

Performance of Reed-Solomon Codes in AWGN Channel

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Abstract

The fundamental reason for the use of error-correction control codes is to minimize the error which occurs due to the addition of noise over the noisy channel. These codes are capable of detecting the errors and also rectifying them to achieve minimum error possible. This paper presents the performance of a Reed-Solomon code in an AWGN channel for various modulation schemes in a simulation environment.

Index Terms: Quadrature Amplitude Modulation, Phase Shift Keying, Frequency Shift Keying, Symbol Error Rate.

Introduction

The process of digital communication involves transmission and reception of a message with zero error. If there is an error during the transmission, the need of retransmission of message over noisy channel having a fixed level of noise is required for effective communication. [1] Now to overcome this problem of retransmission of data for every error forward error correction is used which is known as reed solomon codes. This code eliminates the problem of retransmission when error occurs because this code is enable to correct retrieval of message despite errors in the received sequences. This code have good properties for many small fields used in practice, and have efficient decoding algorithms, making them indispensable to engineers for the past several decades. It has a great power and utility and now found in many applications from compact disc players to deep-space applications. Reed-Solomon codes are perhaps the most commonly used in all forms of transmission and data storage for forward Error correction (FEC). Many algorithms are created for this code.

Now, we are trying to use MATLAB simulation program for this code. We are trying to check error correcting probability of code and complexity of this code. Our main goal is to find the performance of Reed-Solomon codes and to develop a new program which is able to decode this code and correct the errors. Finally, we will look at the performance of the decoding process which will evaluate the Symbol error rate (SER).

To evaluate the SER of this code we used various modulation scheme like QAM, FSK, PSK in AWGN channel. In addition, the performance evaluation of the error-correction codes in a special channel is complex because of the complicated conditions of the error patterns. SER in a system is a very important index. In error control system, the SER is caused by uncorrectable errors and for incomplete decoder, where the number of errors is beyond the error correcting capability. The decoder sometimes declares that it has an uncorrectable message and sometimes decodes the word to other code word. We used MATLAB as simulation program for checking the SER of reed soloman code with three moduation scheme in AWGN channel.

In this paper, section 2 covers Reed Solomon codes and some fundamentals of some modulation scheme such as QAM,FSK,PSK. Section 3 covers simulation process using MATLAB as platform for simulation. section 4 covers result of simulation .In section 5 conclusion are made .and section 6 covers future work for this project.

Reed Solomon Code and Modulation Scheme

Reed Solomon Code

Reed-Solomon codes are nonbinary cyclic codes with symbols made up of m-bit sequences, where m is any positive integer having a value greater than 2. R-S (n, k) codes on m-bit symbols exist for all n and k for which

$$0 < k < n < 2^m + 2 \quad (1)$$

where k is the number of data symbols being encoded, and n is the total number of code symbols in the encoded block. For the most conventional R-S (n, k) code,

$$(n,k) = (2^m - 1, 2^m - 1 - 2t) \quad (2)$$

where t is the symbol-error correcting capability of the code, and n - k = 2t is the number of parity symbols. An extended R-S code can be made up with n = 2^m. Reed-Solomon codes achieve the largest possible code minimum distance for any linear code with the same encoder input and output block lengths. For nonbinary codes, the distance between two code words is defined (analogous to Hamming distance) as the number of symbols in which the sequences differ.

For Reed-Solomon codes, the code minimum distance is given by [2]

$$d_{min} = n - k + 1 \quad (3)$$

Reed-Solomon codes are also a linear code and cyclic. It is particularly good at dealing with bursts of errors because, although a symbol may have all its bits in error, this counts as only one symbol error in terms of the correction capability of the code.

Choosing different parameters for a code provides different levels of protection and affects the complexity of implementation. Thus, a R-S (n, k)code implies that the encoder takes in k symbols and add $n-k$ symbols to make it n symbols codeword. Reed Solomon codes are based on galois fields.

