

## Systematic Comparison of Different PAPR Reduction Methods in OFDM Systems

R. Divya Kanti<sup>[1]</sup> and R.V. Ch. Sekhar Rao<sup>[2]</sup>

<sup>[1]</sup> Dept of ECE, Lendi Institute of Engineering and technology,  
Vizianagaram, Andhra Pradesh, India

<sup>[2]</sup> Dept of ECE, Lendi Institute of Engineering and technology,  
Vizianagaram, Andhra Pradesh, India  
[divya.kanti74@gmail.com](mailto:divya.kanti74@gmail.com), [sekhar\\_rao\\_81@yahoo.co.in](mailto:sekhar_rao_81@yahoo.co.in)

### Abstract

Selected mapping (SLM) is a well-known technique for peak to-average-power ratio (PAPR) reduction of orthogonal frequency-division multiplexing (OFDM) systems. In this technique, different representations of OFDM symbols are generated by rotation of the original OFDM frame by different phase sequences, and the signal with minimum PAPR is selected and transmitted. To compensate for the effect of the phase rotation at the receiver, it is necessary to transmit the index of the selected phase sequence as side information (SI). In this paper, an SLM technique is introduced for the PAPR reduction of space-frequency-block-coded OFDM systems with Alamouti coding scheme. In this paper, we also propose a simple technique for the reduction of high Peak to Average Power Ratio (PAPR), based on Clipping and Differential Scaling, in Orthogonal Frequency Division Multiplexing (OFDM) systems. In this technique, the amplitude of complex OFDM signal is clipped and then scaled in such a way so that the PAPR is reduced without causing much degradation in bit error rate (BER). We have determined the threshold values for clipping and scaling using Monte Carlo Simulations. We have presented PAPR and BER of the system considered using simulations for QPSK constellation.

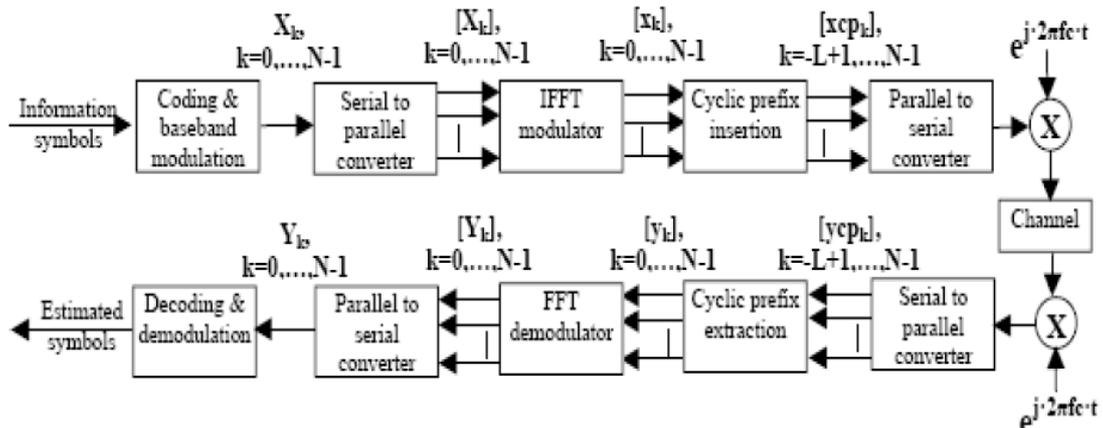
**Index Terms**—Orthogonal frequency-division multiplexing (OFDM), peak-to-average-power ratio (PAPR), selected mapping (SLM), space frequency block coded (SFBC), Clipping, Differential scaling.

### I. INTRODUCTION

In recent many years orthogonal frequency division multiplexing has been used

widely in digital transmission. OFDM has been adopted in several communication systems such as wireless local area networks (WLAN), wireless metropolitan area network (WMAN), digital audio broadcasting (DAB), digital video broadcasting (DVB). OFDM is an potential candidate for the 4<sup>th</sup> generation mobile wireless systems. OFDM is an attractive modulation technique in wireless applications because OFDM system divides frequency selective channel in to several frequency flat subchannels. So that OFDM can get more immunity to multipath fading.

