

Development of a Nonlinear Linearizing Controller Using Input-Output / Input-State Linearization for Implementation in Programmable Logic Controller (PLC)

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Abstract:

This paper describes the design of a nonlinear linearizing controller based on feedback linearizing method to stabilize a nonlinear magnetic levitation system. The linearized model states are selected through input-state linearization, and the mathematical expression of the linearizing controller is obtained. An additional innovative integral LQR is designed to make the closed-loop system more robust. The closed-loop system is implemented in real-time on an IEC 61499 standard-based programmable logic controller using a new model transformation technique. The closed-loop system is simulated in MATLAB/SIMULINK and TwinCAT 3 real-time environment, and the simulation results obtained on both environments are compared.

Key words: IEC, levitation, linearization, nonlinear, PLC, real-time, TwinCAT.

1. INTRODUCTION

Feedback linearization is one of the most attractive nonlinear control design method which is used in most of the researches related to the field of nonlinear control [1,5]. The concept of feedback linearization is to mathematically perform the transformation of the dynamics of a nonlinear system into a fully or partly linear one, so that it is possible to apply linear control methods [2]. The idea of feedback linearization is to get rid of the nonlinearities of a system and impose a desired linear dynamic that can be directly applied to a category of nonlinear systems described in companion, or controllability canonical form.

The implementation of feedback controllers in distributed control system has gained interest in the past decades. In distributed control system configuration, feedback controllers are connected to the rest of the system by network for communication and monitoring. To materialize these features, the International Electro-Technical Commission (IEC) developed an open architecture for the next generation of distributed control and automation: IEC 61499 standard. To demonstrate the functionality of IEC 61499 a magnetic levitation system (Maglev) is selected as a plant to be controlled because it is a highly nonlinear system that suits well with the basic IEC 61499 strategies. The closed loop

control of the Maglev system is done in MATLAB/SIMULINK; the system closed loop is then transformed in basic function blocks based on a full block to block transformation method in IEC 61499 environment designed on Beckhoff TwinCAT 3 software. Once the simulation shows the expected results, then the Function Blocks closed loop system can be deployed to the physical PLC.

This paper shows the development of a nonlinear control algorithm to stabilize a highly unstable magnetic levitation system. The closed-loop system is simulated in MATLAB/SIMULINK environment. The real-time implementation of the control algorithm is done on TwinCAT 3 simulation environment then downloaded into a Beckhoff CX 5020 Programmable Logic Controller (PLC) for testing. The closed loop system is entirely assessed on PLC platform.

The paper is structured as follows: modeling of the system is presented in part 2. The input-state/input-output linearization is described in part 3. Further, the linear control of the Brunovsky system based on ILQR (Integral Linear Quadratic Regulator) method is discussed in part 4. The results of the simulation with both the input-state/input-output linearizing and linear control are presented in part 5. In part 6, the real-time implementation of the closed-loop transformed system based on IEC 61499 methodology is presented. A real-time implementation and simulation of a parallel control system of two levitation systems is proposed in part 7, and finally the conclusion is made based on the results obtained.

2. MODELING AND SIMULATION OF THE MAGNETIC LEVITATION SYSTEM

Magnetic levitation is a fairylike phenomenon which has always captivated the attention of people. It is an interesting highly non-linear and open loop unstable system suitable for control systems applications. Nowadays levitation has the potential to improve numerous sectors such as levitation vehicles, magnetic bearings, aerodynamics and noise mitigation, and fiber reinforced plastics for vehicles and structural concretes [4-5]. Researchers have developed various linear and nonlinear techniques to stabilize this highly nonlinear process [6-7, 10, 16-18-19]. The issue with these

researches is that none of them is implemented in PLC to provide real-time control of the closed-loop system.

The magnetic ball levitation system is made of two sub-systems: electrical and mechanical. The current flowing through the inductor induces a magnetic force that balances the force of gravity and causes the ball to levitate in mid-air [8].

The nonlinear model of the magnetic levitation is developed based on the first principles as follows:

$$\dot{x}_1 = x_2 \quad (1)$$

$$\dot{x}_2 = \bar{g} - \frac{kx_3^2}{mx_1^2} \quad (2)$$

$$\dot{x}_3 = -\frac{R}{L}x_3 - \frac{1}{L}V \quad (3)$$

Where: x_1 is the position of the ball, x_2 the velocity of the ball and x_3 the current across the inductor, \bar{g} is the gravitational force, L is the winding inductance, m is the masse of the ball, R is the winding resistance, V is the input voltage and k is the proportional constant.

