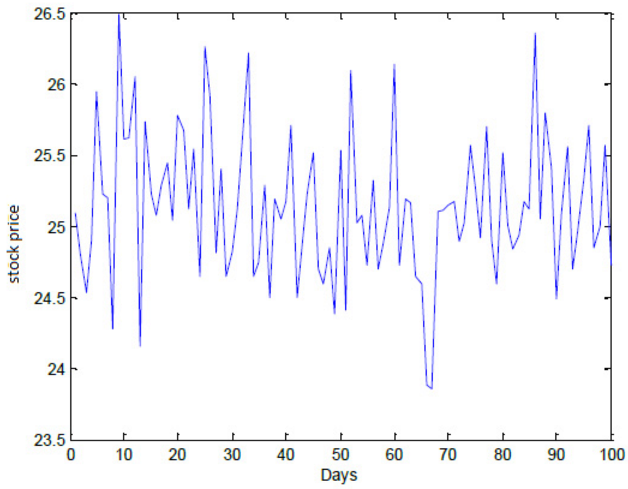

Figure 1: Stock price are dynamics in Heston model

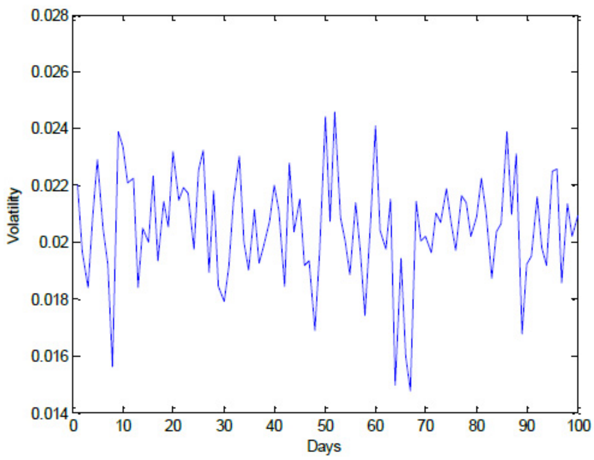

Figure 2: Volatility dynamics in the Heston model

3. The PDE of the heston model for the option price

In this section we derive the PDE that the price of a derivative must solve, where the tradeable security as well as the volatility of the tradeable security follows general stochastic processes. The PDE that governs the prices of derivatives written on a tradeable security with stochastic volatility is derived and we describe how to derive the PDE for the Heston model. This derivation is a special case of a PDE for general stochastic volatility models which is described by Gatheral. Heston is one of the most popular option pricing models. This is due in part to the fact that the Heston model produces call or put prices that are in closed form, up to an integral that must evaluated numerically. In this note we present a complete derivation of the Heston model. In order to price options in a stochastic volatility model, we can apply no-arbitrage arguments, or use the risk-neutral valuation method.

To derive heston PDE let Form a portfolio consisting of one option being priced,denoted by the value $V = V(s, v, t)$, Δ units of the stock S, ψ of another option $U = U(s, v, t)$ that is used to hedge the volatility. The portfolio has value

$$\Pi = V + \Delta S + \psi U \tag{3}$$

where $\Pi = \Pi_t$. Assuming the portfolio is self financing, the change in portfolio value is

$$d\Pi = dV + \Delta dS + \psi U \tag{4}$$

Apply itos Lemma to dV and differentiate with respect to the variables t, S, v we get

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}dS + \frac{\partial V}{\partial v}dv + \frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2}dt + \frac{1}{2}\sigma^2 v\frac{\partial^2 V}{\partial v^2}dt + \sigma vp\frac{\partial^2 V}{\partial v\partial S}dt \tag{5}$$

Applying Itos Lemma to dU produces the identical result, but in U. Combining these two expressions, we can write the change in portfolio value as:

$$d\Pi = dV + \Delta dS + \psi U \tag{6}$$

$$\begin{aligned} d\Pi = &\left\{\frac{\partial V}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2} + p\sigma vS\frac{\partial^2 V}{\partial v\partial S} + \frac{1}{2}v\sigma^2\frac{\partial^2 V}{\partial v^2}\right\}dt \\ &+ \psi\left\{\frac{\partial U}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 U}{\partial S^2} + p\sigma vS\frac{\partial^2 U}{\partial v\partial S} + \frac{1}{2}\sigma^2 v\frac{\partial^2 U}{\partial v^2}\right\}dt \\ &+ \left\{\frac{\partial V}{\partial S} + \psi\frac{\partial U}{\partial S} + \Delta\right\}dS + \left\{\frac{\partial V}{\partial v} + \psi\frac{\partial U}{\partial v}\right\}dv \end{aligned} \tag{7}$$

