

Shaped Beams from Thick Arrays

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Abstract

Several studies on resonant arrays are reported in the literature. Moreover, they are considered for the generation of narrow beams only. No work is reported on the patterns of non-resonant arrays for shaped beams. But these shaped beams are essentially used in modern radars. In view of these facts, intensive investigations are carried out in the present work to produce sector beams from thick arrays. The non-resonant spacing is proposed using a well developed formula. This formula is useful for the determination of space distribution for even and odd elements of the array.

Introducing space distribution so determined, the sector beams of specified width are realized. The radiation patterns are compared with those of specified ones. The patterns are presented in $u(\sin\theta)$ – domain.

Introduction

Array thinning help to reduce power consumption, cost, hardware complexity and weight of the arrays. Several techniques are reported in literature for thinning [1 – 4]. Most of the techniques reported in the literature use unequal spacing. The problem of thinning creates grating lobes as the average spacing becomes large.

It is the practice of using resonant radiating element spacing. The spacing of $\lambda/2$ is considered to be resonant spacing.

Ishimaru et. al reported a method of designing a thinned array using unequal spacing. The patterns are expressed in the series of the angular functions. The improvement in side lobe levels is demonstrated. In radar and radio astronomy applications, it is necessary to obtain very high resolution, which requires large physical size of the antenna. Large physical size means an array with large number of elements.

In the present work, the formulation is reused to extend it for non resonant spacing arrays. The studies include the arrays in which the element spacing is less than $\lambda/2$.

The Space Distribution for Thick Arrays

The space distribution for the case of thick arrays is determined using the concept of source positions described by Ishimaru [5]. For the sake of completeness, it is presented below.

For an array of N elements, the radiation pattern is given by

$$E(\theta) = \sum_{n=1}^N a_n e^{jks_n \sin \theta} \quad (1)$$

Where a_n is the current in the n^{th} element and S_n denotes the position of the element as measured from a reference point 0. For carrying out the transformation of the radiation pattern, $E(\theta)$ is given by

$$E(\theta) = \sum_{n=1}^N f(n) \quad (2)$$

Applying Poisson's sum formula, we have

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} f(v) e^{j2m\pi v} dv \quad (3)$$

i.e.
$$E(\theta) = \sum_{m=-\infty}^{\infty} \int_0^N f(v) e^{j2m\pi v} dv \quad (4)$$

The limit of the integration is from 0 to N because the radiation $E(\theta)$ is the finite sum and $f(V)$ vanishes for $V < 0$ and $V > N$. In fact, any range which covers all the integers from 1 to N may be used. Thus, (4) may also be written as

$$E(\theta) = \sum_{m=-\infty}^{\infty} \int_{\varepsilon}^{\varepsilon+N} f(v) e^{j2m\pi v} dv \quad (5)$$

Where $0 < \varepsilon < 1$

The next step in the formulation is the introduction of a new function, which is called as "Source position function" The "Source position function" is defined by

$$s=s(V) \quad (6)$$

This gives the position of the n^{th} when $V = n$. Thus

$$s_n=s(n) \quad (7)$$

considering V in (6) as a function of s

$$V = V(s) \quad (8)$$

and

$$n = V(s_n) \tag{9}$$

$V(s)$ is called as “source number function” as this yields the numbering of each element when s is at the correct position of the element.

Changing the variable V to s , we obtain

$$E(\theta) = \sum_{m=-\infty}^{\infty} \int_{s_0}^{s_N} f(s) \frac{dv}{ds} e^{j2m\pi v(s)} ds \tag{10}$$

For linear array problem, considering the expression in (1), we write

$$E(\theta) = \sum_{m=-\infty}^{\infty} E_m(\theta) \tag{11}$$

$$E_m(\theta) = \int_{s_0}^{s_N} A(s) \frac{dv}{ds} e^{-j(\psi(s)-2m\pi v(s))} e^{j2ks \sin \theta} ds \tag{12}$$

Where

$$a_n = a(s_n) = A_n e^{-j\psi_n} \tag{13}$$

A_n is the amplitude of the current, ψ_n is the phase of the current in n^{th} element, and $A(s)$ is a function which yields A_n at $s=s_n$ and therefore this may be considered as an envelope of amplitude of each current. $\psi(s)$ is function which gives ψ_n at $s=s_n$. Thus

$$A_n = A(s_n) \tag{14}$$

$$\psi_n = \psi(s_n) \tag{15}$$

The expression (12) represents the radiation pattern of continuous line source, its amplitude is given by

$$A(s) \frac{dv}{ds}, \tag{16}$$

Phase distribution is given by

$$\psi(s) - 2m\pi V(s) \tag{17}$$

Equation (11) represents an infinite series. But it converges rapidly.