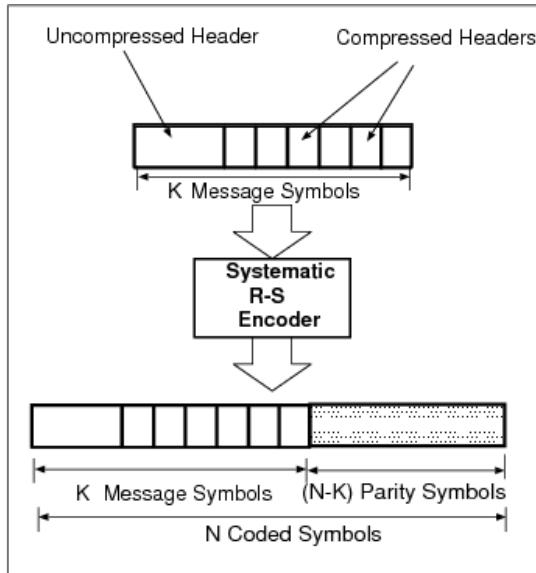


Figure 1: RS Code representation.

Reed Solomon encoding

The generating polynomial for an R-S code takes the following form:

$$g(X) = g_0 + g_1 X^1 + g_2 X^2 + \dots + g_{2r-1} X^{2r-1} + X^{2r} \quad (3)$$

The degree of the generator polynomial is equal to the number of parity symbols. R-S codes are a subset of the Bose, Chaudhuri, and Hocquenghem (BCH) codes; hence, it should be no surprise that this relationship between the degree of the generator polynomial and the number of parity symbols holds, just as for BCH codes. Since the generator polynomial is of degree $2t$ successive powers of that are roots of the polynomial. We designate the roots of $g(X)$ as $\alpha, \alpha^2, \dots, \alpha^n$

It is not necessary to start with the root ; starting with any power of is possible. Consider as an example the (7, 3) double-symbol-error correcting R-S code. We describe the generator polynomial in terms of its $2t = n - k = 4$ roots, as follows:

$$\begin{aligned} g(X) &= (X - \alpha)(X - \alpha^2)(X - \alpha^3)(X - \alpha^4) \\ &= X - \alpha^3 X^3 + \alpha^0 X^2 - \alpha^1 X + \alpha^3 \end{aligned} \quad (4)$$

Following the low order to high order format, and changing negative signs to positive, since in the binary field $+1 = -1$, $g(X)$ can be expressed as follows:

$$g(X) = \alpha^3 + \alpha^1 X + \alpha^0 X^2 + \alpha^3 X^3 + X^4 \quad (5)$$

Consider the finite field with q elements $\text{GF}(2^m)$ the message to be transmitted f , consists of k elements of GF

$$f = (f_0, f_1, f_2, \dots, f_{k-1}), \quad f_i \in \text{GF}(2^m) \quad (6)$$

The message symbol can be considered to be the coefficients of degree $k - 1$ message polynomial which to be encoded,

$$f(x) = f_0 + f_1x + f_2x^2 + \dots + f_{k-1}x^{k-1} \quad (7)$$

While the coefficients of the remainder is $2t$ parity-check digits, where

$$b(x) = b_0 + b_1x + \dots + b_{2t-1}x^{2t-1} \quad (8)$$

Hence, the output codeword to be transmitted is

$$\begin{aligned} v(x) &= f(x) + b(x) \\ &= v_0 + v_1x + v_2x^2 + \dots + v_{n-1}x^{n-1} \end{aligned} \quad (9)$$

Reed Solomon Decoding

During transmission, any codeword received at the receiver is assumed to be corrupted by the noise in the channel that introduces errors in the communication system. Let say if the received corrupted-codeword is assume to be $r(x)$ hence we have

$$r(x) = c(x) + e(x) \quad (10)$$

where $c(x)$ is the original codeword and $e(x)$ is the error-pattern polynomial and can be described as:

$$e(x) = e_{n-1}x^{n-1} + \dots + e_1x + e_0 \quad (11)$$

Each of the coefficients e_{n-1}, \dots, e_0 e is an m -bit error value represented by an element $\text{GF}(2^m)$, with the errors position in the codeword being determined by the degree of x for that term. Clearly, if more than $t = (n-k)/2$ of the e values are non-zero, then the correction capability of the code is exceeded and the errors are not correctable. However, Reed-Solomon algorithm still allow one to detect if there are more than t errors and for this cases, the codeword is declared uncorrectable

QAM

Quadrature amplitude modulation, or *QAM*, is a big name for a relatively simply technique. It is simply a combination of amplitude modulation and phase shift keying.

PSK

Phase-shift keying is a digital modulation scheme that conveys data by changing, or modulating, the phase of a reference signal i.e. the carrier wave.

$$d = \sqrt{4E_s \sin^2(\pi/M)} \quad (12)$$

The above equation shows a distance between the symbols and hence, implies to the error probability of M -PSK modulation technique.