The state-space model of the levitation system is affine according to the control input and can be written in the following way:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} x_2 \\ \bar{g} - \frac{kx_3^2}{mx_1^2} \\ -\frac{R}{L}x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V = \mathbf{x}(t) \quad (4)$$

$$y = C = [x_1(t) \quad x_2(t) \quad x_3(t)]^T,$$

$$C = [1 \quad 0 \quad 0] \quad (5)$$

Equations (4) and (5) can be represented in a standard nonlinear format:

$$\dot{x} = f(x,t) + g(x,t)u, \quad u = V \quad (6)$$

$$y = h(x,t) \quad (7)$$

Where $f(x,t)$, $g(x,t)$, and $h(x,t)$ are nonlinear functions of state.

At equilibrium, the time rate derivatives must strictly be equal to zero, $\dot{x} = 0$.

A benchmark model parameter [9] is used for investigation. The values of the system's parameter are given:

$$m = 8.27 \times 10^{-3} \text{ Kg}$$

$$\bar{g} = 9.81 \text{ N} \times \text{Kg}^{-1}$$

$$R = 1 \Omega$$

$$k = 0.0001$$

$$L = 0.01 \text{ H}$$

$$i = 0.84 \text{ A}$$

$$x_{e1} = 0.012 \text{ m}$$

Figure 1 shows the Simulink diagram of the nonlinear system, and figure 2 shows its open loop response.

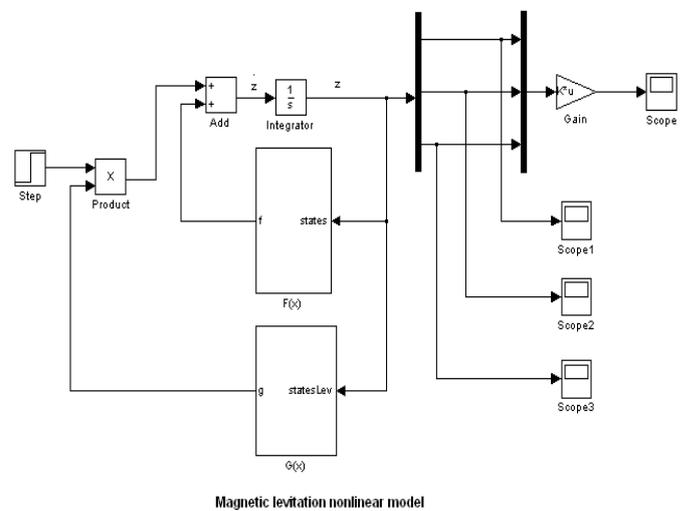


Figure 1: Magnetic levitation system nonlinear state space model in Simulink environment

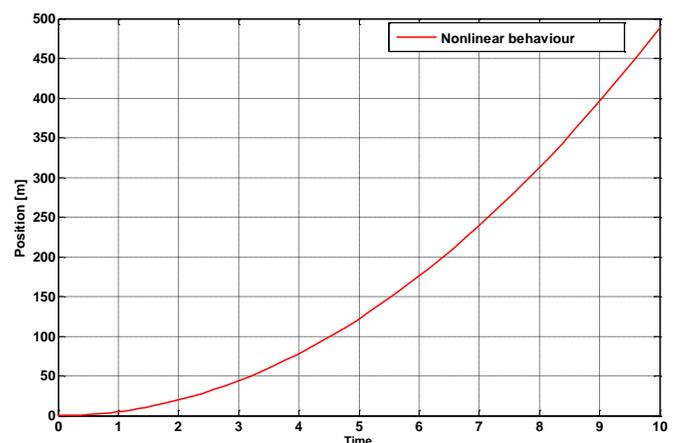


Figure 2: Open loop response of the nonlinear system when the initial position is 0.012[m]

The open loop response of the magnetic levitation nonlinear model shows that under step continuous force, the ball position moves toward infinity.

This analysis confirms that the magnetic ball levitation is a nonlinear open loop unstable system that needs to be controlled effectively.