Rewriting (12) using normalized variables, we have

$$E(u) = \sum_{m=-\infty}^{\infty} (-1)^{m(N-1)} E_m(u) \tag{18}$$

$$E_m(u) = \frac{1}{2} \int_{-1}^1 A(x) \frac{dy}{dx} e^{-j\psi(x)+jm\pi N(y-x)} e^{j(K_a u + m\pi N)x} dx \tag{19}$$

Where

$$u = \sin \theta$$

$2a = S_N - S_0$
 $x = x(y)$ normalized source position function
 $-1 < x < +1$,
 $y = y(x)$ normalized source number function
 $-1 < y < +1$.

Thus, the actual position of the n^{th} element is

$$S_n = ax(y_n) \quad (20)$$

If N is odd, $N = 2M + 1$

$$y_n = \frac{n}{M + (1/2)}, \quad n = 0, \pm 1, \pm 2, \dots, \pm M. \quad (21)$$

If N is even, $N = 2M$

$$y_n = \frac{n - (1/2)}{M} \quad \text{for } n > 0,$$

$$y_n = \frac{n + (1/2)}{M} \quad \text{for } n < 0,$$

$$n = \pm 1, \pm 2, \dots, \pm M. \quad (22)$$

The total length of the array is not $2a$, but

$$L_0 = a[x(y_M) - x(y_{-M})], \quad (23)$$

which is smaller than $2a$.

The expressions (21) and (22) provide element space distribution for odd and even element array respectively. With simple mathematical manipulations it has been possible to obtain a single expression for phase distribution valid for both odd and even element arrays. That is given by

$$x_n = \frac{2n - 1 - N}{N} \quad (24)$$

Here

x_n represents element position along the array.

Results

Using equation (24), the element space distributions are compared and they are presented in tables (1-5). The computations are carried out to obtain the variation of far-field as a function of $\sin\theta$. The patterns are presented in figures (1-10). The pattern characteristics of the isotropic radiators, dipoles and waveguide in terms of beam-width and side lobe levels for thick arrays are presented in tables (6-7).

Table 1 Thick arrays (dipole) $N=21$, $2L/\lambda = 8$

n	x_n
1	-0.9523
2	-0.8571
3	-0.7619
4	-0.6667
5	-0.5714
6	-0.4761
7	-0.3809
8	-0.2857
9	-0.1904
10	-0.0952
11	0.0000
12	0.0952
13	0.1904
14	0.2857
15	0.3809
16	0.4761
17	0.5714
18	0.6667
19	0.7619
20	0.8571
21	0.9523

Table 2 Thick arrays(dipole) N=41, $2L/\lambda = 16$

n	x_n
1	-0.9756
2	-0.9268
3	-0.8780
4	-0.8292
5	-0.7804
6	-0.7317
7	-0.6829
8	-0.6341
9	-0.5853
10	-0.5365
11	-0.4878
12	-0.4390
13	-0.3902
14	-0.3414
15	-0.2926
16	-0.2439

17	-0.1951
18	-0.1463
19	-0.0975
20	-0.0487
21	0.0000
22	0.0487
23	0.0975
24	0.1463
25	0.1951
26	0.2439
27	0.2926
28	0.3414
29	0.3902
30	0.4390
31	0.4878
32	0.5365
33	0.5853
34	0.6341
35	0.6829
36	0.7317
37	0.7804
38	0.8292
39	0.8780
40	0.9268
41	0.9756

Table 3 Thick arrays (dipole) $N=61$, $2L/\lambda = 25$

n	x_n	n	x_n
1	-0.9836	32	0.0327
2	-0.9508	33	0.6667
3	-0.9180	34	0.0983
4	-0.8852	35	0.1311
5	-0.8524	36	0.1639
6	-0.8196	37	0.1967
7	-0.7868	38	0.2295
8	-0.7540	39	0.2622
9	-0.7213	40	0.2950
10	-0.6885	41	0.3278
11	-0.6557	42	0.3606
12	-0.6229	43	0.3934
13	-0.5901	44	0.4262
14	-0.5573	45	0.4590

15	-0.5245	46	0.4918
16	-0.4918	47	0.5245
17	-0.4590	48	0.5573
18	-0.4262	49	0.5901
19	-0.3934	50	0.6229
20	-0.3606	51	0.6557
21	-0.3278	52	0.6885
22	-0.2950	53	0.7213
23	-0.2622	54	0.7540
24	-0.2295	55	0.7868
25	-0.1967	56	0.8196
26	-0.1639	57	0.8524
27	-0.1311	58	0.8852
28	-0.0983	59	0.9180
29	-0.6667	60	0.9508
30	-0.0327	61	0.9836
31	0.0000		