In order for the portfolio to be hedged against movements in the stock and against volatility, the last two terms in Equation (6) involving dS and dv must be zero. This implies that the hedge parameters must be

$$\psi = -\frac{\dfrac{\partial V}{\partial v}}{\dfrac{\partial U}{\partial v}}, \Delta = -\psi\frac{\partial U}{\partial S} - \frac{\partial V}{\partial S} \tag{8}$$

Moreover, the portfolio must earn the risk free rate,r.Hence $d\Pi = r\Pi dt$. Now with the values of Δ and ψ from Equation (7) the change in value of the riskless portfolio is

$$\begin{aligned} d\Pi = &\left\{\frac{\partial V}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2} + p\sigma vS\frac{\partial^2 V}{\partial v\partial S} + \frac{1}{2}v\sigma^2\frac{\partial^2 V}{\partial v^2}\right\}dt \\ +\psi &\left\{\frac{\partial U}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 U}{\partial S^2} + p\sigma vS\frac{\partial^2 U}{\partial v\partial S} + \frac{1}{2}\sigma^2 v\frac{\partial^2 U}{\partial v^2}\right\}dt \end{aligned} \tag{9}$$

In risk less portfolio we have

$$d\Pi = r\Pi dt \tag{10}$$

which we write as

$$d\Pi = X + \psi Y$$

$$X + \psi Y = r(V + \Delta S + \psi U)$$

Substituting for ψ and re-arranging, produces the equality

$$\frac{X - rV + rS\frac{\partial V}{\partial S}}{\frac{\partial V}{\partial v}} = \frac{Y - rU + rS\frac{\partial U}{\partial S}}{\frac{\partial U}{\partial v}} \tag{11}$$

The left-hand side of Equation (11) is a function of V only, and the right-hand side is a function of U only. This implies that both sides can be written as a function f(S, v, t) of S, v, and t. Following Heston, specify this function as $f(S, v, t) = -K(\theta - v) + \lambda(t, s, v)$ where $\lambda(t, s, v)$ is market price of volatility risk. Substitute f(S v,t) into the left-hand side of Equation (11), substitute for Y, and rearrange to produce the Heston PDE for the option U expressed in terms of the price S

$$\begin{aligned} &\frac{\partial U}{\partial t} + \frac{1}{2}vS^2\frac{\partial^2 U}{\partial S^2} + p\sigma vS\frac{\partial^2 U}{\partial v\partial S} + \frac{1}{2}v\sigma^2\frac{\partial^2 U}{\partial v^2} - rU \\ &\quad + rS\frac{\partial U}{\partial S} + [K(\theta - v) - \lambda(t, s, v)]\frac{\partial U}{\partial v} = 0 \end{aligned} \tag{12}$$

Next we can derive a closed form of a PDE in terms log price,let $x = lnS$ and describe the PDE in terms of x, v and t instead of S, v, and t.This leads to a simpler form of the PDE. We need the following derivatives,which are direct to derive

$$\frac{\partial U}{\partial s}, \frac{\partial^2 U}{\partial v\partial s}, \frac{\partial^2 U}{\partial s^2}$$

Insert into the Heston PDE Equation (12). All the S terms eliminated and we obtain the Heston PDE in terms of the log price $x = lnS$ and Heston assumes that the price are risk-neutral.The reason for this term is that in reality most investors are found to be risk averse in experimental settings [1]. Moreover, Lamoureux and Lastrapes find evidence from observed option prices that the efficient-market hypothesis and investor risk-neutrality cannot hold simultaneously [2]. Often λ is assumed zero, however, Then the price is said to be given under the risk-neutral measure, i.e, under the assumption that investors are risk-neutral. In the following formulations we will assume $\lambda = 0$.

$$\frac{\partial U}{\partial t} + \frac{1}{2}v\frac{\partial^2 U}{\partial x^2} + p\sigma v\frac{\partial^2 U}{\partial v \partial x} + \frac{1}{2}v\sigma^2\frac{\partial^2 U}{\partial v^2} - rU + \left(r - \frac{v}{2}\right)\frac{\partial U}{\partial x} + [K(\theta - v)]\frac{\partial U}{\partial v} = 0 \tag{13}$$

The partial differential Equation (13) shows that it is a governing the option price.