Subcarrier spacing in OFDM is minimum frequency separation to maintain orthogonality of corresponding time domain waveform, then the available band width is used efficiently. OFDM is one of the multicarrier modulation technique. This is attractive technique for high speed data transmission. Unlike single carrier systems, OFDM communication systems do not rely on increased symbol rates in order to achieve higher data rates. OFDM is a multicarrier digital modulation scheme. In OFDM carrier spacing is carefully selected so that each carrier is orthogonal to the other subcarrier. Two signals are orthogonal if their dot product is '0'. OFDM systems break the available bandwidth into many narrower sub-carriers and transmit the data in parallel streams. Each subcarrier is modulated using varying levels of QAM modulation, e.g. QPSK, QAM, 64QAM or possibly higher orders depending on signal quality. Each OFDM symbol is therefore a linear combination of the instantaneous signals on each of the sub-carriers in the channel. This scheme facilitates efficient use of bandwidth and reduced Inter Symbol Interference (ISI). But another problem is high Peak to Average Power Ratio (PAPR) OFDM symbols. To counter this we use a modified scheme called Single Carrier FDMA (SC-FDMA). The advantages are reduced PAPR and frequency domain equalization.



**Fig.1. OFDM BLOCK DIAGRAM**

*Advantages:* Due to increase in symbol duration, there is a reduction in delay spread. Addition of guard band almost removes the ISI and ICI in the system. Conversion of the channel into many narrowly spaced orthogonal sub – carriers render it immune to frequency selective fading. As it is evident from the spectral pattern of an OFDM system, orthogonally placing the sub – carriers lead to high spectral efficiency. Can be efficiently implemented using IFFT.

*Disadvantages:* These systems are highly sensitive to Doppler shifts which affect the carrier frequency offsets, resulting in ICI. Presence of a large number of sub-carriers with varying amplitude results in a high Peak – to – Average Power Ratio (PAPR) of the system, which in turn hampers the efficiency of the RF amplifier.

Limitations of OFDM technique is the large PAPR!

1. Which makes the designer in leaving high backoffs for amplifiers and hence limiting the power amplifier Performance
2. Increasing the cost of the systems
3. Degrade the bit error rate (BER) due to inter-modulation noise occurring in the non-linear amplifier.
4. use of higher resolution analog-to-digital-converters to prevent the signal from being clipped or carrier intermodulation to occur. Hence, the need to reduce the PAPR of such systems.

## II. PEAK –TO-AVERAGE POWER RATIO

OFDM is one of the many multicarrier modulation techniques, which provides high spectral efficiency, low implementation complexity, less vulnerability to echoes and non – linear distortion. Due to these advantages of the OFDM system, it is vastly used in various communication systems. But the major problem one faces while implementing this system is the high peak – to – average power ratio of this system. A large PAPR increases the complexity of the analog – to – digital and digital – to – analog converter and reduces the efficiency of the radio – frequency (RF) power amplifier. Regulatory and application constraints can be implemented to reduce the peak transmitted power which in turn reduces the range of multi carrier transmission. This leads to the prevention of spectral growth and the transmitter power amplifier is no longer confined to linear region in which it should operate. This has a harmful effect on the battery lifetime. Thus in communication system, it is observed that all the potential benefits of multi carrier transmission can be out - weighed by a high PAPR value. Presence of large number of independently modulated sub-carriers in an OFDM system the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as Peak-to-Average Power Ratio. Coherent addition of N signals of same phase produces a peak which is N times the average –signal.

The major disadvantages of a high PAPR are-

1. Increased complexity in the analog to digital and digital to analog converter.
2. Reduction is efficiency of RF amplifiers.