Table 4 Thick arrays (dipole) $N=81$, $2L/\lambda = 35$

n	x_n	n	x_n
1	-0.9876	42	0.0246
2	-0.9629	43	0.0493
3	-0.9382	44	0.0740
4	-0.9153	45	0.0987
5	-0.8888	46	0.1234
6	-0.8641	47	0.1481
7	-0.8395	48	0.1728
8	-0.8148	49	0.1975
9	-0.7901	50	0.2222
10	-0.7654	51	0.2469
11	-0.7407	52	0.2716
12	-0.7160	53	0.2962
13	-0.6913	54	0.3209
14	-0.6667	55	0.3456
15	-0.6419	56	0.3703
16	-0.6172	57	0.3950
17	-0.5925	58	0.4197
18	-0.5679	59	0.4444
19	-0.5432	60	0.4691
20	-0.5185	61	0.4938
21	-0.4938	62	0.5185
22	-0.4691	63	0.5432

23	-0.4444	64	0.5679
24	-0.4197	65	0.5925
25	-0.3950	66	0.6172
26	-0.3703	67	0.6419
27	-0.3456	68	0.6667
28	-0.3209	69	0.6913
29	-0.2962	70	0.7160
30	-0.2716	71	0.7407
31	-0.2469	72	0.7654
32	-0.2222	73	0.7901
33	-0.1975	74	0.8148
34	-0.1728	75	0.8395
35	-0.1481	76	0.8641
36	-0.1234	77	0.8888
37	-0.0987	78	0.9153
38	-0.0740	79	0.9382
39	-0.0493	80	0.9629
40	-0.0246	81	0.9876
41	-0.0000		

Table 5 Thick arrays (dipole) $N=101$, $2L/\lambda =45$

n	x_n	n	x_n	n	x_n
1	-0.9900	35	-0.3168	69	0.3564
2	-0.9702	36	-0.2970	70	0.3762
3	-0.9504	37	-0.2772	71	0.3960
4	-0.9306	38	-0.2574	72	0.4158
5	-0.9108	39	-0.2376	73	0.4356
6	-0.8910	40	-0.2178	74	0.4554
7	-0.8712	41	-0.1980	75	0.4752
8	-0.8514	42	-0.1782	76	0.4950
9	-0.8316	43	-0.1584	77	0.5148
10	-0.8118	44	-0.1386	78	0.5346
11	-0.7920	45	-0.1188	79	0.5544
12	-0.7722	46	-0.0990	80	0.5742
13	-0.7524	47	-0.0792	81	0.5940
14	-0.7326	48	-0.0594	82	0.6138
15	-0.7128	49	-0.0396	83	0.6336
16	-0.6930	50	-0.0198	84	0.6534
17	-0.6732	51	0.0000	85	0.6732
18	-0.6534	52	0.0198	86	0.6930
19	-0.6336	53	0.0396	87	0.7128
20	-0.6138	54	0.0594	88	0.7326

21	-0.5940	55	0.0792	89	0.7524
22	-0.5742	56	0.0990	90	0.7722
23	-0.5544	57	0.1188	91	0.7920
24	-0.5346	58	0.1386	92	0.8118
25	-0.5148	59	0.1584	93	0.8316
26	-0.4950	60	0.1782	94	0.8514
27	-0.4752	61	0.1980	95	0.8712
28	-0.4554	62	0.2178	96	0.8910
29	-0.4356	63	0.2376	97	0.9108
30	-0.4158	64	0.2574	98	0.9306
31	-0.3960	65	0.2772	99	0.9504
32	-0.3762	66	0.2970	100	0.9702
33	-0.3564	67	0.3168	101	0.9900
34	-0.3366	68	0.3366		

