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# Detection of SINR Interference in MIMO Transmission using Power Allocation

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**Abstract**

Low-complexity and reduced SINR interference cancellation of MIMO systems has attracted recent research attention. Under the bit error rate minimization criterion, an efficient detection ordering scheme for ordered successive interference cancellation detector is achieved for multiple antenna (MIMO) systems using power allocation (PA) scheme. In this paper, we derive a relation that makes the channel gains converge to their geometric mean from the convexity of the Q-function. Based on this approach, first, we design the fixed ordering algorithm, for which the geometric mean is used for a constant threshold. Further, the performance can be improved by modifying the scheme employing adaptive thresholds is developed using the correlation among the ordering results. In the proposed method, theoretical analysis and simulation results show that the ordering schemes using QR-decomposition reduce the computational complexity compared to the conventional method, and also bit-error-rate (BER) performance can be improved.

**Keywords:** MIMO, SINR, OSIC, QR-decomposition, Power allocation.

## Introduction
The interest of multiple-input multiple-output (MIMO) system analysis has been an active area of research for their great potential of enhancing the system's performance [1], [2]. The V-BLAST architecture, also referred to as the BLAST-ordered successive interference cancellation (B-OSIC) detector, proposed in [3] and [4,] that exploits this potential. In a B-OSIC receiver, first, the data stream with the strongest signal-to-interference-noise ratio (SINR) is selected and is subtracted from the received signal. For equal power allocation (PA) across the transmit antenna array, it is optimal in terms of bit error rate (BER) or equivalently minimum-mean-square

error (MMSE) [5]. When the information about the channel is obtained at the transmitter, further improved performance can be achieved using proper PA schemes. Based on the notion that the data stream with the smallest SINR degrades the overall error performance, PA schemes for the B-OSIC have been suggested in [6] and [7] which reduce the computational complexity. Most of the PA schemes are focusing on the transmitter-side processing strategies, but not at the receiver and the detection ordering scheme have not been fully explained.

In this paper, we derive a new detection ordering strategy and schemes, which is distinct from previous studies. To obtain a QR-factorization based approach will be employed in our study [6]. First, we derive the BER/minimum-mean-square error (MMSE) minimization condition from the convexity of the Q-function in the PA scheme. It is evident that the ordering strategy makes the channel gains converge to their geometric mean and achieves the error performance could be improved. Based on this approach, we develop the two ordering algorithms, which are identical except for selection of threshold. The first algorithm determines the detection-order using the geometric mean as a constant threshold, and by taking the previous ordering results, next (modified) ordering scheme for robust convergence adaptively updates the threshold. The comparison of the cumulative distribution is conducted to confirm the superiority of the adaptive design. In this proposed ordering schemes using QR-decomposition reduce the computational complexity and also better BER performance could be achieved with compared to the conventional methods.

## Mimo System Description

Let us consider a MIMO system with $N_t$ transmit antennas and $N_r$ receive antennas. The flat-fading MIMO channel is expressed by the $N_r$ X $N_t$ matrix H with the element $h_{ji}$ representing the channel gain from $i$th transmit antenna to $i$th receive antenna. The $N_r$ X *1* received signal vector $y = [y_1, \dots, y_{N_t}]^T$ is written as

$$y = \sqrt{\frac{E_s}{N_t}} HPx + n \tag{1}$$

where $x=[x_1, \dots, x_{N_t}]^T$ denotes $N_t$ X *1* the transmitted signal vector, and $n=[n_1, \dots, n_{N_r}]^T$ is the $N_r$ – dimensional noise vector with elements following complex zero mean Gaussian distribution with variance of $\sigma^2_n$. $E_s$ is the total transmitted signal energy on $N_t$ transmit antennas and $P=\sqrt{N_t}.\mathrm{diag}(P_1, P_2, \dots, P_{N_t})$ denotes the diagonal PA of precoding matrix.