4. Floating strike lookback put option price

The payoffs of the floating strike lookback put options depend on the maximum asset price reached during the life of the option and underlying asset price observed at the maturity. Based upon the fundamental theorem of asset pricing (Shreve (2000)), the no-arbitrage price of a European lookback put option with floating strike is given by

$$U^f(t, x, x^*, v) = E^Q[e^{-r(T-t)}H^f(X_T, X_T^*)|X_t = x, X_t^* = x^*, V_t = v] \tag{14}$$

where X_T^* is the maximum asset price observed during the life of the option and H^f is the payoff of the put option.In this section we consider a floating strike lookback put option, where the underlying asset price is assumed to follow the SDE.Then $H^f(X_T, X_T^*) = X_T^* - S_T$ (payoff put option), the risk-neutral price of the floating strike lookback put option, denoted by $U^f(t, x, x^*, v)$ at the time t, $t \in [o, T]$ for $X_t = x$, $X_t^* = x^*$ and $V_t = v$ is given as

$$U^f(t, x, x^*, v) = E^Q[e^{-r(T-t)}H^f(X_T - X_T^*)|X_t = x, X_t^* = x^*, V_t = v] \tag{15}$$

In this section $H^f = (X_T^* - X_T)$ floating strike chosen.Transforming the governing equation (13) in terms of the differential operators explained by

$$£_1 = p\sigma v\frac{\partial}{\partial v} + p\sigma v\frac{\partial^2}{\partial x \partial v} + k(\Theta - v)\frac{\partial}{\partial v} + \frac{1}{2}\sigma^2 v\frac{\partial^2}{\partial v^2}$$

$$£_2 = \frac{\partial}{\partial t} + \frac{1}{2}v\frac{\partial^2}{\partial x^2} + \left(r - \frac{v}{2}\right)\frac{\partial}{\partial x}$$

and the integral problem (15) can be transformed into the PDE problem as follows.since both $x_t^*$and x_t are continuous and non decreasing.So that the quadratic variance and covariance of the $[x_t^*, x_t^*]$ and $[x_t^*, x_t]$ satisfy the following conditions

$$[x_t^*, x_t^*] = \lim_{\Pi\to 0} \Sigma_{i=0}^m([x_{ti+1}^* - x_{ti}^*])^2 \le x_t^* \lim_{\Pi\to 0} max(x_{ti+1}^* - x_{ti}^*) = 0$$

and

$$[x_t, x_t^*] = \lim_{\Pi\to 0} \Sigma_{i=0}^m(x_{ti+1} - x_{ti})^2(x_t^*, x_{ti+1}^* - x_t^*, x_{ti}^*)$$

$$\leq \lim_{\Pi \to 0} max(x_{ti+1} - x_{ti}) = 0$$

for some partition $\Pi = \{0 = t_0 \leq t_1 \leq t_2 \cdots = t\}$. This gives the integral involved with the $dx_t dx_t^*$ and $dx_t^* dx_t^*$ can be zero. so from itos formula we can get

$$d(e^{-rt}U^f) = e^{-rt}(£_1 + £_2)U^f dt + U_x^f dx_t$$

Then we have

$$E[\int_t^T e^{-rs}(£_1 + £_2)U^f ds + \int_t^T U_x^f dx_s | x_t = x, x_t^* = x^*]$$

for $t, T \in [0, \infty]$ since $e^{-rt}U^f$ is a martingale and the second part of the conditional expectation is zero on the $0 < x < x^*$ so that the PDE for the U^f can obtained on the interval $0 < x < x^*$ and by using mean value theorem and taking the $T \to t$ we can get

$$\begin{aligned} (£_1 + £_2)U^f(t, x, x^*, v) = 0, 0 \leq t \leq T, 0 \leq x \leq x^* \\ U(T, x, x^*, v) = x^* - x \\ \frac{\partial U}{\partial x^*}(t, x, x^*, v) \mid_{x=x^*} = 0 \end{aligned} \tag{16}$$

Here, the final condition follows from the definition (16) directly and the assumption on the continuity of partial derivatives leads to the boundary condition.