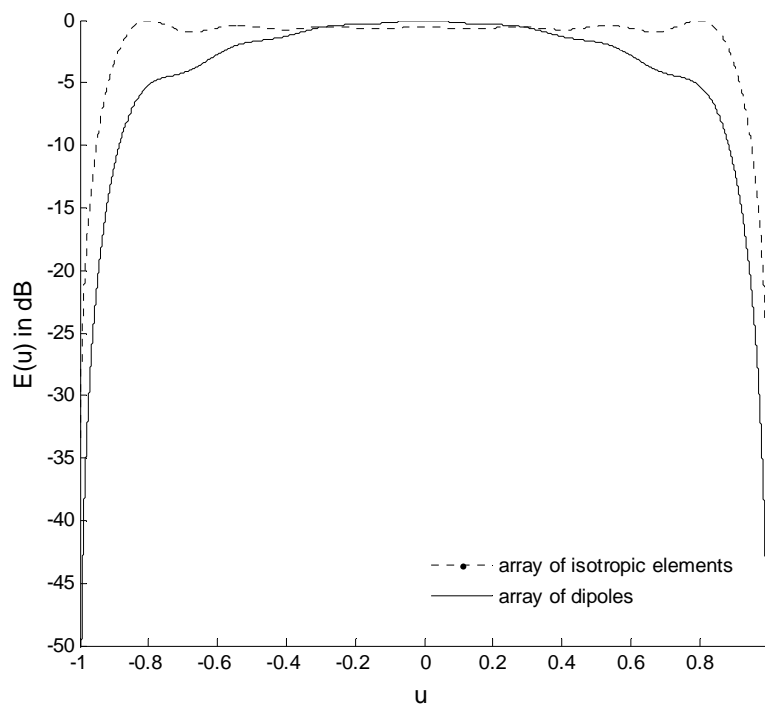


Figure 1 : Radiation pattern for $N = 21$ and $2L/\lambda = 8$

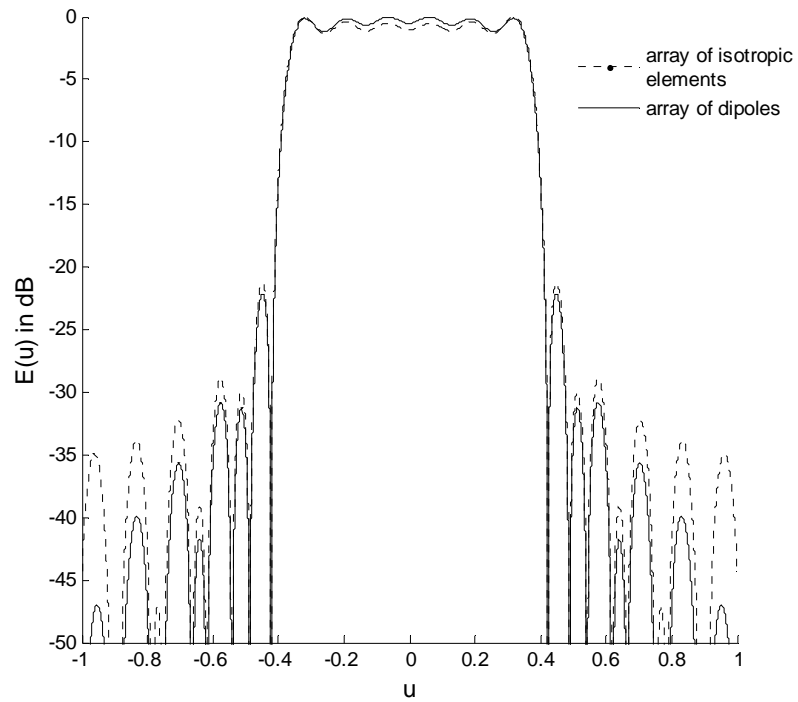


Figure 2: Radiation pattern for $N = 41$ and $2L/\lambda = 16$

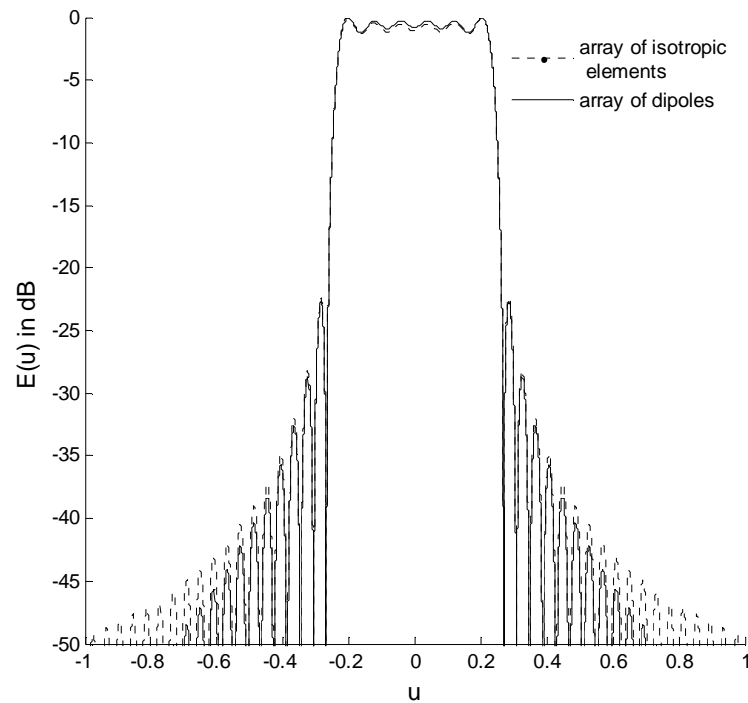


Figure 3: Radiation pattern for $N = 61$ and $2L/\lambda = 25$

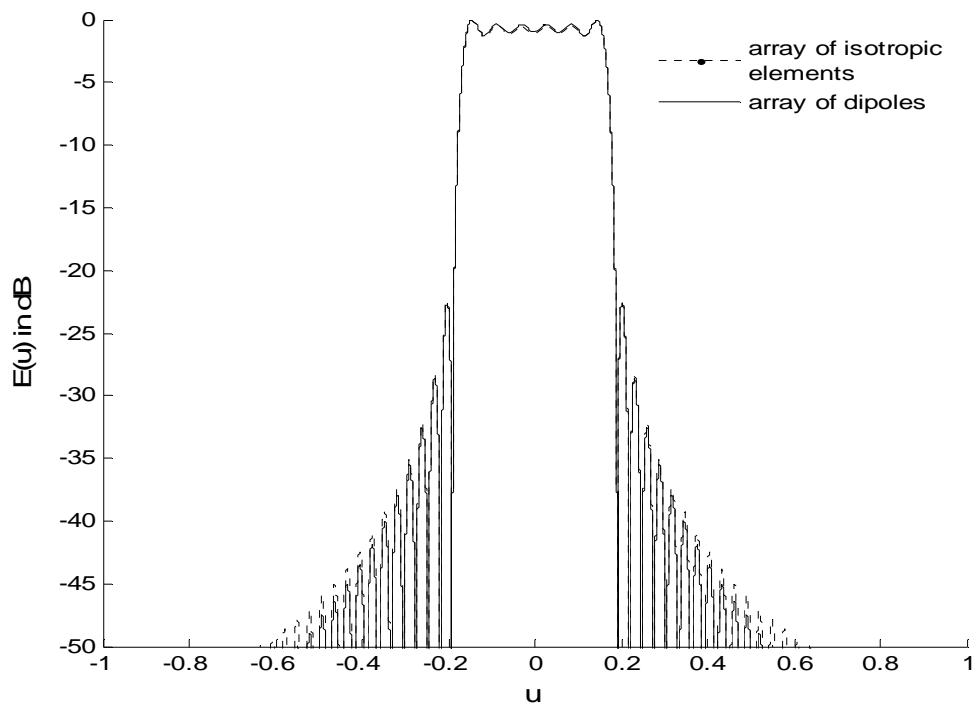


Figure 4: Radiation pattern for $N = 81$ and $2L/\lambda = 35$

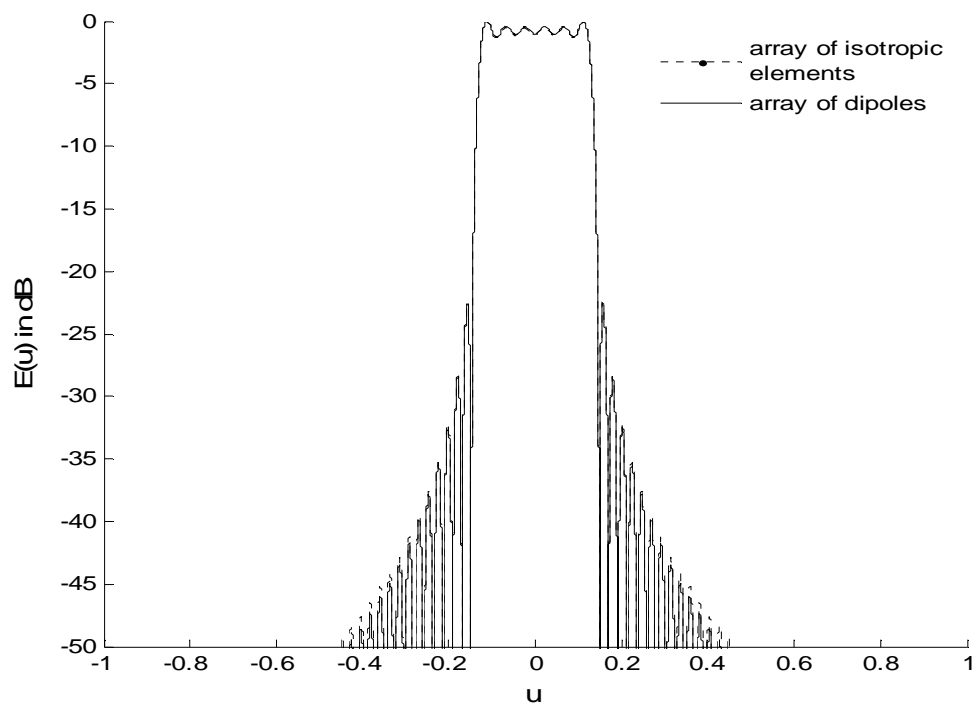


Figure 5: Radiation pattern for $N = 101$ and $2L/\lambda = 45$

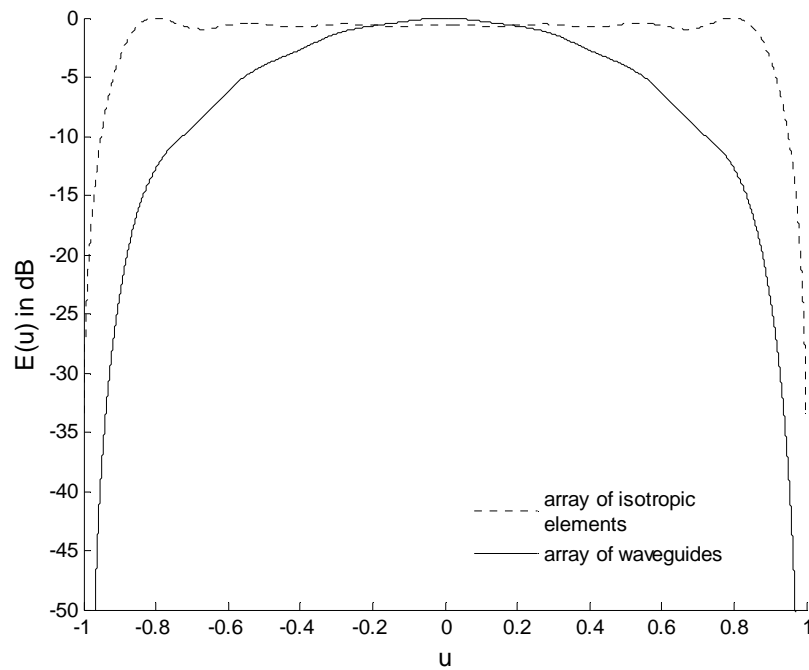