To derive the system model for the MMSE-QR detector, an $(N_r + N_t)$ X $N_t$ augmented channel matrix $\hat{H}$, an $(N_r + N_t)$ X *1* extended receive vector $\bar{y}$ and an $N_t$ X *1* zero matrix $0_{N_t,1}$ can be written as [8]–[10]

$$\bar{H} = \begin{bmatrix} H \\ \sigma_n I_{N_r} \end{bmatrix} \xrightarrow{\text{ordering}} \bar{Q}\bar{R} \quad \text{and} \quad \bar{y} = \begin{bmatrix} y \\ 0_{N_t,1} \end{bmatrix} \tag{2}$$

The upper triangular matrix $\bar{\mathbf{R}}$, which is defined by the detection-order, determines the SINR [9], and the post-detection SINR $\rho_k$ of the $k$th data stream is given as [2]

$$\rho_k = \frac{E_s}{\sigma_n^2} P_k^2 R_{k,k}^2 - 1, \qquad k = 1, 2, \ldots, N_t. \tag{3}$$

The QR-decomposition based OSIC detection for BER-minimized PA transmission can be performed using the architecture shown in Fig.1. Transmission power $P_k$ is assigned to each data stream based on the feedback information of the diagonal elements $R_{k,k}$. The independently encoded symbols are processed through a diagonal PA matrix and then transmitted from $N_t$ data streams. The QR-OSIC receiver detects the transmit symbols sequentially in accordance with the designated detection-order.

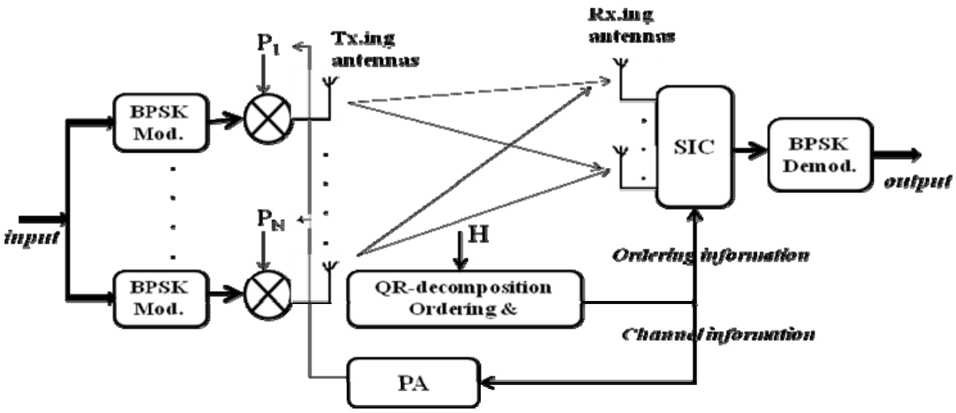


**Figure 1:** MIMO system transmission model with PA and QR-OSIC detector.

## Proposed Algorithms

Theoretical analysis for BER/MMSE performance is explained in section 3.1. The channel gains and the transmission power are affecting the derivation of post detection SINR and also the error rate. The proposed ordering strategy is derived and the efficient ordering algorithms for the QR-OSIC receiver are presented in Section 3.2, from the properties of the Q-function and ordering results.

### Description of the Bit-Error-Rate Performance

A power allocation (PA) scheme is assumed for the average BER minimization under the QR-decomposition of the channel matrix and no error propagation in successive cancellation of the data streams has been proposed in [6]. For BPSK modulation, the PA scheme can be expressed as

$$\begin{aligned} &\text{minimize} \quad \frac{1}{N_t}\sum_{k=1}^{N_t} Q(\sqrt{2}\gamma_s P_k R_{k,k}) \approx \frac{1}{N_t}\sum_{k=1}^{N_t} Q(\sqrt{2\rho_k}) \\ &s.t \quad \frac{1}{N_t}\sum_{k=1}^{N_t} P_k^2 = 1, \quad 0 < {}^{\forall}P < 1 \\ &R_{k,k} \geq 0, \quad k \in \{1, \ldots, N_t\} \end{aligned} \tag{4}$$

where $Q(x) = \sqrt{1/2\pi}\int_x^\infty e^{-(t^2/2)}dt$ and $\gamma_s = \sqrt{E_s/\sigma_n^2}$.