Definition: zeroth-order deformation equation. Let $p \in [0, 1]$denote the embedding parameter and $U_0(t, x, x^*, v)$ be the initial approximation of the $U(t, x, x^*, v)$ such that as p increases from 0 to 1 ,$U(t, x, x^*, v)$ varies continuously from the initial approximation $U_0(t, x, x^*, v)$,such kind of the continuous variation or deformations are defined by the zero order deformation equation.
Applying the definition and Following the same vein as Park and Kim [4] method, the homotopy analysis method is to solve $U(t, x, x^*, v)$ from (16) we can construct a homotopy of the of (16).To construct let us consider $U(t, x, x^*, v, p)$ denoting the solution of a PDE problem given by $H(t, x, x^*, v, p)$ is equal to zero with the final and boundary condition of (16), where H, called a homotopy, is defined by

$$\begin{aligned} H(t, x, x^*, v, p) = (1 - p)(£_2 U(t, x, x^*, v, p) - £_2 U_0(t, x, x^*, v)) \\ +p(£_1 + £_2)U(t, x, x^*, v, p), p \in [0, 1] \end{aligned} \tag{17}$$

Here $U_0(t, x, x^*, v)$ is the initial value approximation from Black-scholes formula for the lookback put option price with the constant volatility.The BlackâŁ"Scholes formula is well-known and, for instance, see Wilmott (2006). By this choice of U_0, the homotopy problem becomes

$$H(t, x, x^*, v, p) = £_2 U(t, x, x^*, v, p) + p£_1 U(t, x, x^*, v, p) = 0 \tag{18}$$

with the final and the boundary condition of (16) we can apply the homotopy analysis method by the considering a Taylor series

$$U(t, x, x^*, v, p) = \sum_{n=0}^{\infty} U_n(t, x, x^*, v,) p^n, \tag{19}$$

where U_n denote a Taylor coefficient. Note that floating strike lookback put option price U_f is then given by

$$U_f(t, x, x^*, v,) = \lim_{p \to 1} U(t, x, x^*, v, p) = \sum_{n=0}^{\infty} U_n(t, x, x^*, v) \tag{20}$$

Inserting equation (19) into (18) and using a standard perturbation argument, we obtain formally a hierarchy of PDE problem as follows we get

$$\begin{aligned} £_2 U_n(t, x, x^*, v) + £_1 U_{n-1}(t, x, x^*, v) &= 0, \\ U_n(T, x, x^*, v) &= 0, \\ \frac{\partial U}{\partial x^*}(t, x, x^*, v) \mid_{x=x^*} &= 0 \end{aligned} \tag{21}$$

for all $n = 1, 2, 3, \dots$.

To find the solution of the equation (21) we use two lemmas: i.e. a lemma about a Feynman-kac formula for floating strike lookback put option price and lemma about the joint probability density of the two Gaussian processes.For the convenience, we use the notation

$$E^{x,x^*}[.] := E^Q[.|S_t = x, S_t^* = x^*]$$

whereS_t and S_t^* are the solution given by

$$S_t = rS_t dt + \sqrt{V_t} S_t dW_t^t$$

$$S_t^* = max_{u \le t} S_u$$

respectively, for some$\sqrt{V_t} \in R^+$.