Figure 6: Radiation pattern for $N = 21$ and $2L/\lambda = 8$

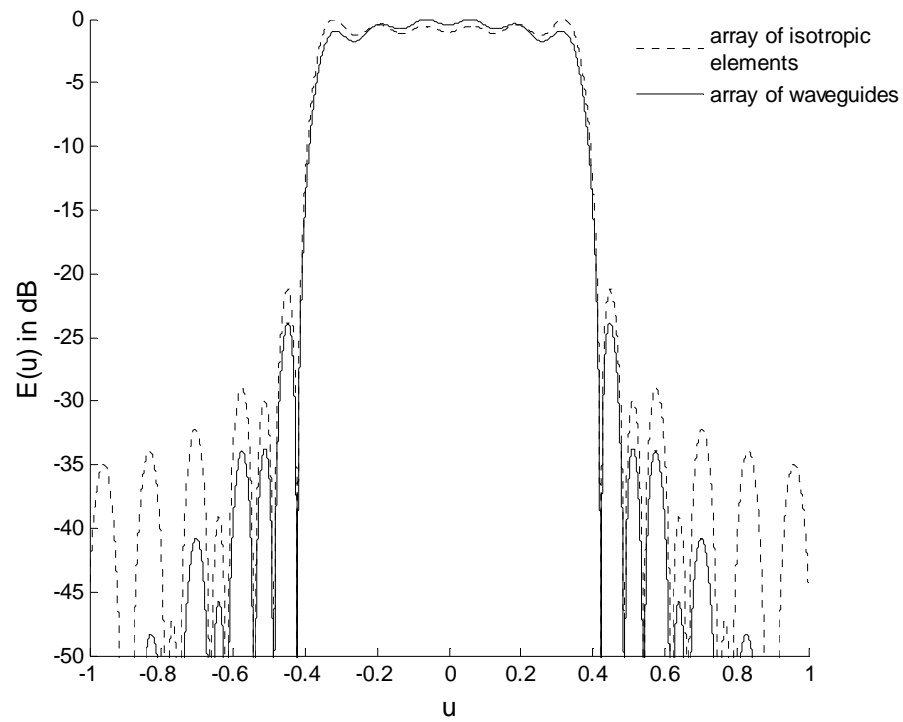


Figure 7: Radiation pattern for $N = 41$ and $2L/\lambda = 16$

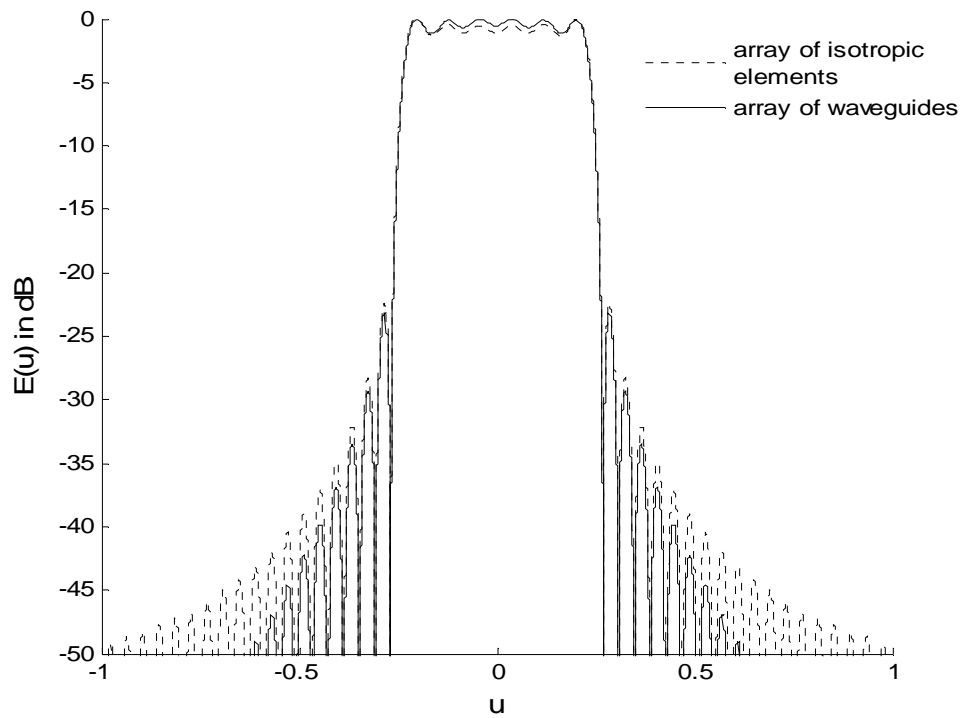


Figure 8: Radiation pattern for $N = 61$ and $2L/\lambda = 25$

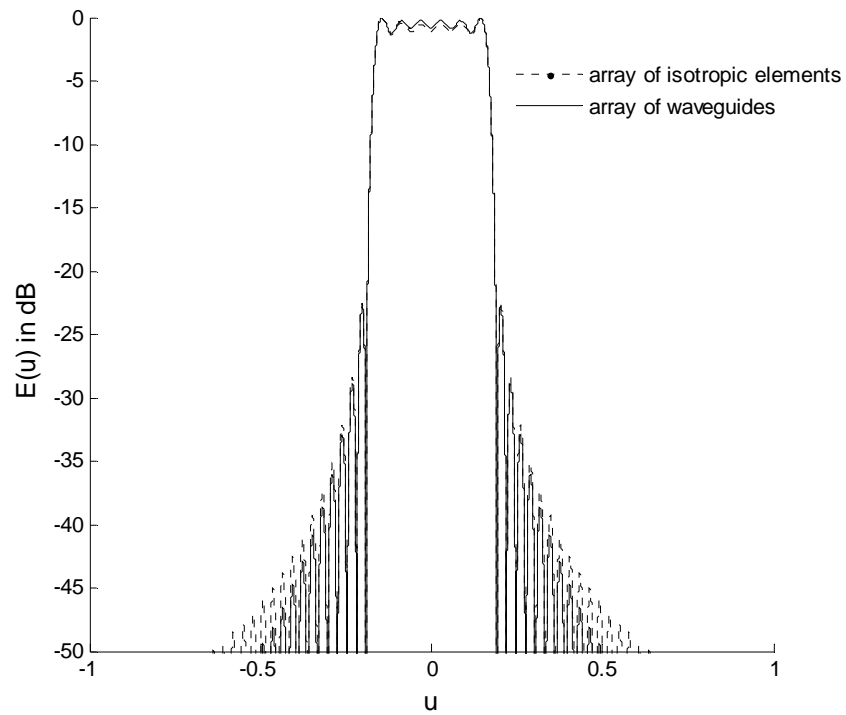


Figure 9: Radiation pattern for $N = 81$ and $2L/\lambda = 35$

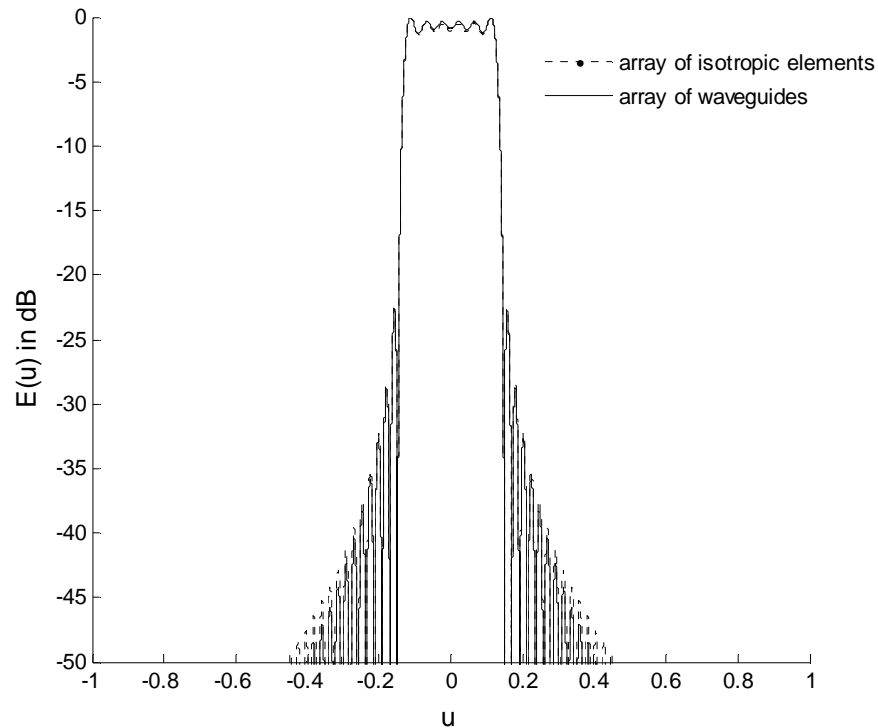


Figure 10 : Radiation pattern for $N = 101$ and $2L/\lambda = 45$

Table 6: Array of Isotropic Radiators, Dipoles and Waveguide

S. No	N	$2L/\lambda$	Beam Width (rad)			Side lobe Level(dB)		