Multi-carrier phenomena is considered to be one of the major development in wireless communication and among them OFDM is becoming the important standard. However, high PAPR is the major drawback of OFDM, which results in lower power efficiency hence impedes in implementing OFDM. To overcome the low power efficiency requires not only large back off and large dynamic range digital-to-analog converter (DAC) but also highly efficient high power amplifiers (HPA) and linear converters. These demands result in costly hardware and complex systems. Therefore

to lessen the difficulty of complex hardware design it has become imperative to employ efficient PAPR reduction techniques. Let the data block of length  $N$  be represented by a vector

$$X = [X_0, X_1, \dots, \dots, \dots, X_{N-1}]^T \quad (1)$$

The complex data block for the OFDM signal to be transmitted is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi n \Delta f t}, 0 \leq t \leq NT \quad (2)$$

The PAPR of the transmitted signal is defined as

$$PAPR = \frac{\max |x(t)|^2}{\frac{1}{NT} \int_0^{NT} |x(t)|^2 dt} \quad (3)$$

Expressing in decibels,

$$papr_{db} = 10 \log_{10} (papr) \quad (4)$$

The crest factor or peak-to-average ratio (PAPR) or peak-to-average power ratio (PAPR) is a measurement of a waveform, calculated from the peak amplitude of the waveform divided by the RMS value of the waveform.

$$C = \frac{|X|_{peak}}{X_{rms}} \quad (5)$$

Reducing the  $\max |x(t)|$  is the principle goal of PAPR reduction techniques. Since, discrete-time signals are dealt with in most systems, many PAPR techniques are implemented to deal with amplitudes of various samples of  $x(t)$ . Due to symbol spaced output in the first equation we find some of the peaks missing which can be compensated by oversampling the equation by some factor to give the true PAPR value.

#### **Cumulative distribution function :**

The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. By implementing the Central Limit Theorem for a multi-carrier signal with a large number of sub-carriers, the real and imaginary part of the time-domain signals have a mean of zero and a variance of 0.5 and follow a Gaussian distribution. So Rayleigh distribution is followed for the amplitude of the multi-carrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system. The CDF of the amplitude of a signal sample is given by

$$F(Z) = 1 - \exp(-Z) \quad (6)$$

The CCDF of the PAPR of the data block is desired is our case to compare outputs of various reduction techniques. This is given by

$$\begin{aligned} P(papr > z) &= 1 - P(PAPR \leq Z) \\ &= 1 - F(Z)^N \\ &= 1 - (1 - \exp(-Z))^N \end{aligned}$$

They are many PAPR reduction techniques. Those are selected mapping, clipping and differential scaling.

### III. SELECTED MAPPING ALGORITHM

Selected mapping (SLM) is a promising PAPR reduction technique. Although SLM is also a scrambling technique, the main idea of SLM is quite different from PTS. It selects the most favorable signal from a set of phase rotated candidate data blocks generated by transmitter, which all represent the same information as the original data block. A block diagram of SLM scheme is shown in fig. 2. we get  $U$  different time domain candidate signals with different PAPR values. Among them, the one with the lowest PAPR is selected for transmission. This selecting can be mathematically expressed as  $x = \arg \min \{PAPR(x_u)\}$

SLM technique generates several OFDM symbols as candidates and then select the one with the lowest PAPR for the actual transmission. Conventionally, the transmission of side information is needed so that the receiver can use the side information to determine which candidate is selected in the transmission and then recover the information. SLM technique do introduced some additional complexity, but with loss in efficiency