We assume $\bar{R}_{k,k} \geq 0$ because it is defined as the norm of the *k*th column of the augmented channel matrix [8]. For general constellations, the average BER of the PA can be approximated with a constellation-specific constant [7], [11].

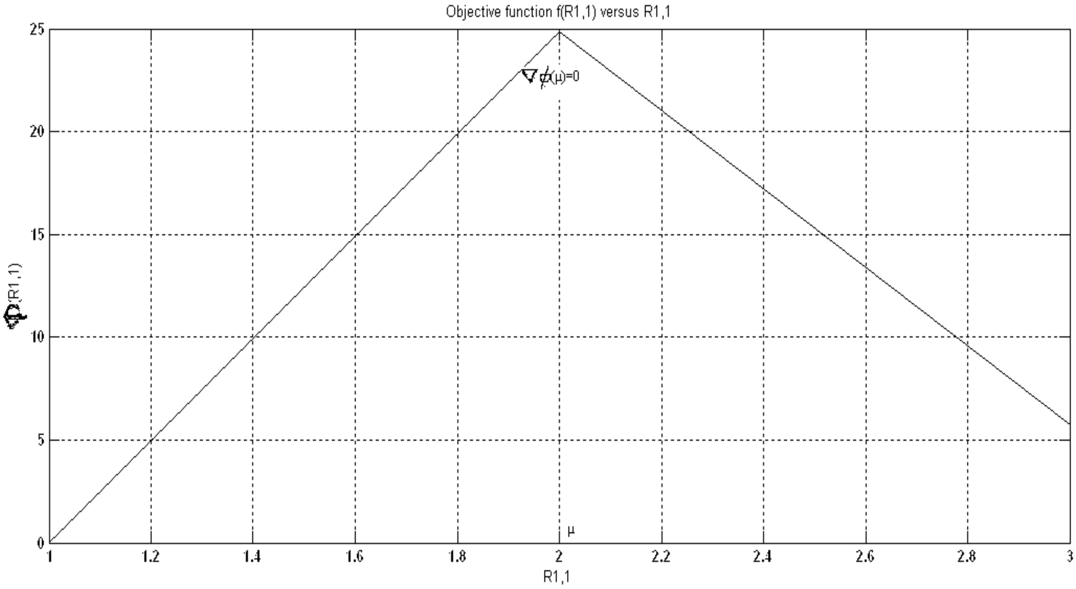


**Figure 2:** Graph of objective function $\phi(\bar{R}_{1,1})$ versus $\bar{R}_{1,1}$

The average BER as well as the post-detection SINR $\rho_k$ is determined by the allocated power $P_k$ and the channel gain $\bar{R}_{k,k}$, can be observed in equation (4). Because of the convexity property of the Q-function, the resulting BER is minimized by (i) the detection ordering of the QR-OSIC receiver such that all diagonal elements of the matrix $\bar{\mathbf{R}}$ are equal to their geometrical average $\mu = \sqrt[N_t]{\det(\bar{R})} = \sqrt[N_t]{\prod_{k=1}^{N_t} \bar{R}_{k,k}}$ and alternatively (ii) the PA scheme at the transmitter which makes the product of two variables $P_k$ and $\bar{R}_{k,k}$ identical for all data streams. As the real MIMO channel is characterized by several spatio-temporal properties, the condition (i) is not practical in spite of its optimality. On the other hand, in (ii), different detection-order leads to different $\bar{R}_{k,k}$, and $P_k$ hence should be also differently assigned. This indicates that an appropriate detection ordering strategy incorporates with the PA scheme can achieve the improved BER performance.