3. INPUT-STATE/INPUT-OUTPUT LINEARIZATION OF THE CLOSED-LOOP MAGNETIC LEVITATION SYSTEM

In input-output linearization, the output y is indirectly related to the control u through the state variable x and the nonlinear state equations. The relationship between the output y and the input u can be created by differentiating the output function y repeatedly until the input u appears. Then it is necessary to design u to cancel the non-linearity and to receive stable linearized closed loop system [10].

The nonlinear system is represented by the equations (6) and (7). The relationship between the input and the output is established at the third derivative of the output thus the relative degree is 3. The procedure to find the derivatives of the output is:

First derivative:

$$\begin{aligned} y &= x_1 \\ \dot{y} &= L_f h(x) + L_g h(x)u(t) = x_2 + 0.u(t) \end{aligned} \quad (8)$$

Where $L_f h(x)$ and $L_g h(x)u(t)$ are Lie derivatives [20].

Second derivative:

$$\begin{aligned} \dot{y} &= x_2 \\ \ddot{y} &= L_f^2 h(x) + L_g L_f h(x)u(t) = g - \frac{kx_3^2}{mx_1^2} + 0.u(t) \end{aligned} \quad (9)$$

Third derivative:

$$\begin{aligned} \ddot{y} &= g - \frac{kx_3^2}{mx_1^2} + 0.u(t) \\ \dddot{y} &= L_f^3 h(x) + L_g L_f^2 h(x)u(t) = \\ &= \frac{2k}{m} \left(\frac{x_2 x_3^2}{x_1^3} \right) + \frac{2kR}{mL} \left(\frac{x_3^2}{x_1^2} \right) - \frac{2k}{mL} \left(\frac{x_3}{x_1^2} \right) u(t) \end{aligned} \quad (10)$$

The third differentiation shows a direct relationship between the input and output. The system's relative degree is 3, because the output must be differentiated three times to find its relationship with the input.

This case is of interest as the relative degree of the system is identical to the order of the system ($r = n$), therefore the

input-output linearization implementation yields input-state linearization [11].

According to theory [12], the new coordinates of the system have the following representation:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ g - \frac{k}{m} \left(\frac{x_3^2}{x_1^2} \right) \end{bmatrix} \quad (11)$$

Equations 8, 9 and 10 form the new vector of the transformed coordinate system, and it is represented as:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dddot{y} \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{kx_3^2}{mx_1^2} \\ \frac{2k}{m} \left(\frac{x_2 x_3^2}{x_1^3} \right) + \frac{2kR}{mL} \left(\frac{x_3^2}{x_1^2} \right) - \frac{2k}{mL} \left(\frac{x_3}{x_1^2} \right) u \end{bmatrix} \quad (12)$$

The system (12) can be linearized through proper selection of the control u .

The nonlinear linearizing control law is based on defining of the right side (12) to be equal to $[0 \ 0 \ v]^T$ where v is the input of the transformed linear system.

$$u(t) = \frac{-L_f^3 h(x) + v}{L_g L_f^2 h(x)} = \frac{\frac{2k}{m} \left(\frac{x_2 x_3^2}{x_1^3} \right) + \frac{2kR}{mL} \left(\frac{x_3^2}{x_1^2} \right) - v}{\frac{2k}{mL} \left(\frac{x_3}{x_1^2} \right)} \quad (13)$$

v can be considered as a linear control signal used to improve the performance of the linearized system.

Under input-output linearizing control (13) the nonlinear system is linearized and has the expression:

$$\dot{z} = \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \quad (14)$$

Equation (14) is in a special format called Brunovsky form. The last row is of special interest because it is the nonlinear feedback controller.

To make the linearized system stable and follow a required trajectory, a linear controller must be designed. In next section, a linear quadratic regulator is designed to stabilize the Brunovsky system.

The transition characteristics of the dynamic output behaviour of the closed-loop system are given below in Table 1.

Table 1 summarizes the simulation results obtained on Simulink environment.

Table 1: Simulink results

Set point	Characteristics	Magnetic levitation Simulink results
0.75m	Time Delay	0
	Overshoot	0
	Rising Time	2s
	Steady State Error	0.002m
	Settling Time	2.2s
0.85m	Time Delay	0
	Overshoot	0
	Rising Time	1.8s
	Steady State Error	0.001m
	Settling Time	2s

6. REAL-TIME IMPLEMENTATION OF THE CLOSED-LOOP SYSTEM

Beckhoff PLC has developed a model transformation software that allows full transformation from Simulink to partial IEC 61499 function blocks model. The code generation to move the closed-loop system model from a Simulink program to a TwinCAT 3 one is as follow [14]:

- Design and build the system model on Simulink
- Automatic generate of C/C++ code by the Simulink Coder of the developed model
- Compilation with Visual Studio C Compiler
- Parameterize the generated code in the TwinCAT System Manager
- Download and extend the developed model in the TwinCAT 3 runtime.