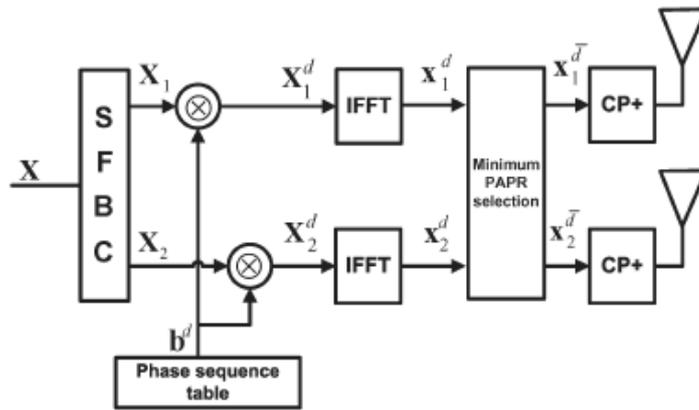


Fig.2. Block diagram of SLM method for PAPR reduction

The technique of selected mapping (SLM) for PAPR reduction was proposed in 1996. In SLM from a set of candidate signals which are generated to represent the same information, the signal with lowest PAPR is selected and transmitted. The information about this selection also needs to be explicitly transmitted along with the selected signal as side information.

Selected mapping algorithm is as follows:

1. The sequence of data bits are mapped to constellation points QPSK to produce sequence symbols  $X_0, X_1, X_2, \dots$
2. These symbol sequences are divided into blocks of length  $N$ .  $N$  is the number of subcarriers.
3. Each block  $X = [X_0, X_1, X_2, \dots, X_{N-1}]$  is multiplied (point wise multiplication) by  $U$  different phase sequence vectors

$$B^{(u)} = [B_0^{(u)}, B_1^{(u)}, \dots, \dots, B_{N-1}^{(u)}]^T$$

where each row of the normalized Riemann matrix  $B$  is taken as  $B(u)$ ,  $u=1, 2, \dots, U$ .

4. A set of  $U$  different OFDM data blocks

$$X^{(u)} = [X_0^{(u)}, X_1^{(u)}, \dots, X_{N-1}^{(u)}]^T$$

Are formed, where

$$X_n^{(u)} = X_n \cdot B_n^{(u)} \quad n = 0, 1, \dots, N-1, \\ u = 1, 2, \dots, U$$

5. Transform into time domain to get

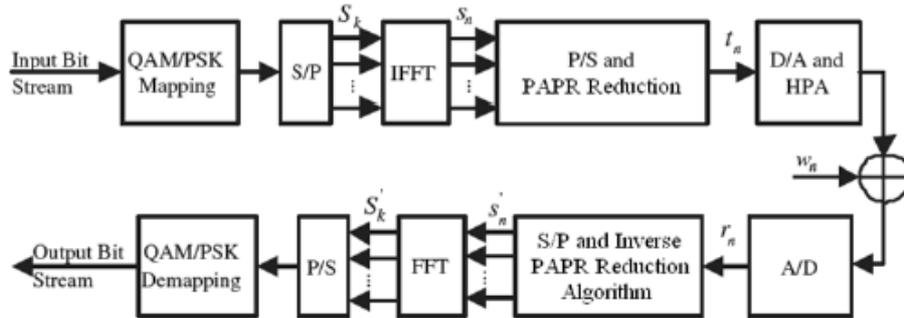
$$X^{(u)} = IDFT\{X^u\}$$

6. Select the one from  $X^{(u)}$   $u = 1, 2, \dots, U$  which has the minimum PAPR and transmit. Block diagram of SLM technique is given in figure 2.

We use MATLAB simulations to evaluate the performance of the different phase sequences for SLM technique. As a performance measure, complementary cumulative density function (CCDF) of PAPR is used. Mean and Variance of PAPR of the whole data blocks is taken as second criteria for performance measure among different phase sequence sets. OFDM system with 128 subcarriers is simulated with QPSK. The main disadvantage in SLM is low data transfer. So we go for Clipping and Differential Scaling.

#### IV. CLIPPING AND DIFFERENTIAL SCALING

We propose in this section a new technique called Clipping and Differential Scaling. The probability distribution of amplitudes of the OFDM signal follows Rayleigh distribution and thus the probability of high peaks is very less.