**Proposed Detection Ordering method and Algorithms**

The average BER minimization problem (4) can be simplified to maximize the product of two variables $P_k$ and $\bar{R}_{k,k}$ since the Q-function has convex and decreasing properties.

$$\text{maximize} \quad P_1\bar{R}_{1,1}$$
$$\text{s.t} \quad P_1\bar{R}_{1,1} = P_2\bar{R}_{2,2} = \ldots = P_{N_t}\bar{R}_{N_t,N_t}$$

$$\sum_{k=1}^{N_t} P_k^2 = 1, \qquad 0 < {}^{\forall}P < 1 \tag{5}$$

Using the following properties of $P_1 R_{1,1} = P_2 R_{2,2} = \sqrt{1-P_1^2}(\det(\mathbf{R})/R_{1,1}), P_1^2 = (\det^2(\mathbf{R})/(R_{1,1}^4 + \det^2(\mathbf{R})))$ and max $P_1 R_{1,1} \rightleftharpoons \max P_1^2 R_{1,1}^2$ , the problem for two transmit antennas can be written as

$$\text{maximize} \quad \frac{R_{1,1}^2 \det^2(\mathbf{R})}{R_{1,1}^4 + \det^2(\mathbf{R})} = \emptyset(R_{1,1})$$
$$s.t \quad P_1 R_{1,1} = P_2 R_{2,2}, \; P_1^2 + P_2^2 = 1 \tag{6}$$

To find the direction of increasing, a plot of the objective function $\emptyset(R_{1,1})$ versus $R_{1,1}$ is given in Fig. 2. It is observed that $\emptyset(R_{1,1})$ increases as $R_{1,1}$ tends to μ. When differential calculus is applied to $\emptyset(R_{1,1})$, we also obtain

$$2R_{1,1}\left(R_{1,1}^4 - \det^2(\mathbf{R})\right) = 0$$
$$R_{1,1} = \sqrt{\det(\mathbf{R})} = \mu \tag{7}$$

Note that $\rho_k \propto P_k^2 R_{k,k}^2$ is monotonically (almost linear) increasing as $R_{k,k}$ approaches to μ. And the ordering strategy that makes $R_{k,k}$ converge to μ achieves higher post-detection SINR, which also further improves the overall BER performance. It can be extended to the system with $N_t$ transmit antennas, from (4). To satisfy the derived strategy, we establish the fixed ordering algorithm, the architecture of which arranges the channel gains to minimize $|R_{k,k} - \mu|$ for all $k$

$$k_l = \arg\min_w |R_{w,w} - \mu|$$
$$s.t \quad w \notin \{k_l, \ldots, k_{l-1}\}, \quad \mu = \sqrt[N_t]{\det(\mathbf{R})} \tag{8}$$

where the list of $N_t$ elements $\{1, 2, \ldots, N_t\}$ are rearranged with the parenthesized subscript implying the reverse order in which the elements are to be detected and the ordered set k = $\{k_1, k_2, \ldots, k_{N_t}\}$ is a permuted sequence of them [8], [10]. The modified ordering algorithm employing adaptive algorithm can be developed using the correlation among ordering results for robust convergence. For instance in $N_t = 3$, system, selecting an element 1 as $k_1$ will, in general, result in a different $R_{1,1}$ than if element 2 or 3 was selected. It also affects the remaining sets which decide $k_2$, $k_3$. Mostly, the channel gains are constrained via $\mu = \sqrt[N_t]{\prod_{k_l=1}^{N_t} R_{k_l k_l}}$ . From the above properties, we propose the adaptive ordering design which continually renews the thresholds by controlling the weights with reference to previously determined channel gains. By substituting the variable thresholds into the fixed method, we get

$$k_l = \arg\min_w |R_{w,w} - \mu_l|$$
$$s.t \quad \mu_l = \mu, \quad \mu_{l+1} = \sqrt[\frac{N_t-l+1}{N_t-l}]{\mu_l / R_{l,l}^{N_t-l+1}} \tag{9}$$

where $\mu_l$ denotes the threshold for $k_l$. The adaptive ordering algorithm can be considered as the reduced-sized fixed ordering process extracting the already decided gains thus it plays a large part in balancing among ordering results. If the sign of $R_{k,k}$- μ is distributed to one side serially, the adaptive ordering algorithm enables the following channel gain to be on the opposite side by adjusting $\mu_{l+1}$. This allows more channel gains to converge to μ. To identify it, the cumulative distributions of $R_{k,k}$- μ with four transmit/receive antennas are drawn in Fig. 3. The small gap between two similar schemes is noticeable because the adaptive algorithm is equivalent to the fixed one for slight differences in $| R_{k,k}$- μ.