Figure 6 shows the code generation process [15].

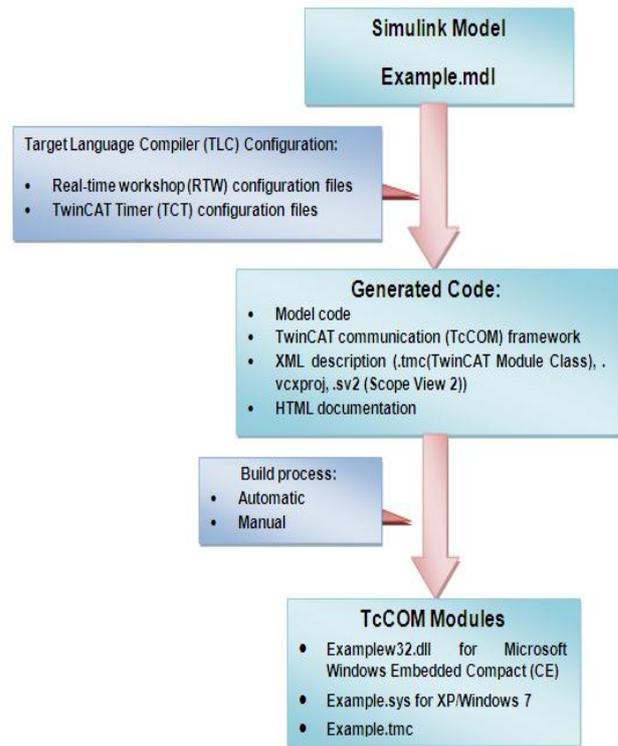


Figure 6: Code Generation process [15]

The CX5020 serves as a real-time platform to execute the application developed in TwinCAT3. The most important factor to link the development Personal Computer (PC) and the PLC is the communication driver module. Ethernet is used as a mean of communication to link the personal computer used as development platform and the PLC via fast duplex communication. Ethernet module scans the PLC continuously for values of the input variables and writes them in an appropriate database in TwinCAT 3. Figure 7 shows the closed-loop algorithm for real-time communication.

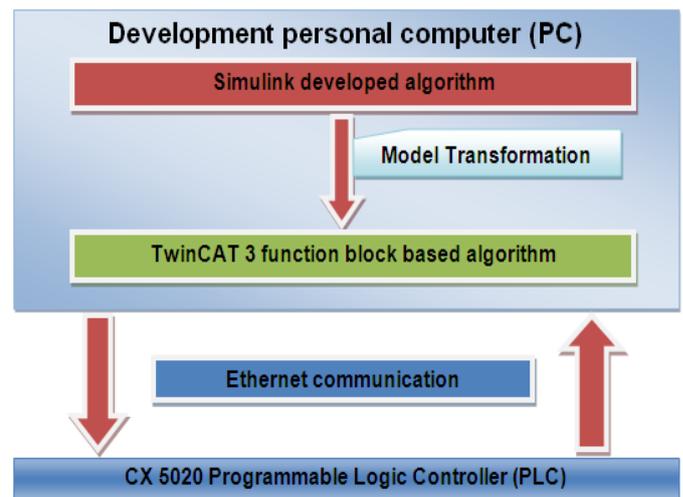


Figure 7: Closed-loop algorithm for real-time communication

The transformation software automatically translates each state flow block into customized basic function block with exactly: the same inputs, outputs and parameters as their Simulink counter parts as shown in figure 8. The amount of time for performing model transformation depends on the complexity of the model in Simulink [3]. The transformation technique has some limitations such as:

- Function Blocks are not event driven.
- The Simulink solver type is set at fixed step instead of variable step.

The real-time simulation has a set of priority of 5 and a 10ms cycle time, figures 9 and 10 show the response of the system in the PLC CX 5020.