FSK

Frequency-Shift Keying is a modulation technique that holds data in the frequency of the transmitting signal i.e. the carrier signal.

$$d = \sqrt{4NE_b} \quad (13)$$

The above equation denotes the distance between the symbols in an M -FSK modulation

Matlab Simulation

Simulation of this project contain implementation of coding and decoding of reed Solomon code with QAM, PSK,FSK modulation scheme in AWGN channel. The general block diagram of the entire process can be summed up into a single block diagram as shown below

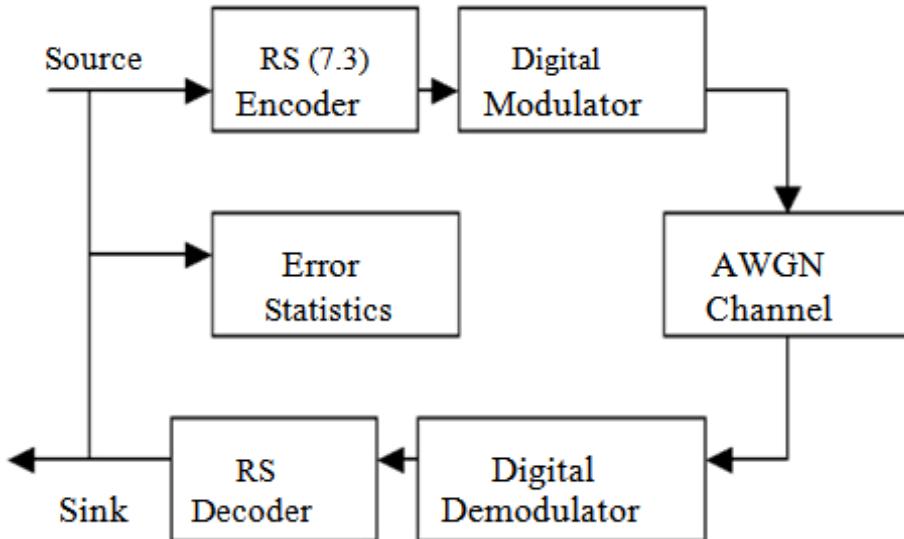


Figure 2: Block diagram of the system.

If we consider k -information symbols as the input message which set to be random, the channel encoder will map each of the information sequence into a unique n -symbols sequence, known as *code word*. The amount of redundancy bit in this manner is depends on the requirement of the system. It is also measured by the ratio, n/k . Meanwhile, the code rate for a system is k/n . In this case the ratio remains the same as $k = 3$ and $n = 7$. This codeword will then be passed to the digital modulator

and for this project; the modulation scheme used is *8-QAM, 8-FSK and 8-PSK*. The primary purpose of digital modulation is to map each of the information sequence into signal waveforms. In addition, AWGN channel will be the physical medium used for data transmission. For every transmission through a channel, there exist corruptions in a random manner on the transmitted signal by a variety of possible cause such as man-made noise, atmospheric noise and etc. At the receiver, the digital demodulator processes the channel-corrupted signal waveforms and reduces it into a sequence of data symbols. As a final step, the channel decoder will decode and attempts to reconstruct the received output sequence to get back the original information data. However, the capability of correcting errors are depends on the system.

Thus when we consider the output symbols, there we find some of the bits in error due to the stray noise that corrupt the signal on comparing with the input signal to the system. If a communication system uses several bits to represent a single symbol, then counting bit errors is different from counting symbol errors. In either the bit or symbol counting case, the error rate is the number of errors divided by the total number of symbols transmitted. The *symerr()* function compares two symbol messages and computes the number of symbol errors and the symbol error rate. The error rate decreases after decoding because the decoder corrects some of the errors.

Results

In this section we will consider the performance of R-S (7,3) codes for various different modulation schemes such as 8-QAM, 8-FSK and 8-PSK when data transmission takes place through an *Additive Noise Gaussian Channel* i.e. AWGN channel. The R-S (7,3) code is defined over GF(8), is selected for the system. Each symbol of the codes can be mapped into one point of the signal set of 8-QAM and one channel symbol represent one code symbol. Similar steps for 8-PSK and 8-FSK are also carried out.