In Table 1, the process of the proposed detection ordering algorithms are summarized. Here, $A_{(:,m)}$ indicating the *m*th column of matrix A , $A_{(l,m)}$ indicating the *l*th row and *m*th column's element of matrix A and vector k denoting the permutation of the columns of $\bar{H}$.

The complexity comparison between the B-OSIC and the QR-OSIC receiver is not discussed in this paper. Fortunately, the efficiency of the QR-OSIC receiver which reduces the computational complexity by an order of magnitude is proven in [5]. In a B-OSIC detector with $N_t = N_r$, the total numbers of multiplications and additions are (43/12) $N_t^4$ + (22/3) $N_t^3$ + $\mathcal{O}(N_t^2)$ and (43/12) $N_t^4$ + (20/3) $N_t^3$ +$\mathcal{O}(N_t^2)$, respectively. On the other hand, the OSIC receiver using QR-factorization requires (2/3)$N_t^3$ + 7$N_t^2 N_r$+ 2$N_t N_r^2$ +$\mathcal{O}(N_t^2)$ multiplications and additions. Because of the multiple calculations of pseudo-inverse for nulling and ordering, the B-OSIC requires higher computational cost [10]. When $N_t = N_r$, the numbers of multiplications and additions are given with the complex floating point operations (flops).

$$\frac{43}{6} N_t^4 + 14 N_t^3 + \mathcal{O}(N_t^2) \qquad \textit{for} \quad \text{B-OSIC}$$
$$\frac{29}{3} N_t^3 + \mathcal{O}(N_t^2) \qquad \textit{for} \quad \text{QR-OSIC} \tag{10}$$

**Table 1:** Proposed detection ordering algorithm.

| Steps required to implement proposed algorithm |
|---|
| 1. $\mathbf{R} \equiv \mathbf{0}_{N_t}$, $\mathbf{Q} \equiv \mathbf{H}$, $\mathbf{k} = \{1, \ldots, N_t\}$ , $\mu_1 = \mu$ |
| 2. *for* $i = 1, \ldots, N_t$ |
| 3. $\tau_i = \lVert \mathbf{Q}_{(:,i)} \rVert^2$ |
| 4. *end* |
| 5. *for* $l = 1, \ldots, N_t$ |
| 6. $k_l = \arg\min_w \left| \sqrt{\tau_w} - \mu_l \right|$ |
| 7. Fixed : $\mu_{l+1} = \mu_l$ , $\mu_{l+1} = \sqrt[\frac{N_t-l+1}{N_t-l}]{\mu_l / R_{l,l}^{N_t-l+1}}$ |
| 8. $\mathbf{R}_{(:,l)} \rightleftharpoons \mathbf{R}_{(:,k_l)}$ , $\tau_l \rightleftharpoons \tau_{k_l}$ |
| 9. $\mathbf{k}(l) \rightleftharpoons \mathbf{k}(k_l)$ , $\bar{\mathbf{Q}}_{(1:N_r+l-1,\, l)} \rightleftharpoons \bar{\mathbf{Q}}_{(1:N_r+l-1,\, k_l)}$ |
| 10. $\mathbf{R}_{(l,l)} = \sqrt{\tau_l}$ |
| 11. $\bar{\mathbf{Q}}_{(:,l)} = \bar{\mathbf{Q}}_{(:,l)} / \mathbf{R}_{(l,l)}$ |
| 12. *for* $m = l+1, \ldots, N_t$ |