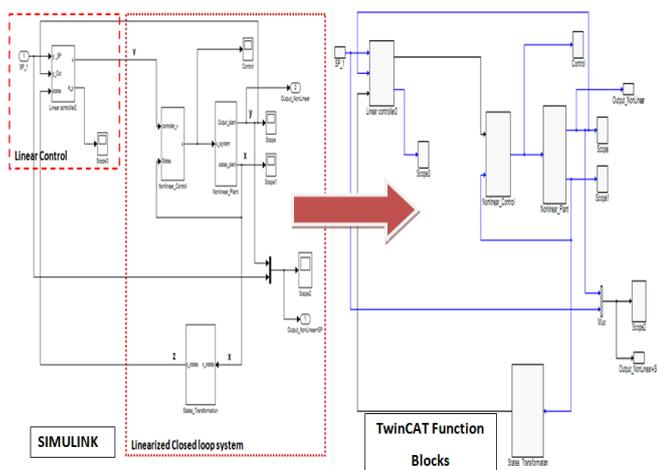


Figure 8: Model transformation from Simulink to TwinCAT PLC function blocks.

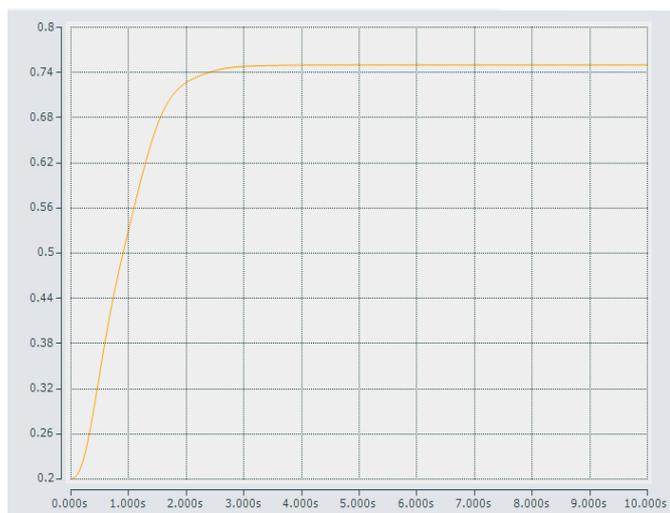


Figure 9: Magnetic levitation real-time position when the set point is $y^{sp} = 0.75[m]$

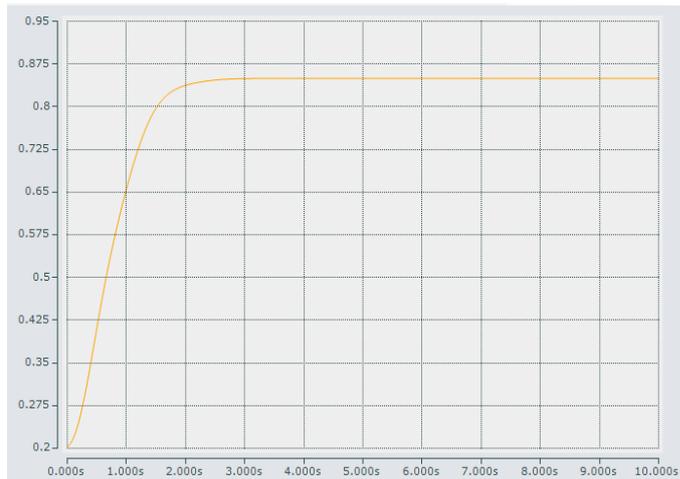


Figure 10: Magnetic levitation real-time position when the set point is $y^{sp} = 0.85[m]$

The comparison of the dynamic output behaviors of the closed-loop systems in Simulink with the PLC real-time results are given below in Table 2.

Table 2: Simulink versus real-time results comparison

Set point	Characteristics	Magnetic levitation Simulink results	Magnetic levitation Real-time results
0.75m	Time Delay	0	0
	Overshoot	0	0
	Rising Time	2s	3s
	Steady State Error	0.002m	0
	Settling Time	2.2s	3.6s
0.85m	Time Delay	0	0
	Overshoot	0	0
	Rising Time	1.8s	2.8s
	Steady State Error	0.001m	0
	Settling Time	2s	3s

Table 2 shows that the magnetic levitation system behaves within the same time delay, overshoot, rising time, steady state error, and settling time in both real-time and Simulink environments. These results prove that the model

transformation technique was successful to transform a closed-loop magnetic levitation system based on input-state feedback linearization from Simulink to function block-based IEC 61499 standard real-time environment.