Lemma 4.1. If $Z(t, x, x^*, v) \in U_b^{1,2}(R^+ \times R^3)$ and solve the PDE problem and also $U_b^{1,2}(R^+ \times R^3)$ is the function space of bounded functions continuously differentiable with respect to $t > 0$ and twice continuously differentiable with respect to $(t, x, x^*, v) \in R^3$.

$$£_2 Z(t, x, x^*, v) = g(t, x, x^*, v), 0 \le t \le T, 0 < x \le x^*$$

$$Z(T, x, x^*, v) = h(x, x^*)$$

,

$$\frac{\partial Z}{\partial x^*}(t, x, x^*, v) \mid_{x=x^*} = 0$$

where g and h satisfy the conditions $g + h = o(e^{x^2+x^*})$ as x and $x^* \to \infty$ then

$$Z(t, x, x^*, v) = E^{x,x^*}[e^{-r(T-t)}h(S_T, S_T^*) - \int_t^T e^{r(t-s)}g(s, S_s, S_s^*, v)ds]$$

Proof. see (7) theorem 2.2. ■

Lemma 4.2. If H_t and H_t^* are the two Gaussian processes defined by

$$H_t = (r - \frac{1}{2}\sigma^2)t + \sigma W_t$$

,

$$H_t^* = max\{(r - \frac{1}{2}\sigma^2)s + \sigma W_s\}$$

, then the joint probability density of a processes (H_t, H_t^*) is given as

$$Q(H_t \in db, H_t^* \in dc) = \frac{2(2c-a)}{\sigma^3 t^{\frac{3}{2}}\sqrt{2\Pi}} e^{\frac{r-\frac{1}{2}\sigma^2}{\sigma^2}} b - \frac{(r-\frac{1}{2}\sigma^2)^2}{2\sigma^2} t - \frac{(2c-b)^2}{2\sigma^2 t} dbdc$$

Proof. see (shreve 2000) theorem 7.2.1. ■

Using the above two lemmas we get the following result on the solution of the PDE problem (21)

Theorem 4.3. Assume that the floating strike lookback put option price $U^f(t, x, x^*, v)$ is represented as

$$U^f(t, x, x^*, v) = \sum_{n=0}^{\infty} U_n^f(t, x, x^*, v),$$

then $U_n^f(t, x, x^*, v)$ is given by

$$U_0^f(t, x, x^*, v) = \left(1 + \frac{\sigma^2}{2r}\right) xN\left(\delta + \left(T - t, \frac{x}{x^*}\right)\right) + e^{-r(T-t)}x^*N\left(-\delta - \left(T - t, \frac{x}{x^*}\right)\right)$$

$$-\frac{\sigma^2}{2r}e^{-r(T-t)}x\left(\frac{x^*}{x}\right)^{\frac{2r}{\sigma^2}} N\left(-\delta - \left(T - t, \frac{x^*}{x}\right)\right) - x$$

for $n = 0$ and

$$U_n^f(t, x, x^*, v) = \int_t^T \int_{ln(\frac{x^*}{x})}^{\infty} \int_{-\infty}^{\infty} \frac{2(2c-b)£_1 U_{n-1}(s, xe^b, xe^c, v)}{\sigma^3 (s-t)^{\frac{3}{2}}\sqrt{2\Pi}}$$

$$\cdot exp\{r(t-s) + \frac{r - \frac{1}{\sigma^2}}{\sigma^2} b - \frac{(r - \frac{\sigma^2}{2})^2}{2\sigma^2}(s-t) - \frac{(2c-b)^2}{2\sigma^2(s-t)}\} dbdcds,$$

where $£_1$ is given by equation (16), for $n \geq 1$, where N denotes the usual cumulative normal distribution,

$$\delta \pm (t, x) = \frac{1}{\sigma\sqrt{t}}(lnx + (r \pm \frac{1}{2}\sigma^2)t)$$

and $\sqrt{v} = \sigma$.

Proof. Since U_0^f is the Black-scholes put option price.thus the PDE of (16) for U_1 satisfies the required conditions of lemma and also U_n for $n > 1$ are smooth to be U_o^2 due to $Q(H_t \in db, H_t^* \in dc) = o(e^{-b^2-c^2})$, then from both lemmas we can obtain