$\mathbf{R}_{(i,m)} = \mathbf{Q}^{H}_{(:,\,i)} \cdot \bar{\mathbf{Q}}_{(:,\,m)}$

$\bar{\mathbf{Q}}_{(:,\,m)} = \bar{\mathbf{Q}}_{(:,\,m)} - \mathbf{R}_{(:,\,m)} \cdot \bar{\mathbf{Q}}_{(:,\,i)}$

$\tau_m = \tau_m - \mathbf{R}^2_{(i,m)}$

*end*

*end*

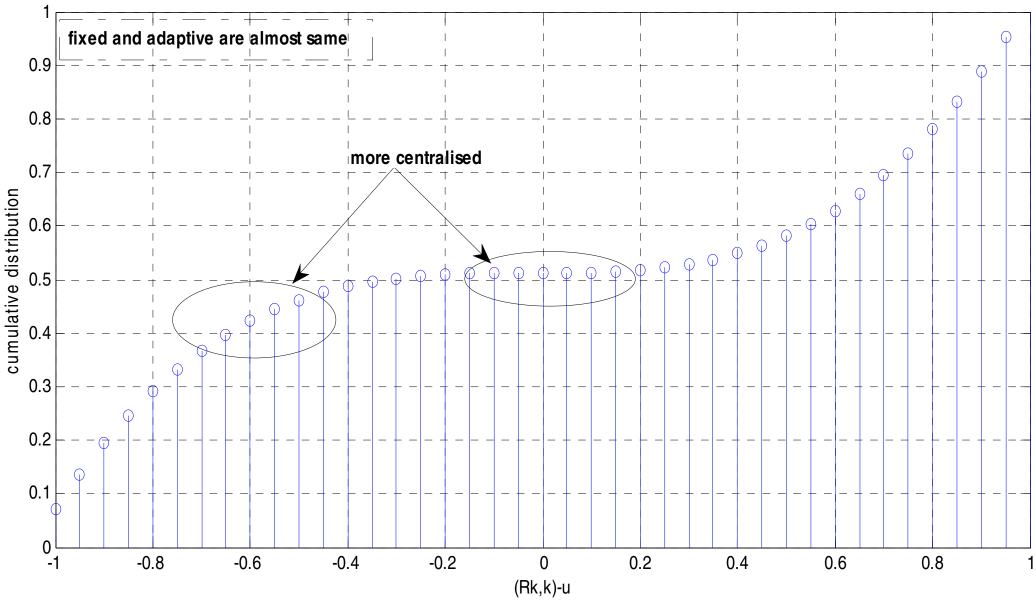


**Figure 3:** Comparison of cumulative distribution of $\bar{R}_{k,k}$ - μ

## Simulation Results

In this paper, an uncoded MIMO system with 3 X 3, 4 X 4 transmit/receive antenna configurations and BPSK modulation considered and simulations are used to obtain the system performance. A quasi-static channel is assumed for each of the MIMO systems and for a specific value of SNR for the performance evaluation, for which the channel gain is constant over a frame and changed independently from frame to frame. To concentrate our point on comparing ordering algorithms, we postulate the perfect channel estimation at the receiver and error-free PA information at the transmitter.

Fig. 4 shows the average BER performance comparison fixed and adaptive threshold algorithms for MIMO systems with 3X3 antenna and the simulation results of 4X4 antenna are shown in Fig. 5. Here, results indicate a system with the BER-minimized PA scheme. The green line indicates the average BER performance for fixed algorithms, where as red line for adaptive algorithm. As explained in previous papers, without the PA, the B-OSIC outperforms the QR-OSIC receiver. Power controlled MIMO systems, the proposed ordering strategy, achieve the reduced computational complexity and the improved error performance. It is sufficient to confirm the superiority of the proposed design because the ordering algorithms of previous studies comply with the strategy of the B-OSIC [5]–[8]. A further performance improvement in the high SNR region can be explained in terms of the error propagation, since the PA scheme is designed under the assumption of the error-free decision in previous detection methods.