**Fig. 3.** OFDM system model with PAPR reduction block

An upper threshold above which the signal amplitudes do not contribute much to the signal is determined as follows. Using simulations, we have determined BER for the modified signals along with PAPR. We select the clipping threshold at which the BER is degraded from  $1.5 \times 10^{-3}$  to  $3.5 \times 10^{-3}$  at SNR of 10dB and the amplitudes above this clipping threshold are clipped. Instead of clipping the signal further to reduce the PAPR, we consider a reversible process - Differential Scaling which would reduce the PAPR but not deteriorate the BER. Since different ranges of amplitudes of the signal are scaled in a different manner, it is called Differential Scaling. We have considered three types of scaling as described below.

*Scale Up*: In this method, we scale up the lower amplitudes of the signal by a factor of  $\beta$ . This leads to increase the average value without affecting the peak values. Therefore, the resulting PAPR reduces. The PAPR reduction function can be defined as

$$\begin{aligned} h(x) &= \alpha xp, \text{ if } x > \alpha xp \\ &= \beta x, \text{ if } x < A \\ &= x, \text{ if } A \leq x \leq \alpha xp \end{aligned}$$

where  $xp$  is the amplitude peak value occurring in an OFDM symbol block,  $\alpha$  is the factor deciding the clipping threshold in terms of percentage of the peak value and  $\beta$  is the scaling factor for the range  $[0, A)$  whose value is greater than one. The values of the parameters used are mentioned at the end of this section.

*Scale Down*: In this method, we scale down the higher amplitudes of the signal by a factor of  $\gamma$ . This leads to decrease the peak value. Although the average value would also fall down, the resulting PAPR reduces. Because the reduction in peak power is greater than the reduction in the average power.

The PAPR reduction function can be defined as

$$\begin{aligned} h(x) &= \alpha xp, \text{ if } x > \alpha xp \\ &= \gamma x, \text{ if } B \leq x \leq \alpha xp \\ &= x, \text{ if } x < B \end{aligned}$$

where  $xp$  is the amplitude peak value occurring in an OFDM symbol block,  $\alpha$  is the factor deciding the clipping threshold in terms of percentage of the peak value and  $\gamma$  is the scaling factor for the range  $[B, \alpha xp]$  whose value is less than one. The values of the parameters used are mentioned at the end.

**TABLE 1** SIMULATION PARAMETERS

Clipping threshold ( $\alpha$ )	0.47
Scale down factor ( $\gamma$ )	0.8
Lower limit for Scale down (B)	1.2
Scale up factor ( $\beta$ )	2
Upper limit for Scale up (A)	0.5

*Scale Up and Down*: In this method, we combine both the above-mentioned approaches i.e. up-scaling and down-scaling. This method exploits the advantages of both the methods. Hence, a PAPR can be reduced considerably. The PAPR reduction function can be defined as

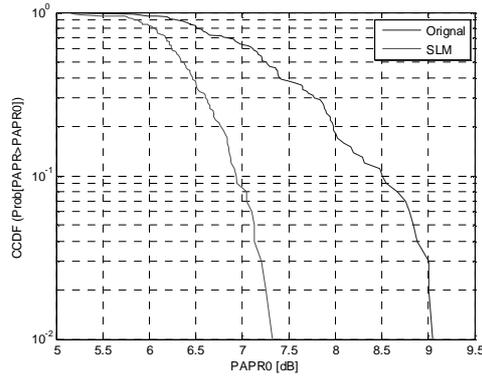
$$\begin{aligned} h(x) &= \alpha xp, \text{ if } x > \alpha xp \\ &= \gamma x, \text{ if } B \leq x \leq \alpha xp \\ &= \beta x, \text{ if } x < A \\ &= x, \text{ if } A \leq x \leq B \end{aligned}$$

where  $xp$  is the amplitude peak value occurring in an OFDM symbol block,  $\alpha$  is the factor deciding the clipping threshold in terms of percentage of the peak value.  $\beta$  is the scaling factor for the range  $[0, A)$  and  $\gamma$  is the scaling factor for the range  $[B, \alpha xp]$ . In order to make all these scaling techniques realizable, a marker needs to be used. The marker is basically a small set of signal values that needs to be transmitted along