$$\begin{aligned}
&U_n(t, x, x^*, v)\\
&= E^{x,x^*}[\int_t^T e^{r(t-s)}£_1 U_{n-1}(s, S_s, S_s^*, v)ds]\\
&= E^{x,x^*}[\int_t^T e^{r(t-s)}£_1 U_{n-1}(s, xe^{H_{s-t}}, xe^{H_{s-t}^*}, v)ds]\\
&= \int_{ln(\frac{x^*}{x})}^{\infty}\int_{-\infty}^{\infty}(\int_t^T e^{r(t-s)}£_1 U_{n-1}(s, xe^b, xe^c, v)ds)Q(H_{s-t} \in db, H_{s-t}^* \in dc)\\
&= \int_t^T\int_{ln(\frac{x^*}{x})}^{\infty}\int_{-\infty}^{\infty} e^{r(t-s)}£_1 U_{n-1}(s, xe^b, xe^c, v)ds)Q(H_{s-t} \in db, H_{s-t}^* \in dc)ds\\
&= \int_t^T\int_{ln(\frac{x^*}{x})}^{\infty}\int_{-\infty}^{\infty} \frac{2(2c-b)£_1 U_{n-1}(s, xe^b, xe^c, v)}{\sigma^3(s-t)^{\frac{3}{2}}\sqrt{2\Pi}}\\
&\quad \cdot exp\{r(t-s) + \frac{r - \frac{1}{\sigma^2}}{\sigma^2}b - \frac{(r - \frac{\sigma^2}{2})^2}{2\sigma^2}(s-t) - \frac{(2c-b)^2}{2\sigma^2(s-t)}\}dbdcds
\end{aligned}$$

this proves the theorem. ■

5. Numerical simulation and result

First, we study the performance of the pricing formula for floating strike lookback put option price with payoff $(max_{t\leq T} S_t - S_T)$ in the heston stochastic volatility model. We present the numerical example of the homotopy approximation result on the floating strike lookback put option price under a stochastic volatility model of the Heston type which is specified by the implementation is done by evaluating the integral of the theorem 4.1,and Monte-Carlo simulation.The parameters $r = 0.02$, $\Theta = 0.04.p = 0$, $v_0 = 0.04$, $k = 0.03$, $x^* = 110$, $x = 100$,and t=0.5 are used for the implementation. We observe in the option values are very close between our pricing formula which is a blue stars and Monte-Carlo simulation which is a red line.

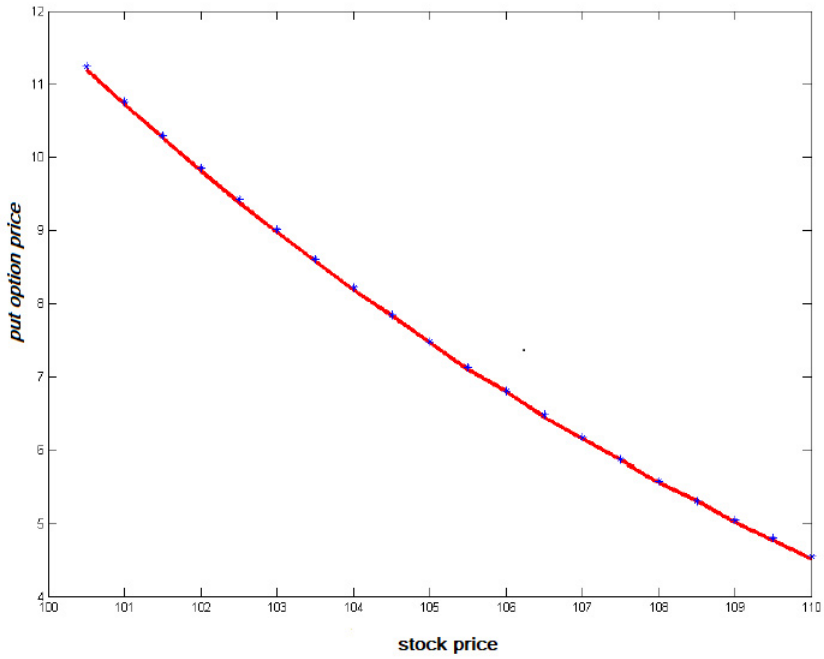

Figure 3: Price of the floating strike lookback put options as a function of stock price in the Heston model.

6. Conclusion

The homotopy analysis method used in this paper offers a simple analytic method for the pricing lookback put options under Heston stochastic volatility model. The price is given by an infinite series whose value can be determined once an initial term is given well. This paper uses the probabilistic argument together with the semi-analytic method called the homotopy analysis method to obtain an approximation formula for the floating strike lookback put option price under Heston stochastic volatility model. The resultant formulas for the option price are very close between our pricing formula and monte-carlo simulation.

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