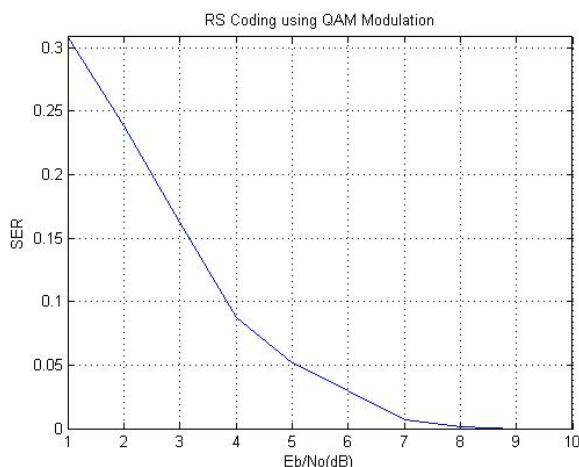
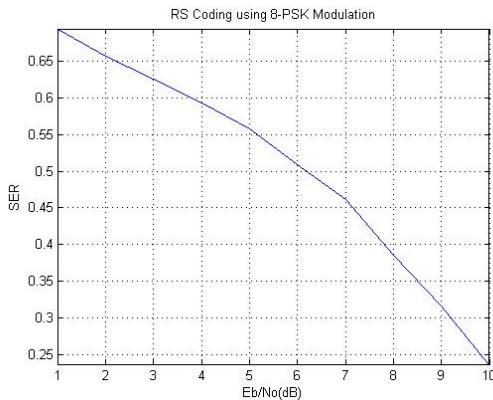
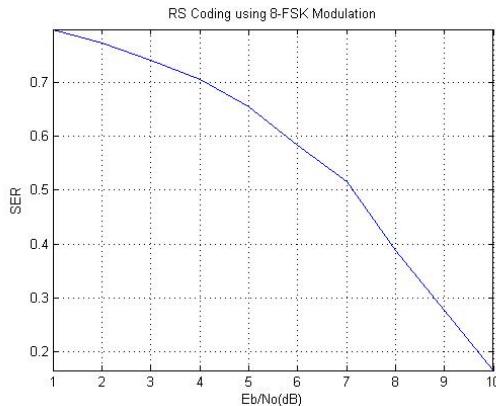


Figure 3: Performance of R-S (7, 3) for QAM.

**Figure 4:** Performance of R-S (7, 3) code for PSK.**Figure 5:** Performance of R-S(7,3) code for FSK.

Figures 3, 4 and 5 display the obtained symbol Error Rate of the R-S (7, 3) for various modulation schemes over AWGN channel.

The three schemes can be easily distinguished for their performance in terms of symbol error rate i.e. SER. Clearly as shown by the diagrams as the signal power increases the error rate decreases, as the signal rises above the noise table level present in the channel. To obtain a better view consider the following table which compare the discrete values of each of the modulation scheme.

Table 4.1: Performance Of R-S (7, 3) In Qam Results

E_b/N_o	0	1	2	3	4
SER	0.308	0.238	0.163	0.087	0.058
E_b/N_o	5	6	7	8	9
SER	0.030	0.007	0.001	0	0

Table 4.2: Performance Of R-S (7, 3) In Psk Results.

E_b/N_o	0	1	2	3	4
SER	0.694	0.656	0.625	0.593	0.557
E_b/N_o	5	6	7	8	9
SER	0.508	0.461	0.385	0.316	0.236

Table 4.3: Performance Of R-S (7, 3) In Fsk Results.

E_b/N_o	0	1	2	3	4
SER	0.798	0.772	0.740	0.704	0.655
E_b/N_o	5	6	7	8	9
SER	0.655	0.583	0.515	0.387	0.227

The above tables depict the performance of R-S code over various modulation schemes. Clearly as the values of E_b/N_o increases the error declines. The notable fact in this is that when we see the decline it is sharper in QAM as compared to any other modulation scheme studied in this report. Further as we compare PSK and FSK, we find that PSK depicts sharper decline compared to FSK over an AWGN channel. The use of R-S codes reduces the error rate as some of the errors are rectified by the code itself. The modulation schemes each have its own error probability over the channel.

Conclusion

The proposed program based on simulation to calculate the SER performance has been proved better for the QAM scheme of modulation over FSK and PSK modulation schemes over the AWGN channel. Thus, the objective of this paper is to see the performance of the code and from the result and analysis it is shown that we had achieved the goal.

Future Work

Study other type of noise that added into the system such as Rayleigh Fading, Rician Fading and Nakagami Fading. This can be done by using different type of channel and see which channel is the best.

Construct a new algorithm for RS codes instead of using existing function in MATLAB program.