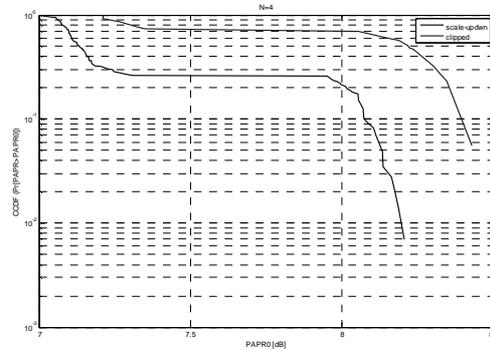
with the information signal. Its job is to keep track of values which have been scaled at the transmitter. The same values would be reversibly scaled at the receiver. The marker may be accommodated like the pilot carriers or sent on another frequency orthogonal to the carriers. Using extensive simulations, the variation in PAPR with the simulation parameters  $A$  and  $\beta$  for the scale-up technique was observed at an SNR of 10 dB. From the 3-D plot obtained, we deduced the optimum values of  $A$  and  $\beta$  for which the PAPR is minimum. The optimum value of  $A$  and  $\beta$  is 0.5 and 2 respectively. Moreover, the BER obtained for the optimum values is  $4 \times 10^{-3}$  whereas the BER for the performance bound at 10 dB SNR is  $2 \times 10^{-3}$ . Thus, there is only a marginal compromise in the BER although we have reduced the PAPR significantly. In the same manner for scale-down technique, the optimum values of  $\gamma$  and  $B$  for which the PAPR is minimum can be obtained. All these values are documented in Table-I.

## V. SIMULATION RESULTS

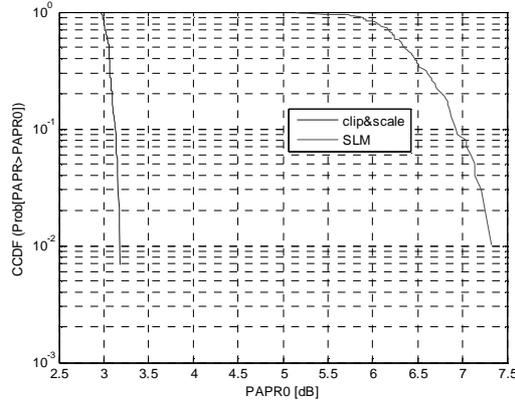
The performance of the proposed method has been evaluated for two different OFDM frame lengths  $N_c = 128$ . The symbols  $X(k)$  are chosen from the QPSK constellations.



**Fig. 4.** PAPR reduction performance of the ordinary SLM, simplified SLM, PII, and clipping and filtering methods for the SFBC-OFDM system with two transmitter antennas and  $N_c = 128$  for different values of  $D$ .



**Fig. 5.** Comparison of PAPR performance (CCDF) of Clipping and Differential Scaling with the CCDF of original OFDM signal.



**Fig.6.** Comparison of PAPR performance (CCDF) of Clipping and Selected Mapping Method with the CCDF of original OFDM signal

## VI. CONCLUSION

In this paper, it has been shown that the simplified method that has been previously proposed for spatially multiplexed OFDM systems is suitable for PAPR reduction of SFBC-OFDM systems. In fact, the simplified SLM does not change the orthogonality of space frequency codes. In this method, the same phase sequence is concurrently applied to the frequency-domain signals for both antennas, and the signal with minimum PAPR has been found and transmitted. So in order to avoid the drawbacks in SLM we have used a simple approach based on Clipping and Differential Scaling to reduce the PAPR of OFDM signals. We have used Clipping along with three different scaling methods, namely up scaling, down scaling and up-down scaling. Using simulations, we obtained the values of threshold for clipping and parameters for scaling with a view to reduce PAPR without degradation in BER. We have presented the PAPR and BER performance for all the techniques considered. The proposed up-down scaling technique is able to achieve PAPR reduction of the order of 8.5 dB from 12 dB PAPR initially. The proposed technique is able to achieve a PAPR of 3.5 dB while maintaining the BER within a margin of 3 times the BER value at the performance bound at an SNR of 10 dB.

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