			Isotropic Radiators	Dipoles	Waveguide	Isotropic Radiators	Dipoles	Waveguide
1	11	4	2.00	2.00	1.9548	--	--	--
2	21	8	2.00	2.00	1.9382	--	--	--
3	31	12	1.14	1.14	1.1370	-22.01	-24.15	-27.59
4	41	16	0.84	0.84	0.8440	-21.26	-22.09	-24.16
5	51	20	0.66	0.66	0.6708	-18.62	-19.43	-20.54
6	61	25	0.52	0.52	0.5376	-22.71	-22.71	-23.13
7	71	30	0.44	0.44	0.4476	-22.11	-22.29	-22.52
8	81	35	0.38	0.38	0.3834	-22.63	-22.50	-22.81
9	91	40	0.32	0.32	0.3350	-22.52	-22.62	-22.75
10	101	45	0.28	0.28	0.2968	-22.52	-22.60	-22.71

Table 7: Array of Isotropic Radiators, Dipoles and Waveguide

S. No	N	2L/λ	Beam Width (rad)			Side lobe Level(dB)		
			Isotropic Radiators	Dipoles	Waveguide	Isotropic Radiators	Dipoles	Waveguide
1	8	5	0.7442	0.7442	0.7378	-24.12	-25.56	-27.43
2	16	10	1.4064	1.4064	1.4018	-18.81	-22.84	-28.17
3	24	15	0.9208	0.9208	0.4591	-21.5	-22.79	-24.88
4	32	20	0.6818	0.6818	0.3409	-22.03	-22.49	-23.51
5	40	25	0.5404	0.5404	0.2713	-22.19	-22.46	-22.89
6	48	30	0.4494	0.4494	0.2257	-22.32	-22.5	-22.78
7	56	35	0.3842	0.3842	0.1922	-22.42	-22.55	-22.72
8	64	40	0.3370	0.3370	0.1679	-22.44	-22.55	-22.63
9	72	45	0.2978	0.2978	0.1489	-22.48	-22.56	-22.67
10	80	50	0.2674	0.2674	0.1338	-22.48	-22.55	-22.63

Conclusions

The proposed distributions in the presented work are found to yield useful radiation beam shapes for thick arrays. However, the thick arrays are found to increase the beam width marginally. As it is well-known that grating lobes appear in thinned arrays, the concept of thickening is introduced in the present work to investigate them. The results presented in this paper are useful for the array designers to produce desired shaped beams.

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Authors Biography



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