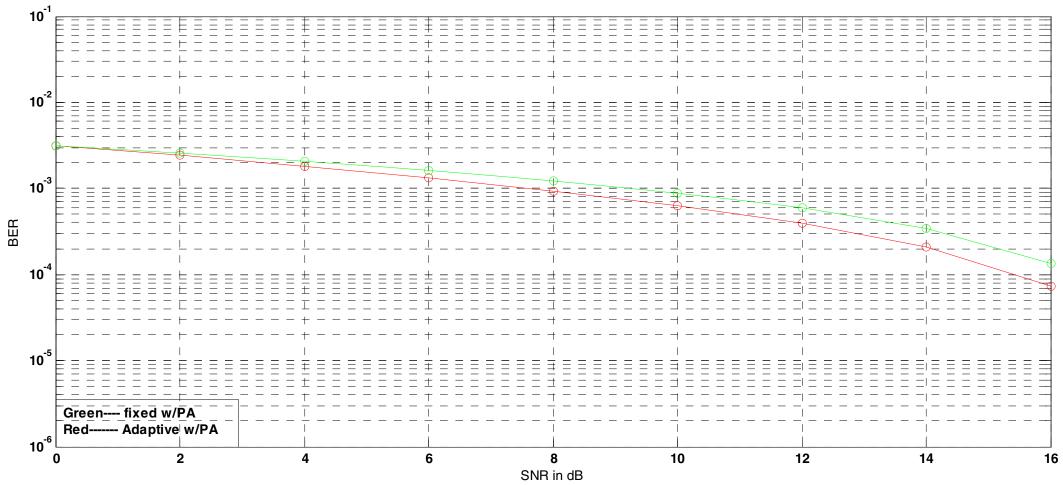


**Figure 4:** Average BER performances of MIMO systems with 3X3 transmit/receive antenna.

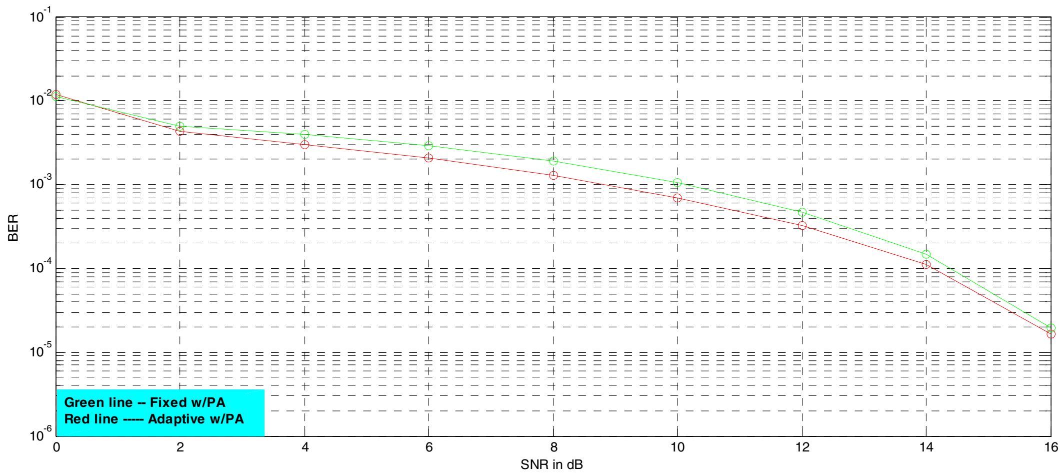


**Figure 5:** Average BER performances of MIMO systems with 4X4 transmit/receive antenna

## Conclusion

In this study, we investigate the QR-OSIC receiver design for the transmitter-side power allocated MIMO system. We develop the efficient detection ordering algorithms in combination with the PA scheme, from the properties of the Q-function and ordering results. In spite of less computational complexity, the proposed ordering schemes reduce the overall BER in comparison with the previously derived B-OSIC scheme. Because of the post-detection SINR increment, the coded systems with the derived approach can also be expected to achieve the improved BER performance.

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