

## A note on Harmonious Coloring of n-sunlet Graph

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### Abstract

A Harmonious coloring of a graph  $G$  is a proper vertex coloring of  $G$ , in which every pair of colors appears on at most one pair of adjacent vertices and the harmonious chromatic number of graph  $G$  is the minimum number of colors needed for the harmonious coloring of  $G$  and it is denoted by  $\chi_H(G)$ . The aim of this paper is to find the harmonious chromatic number of middle graph and central graph of  $n$ -sunlet graph.

**Keywords:** Middle graph, Central graph, Harmonious coloring and Harmonious chromatic number.

**AMS Mathematics Subject Classification:** 05C15, 05C75

### Introduction

Throughout the paper we consider the graph  $G$  as a finite undirected with no loops and multiple edges.

**Definition 1.1:** Let  $G$  be a graph with vertex set  $V(G)$  and edge set  $E(G)$ . The middle graph [6, 12] of  $G$  is denoted by  $M(G)$  and is defined as follows.

The vertex set of  $M(G)$  is  $V(G) \cup E(G)$ . Two vertices  $x, y$  in the vertex set of  $M(G)$  are adjacent in  $M(G)$ , if one of the following cases holds:

- (i)  $x, y$  are in  $E(G)$  and  $x, y$  are adjacent in  $G$ .
- (ii)  $x$  is in  $V(G)$ ,  $y$  is in  $E(G)$  and  $x, y$  are adjacent in  $G$ .

**Definition 1.2:** The central graph [11, 12] of a graph  $G$ , is obtained by subdividing each edge of  $G$  exactly once and joining all the non-adjacent vertices of  $G$ . It is denoted by  $C(G)$ . By the definition  $p_{C(G)} = p + q$ . For any  $(p, q)$  graph here exactly  $p$  vertices of degree  $p - 1$  and  $q$  vertices of degree 2 in  $C(G)$ .

**Definition 1.3:** The  $n$  – sunlet graph  $S_n$  [11 ] is the graph on  $2n$  vertices obtained by attaching  $n$  pendent edge to the cycle graph  $C_n$ .

Structural properties of  $C(S_n)$  and  $M(S_n)$

The  $n$ -Sunlet graph ( $S_n$ ) with  $2n$  vertices, has

No of edges  $q$ ,  $E(S_n) = 2n$

$n$  vertices of degree 3,  $n$  vertices of degree 1

Maximum degree,  $\Delta(S_n) = 3$

Minimum degree,  $\delta(S_n) = 1$

No. of vertices  $p$ ,  $V[C(S_n)] = 4n$

No. of edges  $q$ ,  $E[C(S_n)] = 2n^2+n$

No. of vertices  $p$ ,  $V[M(S_n)] = 4n$

No. of edges  $q$ ,  $E[M(S_n)] = n^2+4n$

$n$  vertices of degree  $2n$ ,  $n$  vertices of degree 1,  $n$  vertices of degree 3 &  $n$  vertices of degree 4

Maximum degree  $\Delta[M(S_n)] = 2n$

Minimum degree  $\delta[M(S_n)] = 1$

### 3 Harmonious Coloring of Firecracker Graph ( $F_{n,3}$ )

**Theorem 1** For  $n$ -sunlet graph, the harmonious chromatic number

$$X_H[M(S_n)] = \begin{cases} 2n, & n > 4 \\ 2n + 3 \text{ or } 9, & n = 3 \\ 2n + 2 \text{ or } 10, & n = 4 \end{cases}$$

**Proof.** Let  $S_n$  be the  $n$  – Sunlet graph having  $2n$  vertices which is obtained by attaching  $n$  pendent edge to the cycle graph  $C_n$ .

i.e  $V(C_n) = (u_1, u_2, \dots, u_n)$

Then, let the pendent edge connected to  $u_i$  be  $u_i v_i$  for  $i = 1, 2, 3, \dots, n$

i.e.  $v_1, v_2, \dots, v_n$  are the pendent vertices of  $S_n$ .

By definition of central graph each edge of graph is subdividing exactly once & joining all non adjacent vertices of graph. So, let the each edge  $u_i v_j$ ,  $u_i u_{i+1}$  and  $u_{n,1}$  of sunlet graph  $S_n$  are subdivided by the vertices  $e_j$ ,  $u_{i,i+1}$  and  $u_{n,1}$  respectively for  $(1 \leq i \leq n-1; 1 \leq j \leq n)$ .

Now assign the coloring to the vertices as follows:

Consider the color class  $C = \{c_1, c_2, c_3, \dots, c_{2n}\}$

Assign the color  $c_i$  to  $v_i$  for  $1 \leq i \leq n$ , assign color  $c_i$  to  $u_i$  for  $(2 \leq i \leq n-1)$  &  $c_n$  to  $u_1$ . Now assign color  $c_{n+i}$  to  $u_{i,i+1}$  for  $(1 \leq i \leq n-1)$  &  $c_{2n}$  to  $u_{n,1}$ . At last color  $c_i$  to  $e_j$  for  $(1 \leq i \leq n; 1 \leq j \leq n)$ .

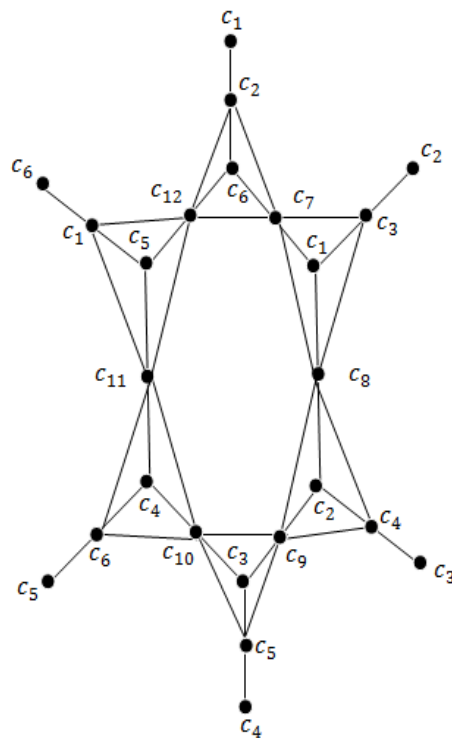


Figure 1:  $M(S_6)$  WITH COLORING

$$X_H [M (S_n)] = 2n$$

Therefore, the total number of colors used in this coloring is  $2n$ . Thus above said coloring is obviously harmonious with minimum number of colors because if we replace any color, which is minimum in number by a color, which is already used then the resulting coloring will be improper or not harmonious. Hence  $X_H [M (S_n)] = 2n$ .

Observation 1 For  $n$ -sunlet graph,  $n > 2$

$$X_H[C(S_n)] = \begin{cases} \frac{5n+3}{2}, & \text{for odd } n \\ \frac{5n+2}{2}, & \text{for even } n \end{cases}$$

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