7. REAL-TIME IMPLEMENTATION AND SIMULATION OF A DISTRIBUTED SYSTEM OF TWO MAGNETIC LEVITATION SYSTEMS

A distributed system of parallel distributed control of two levitation systems based on the same controllers and plants parameters but with different initial conditions and set-points is developed. The following assumptions are made:

- Controller 1 and controller 2 are the combination of Integral Linear Quadratic Regulator (ILQR) and nonlinear input-state/input-output feedback controllers. Both have the same parameters.
- MagLev 1 and MagLev 2 have different initial conditions.
- The set-point 1 and the set-point 2 are different.

The two closed-loop models of the magnetic levitation system controlled by feedback input-state linearization technique are implemented in the CX5020 PLC to achieve real-time distributed control. The set-points and initial conditions are different for both systems, and an algorithm to select the appropriate set-point for each system is developed. Figure 11 shows the block diagram of the distributed control methodology applied.

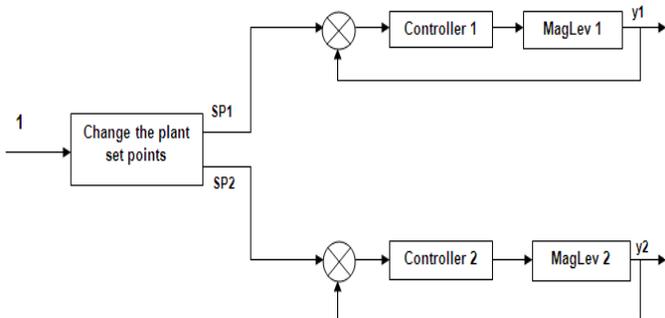


Figure 11: Block diagram of the distributed solution

Table 3 gives the parameters of the simulation of both systems, and Figure 12 shows the function block algorithm developed for the distributed implementation.

Table 3: Parameters of simulation for the distributed system

Magnetic levitation 1		Magnetic levitation 2	
Set points	Initial conditions	Set points	Initial conditions
1.1[m]	[0.15 0 0.8 0]'	1.24 [m]	[0.2 0 0.86 0]'
1.15[m]	[0.2 0 0.86 0]'	1.45[m]	[0.25 0 0.9 0]'
1.2[m]	[0.4 0 0.96 0]'	1.4[m]	[0.45 0 0.97 0]'

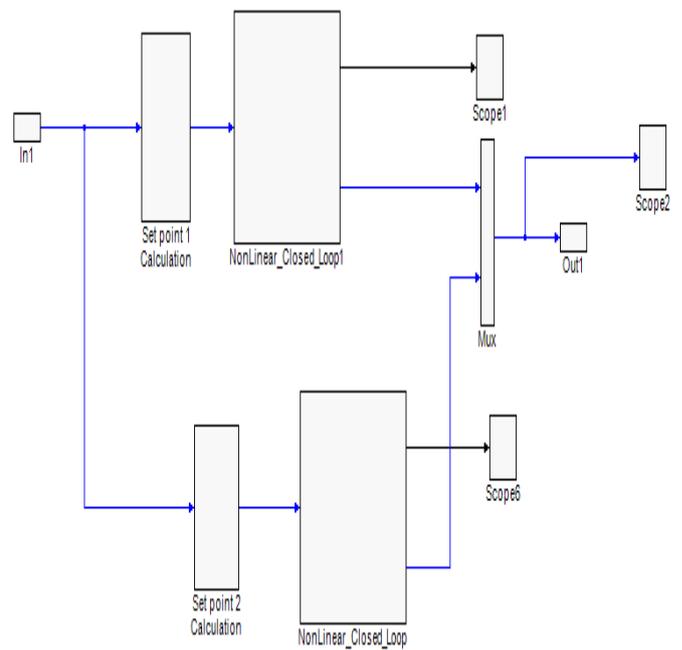


Figure 12: Block diagram algorithm developed on Beckhoff CX5020 for the distributed implementation

The results of the real-time simulation of the distributed system are shown from Figure 13 to Figure 15.

- Simulation when the set-points are: $y_{Maglev1}^{SP1} = 1.1[m]$ and $y_{Maglev2}^{SP1} = 1.24[m]$:

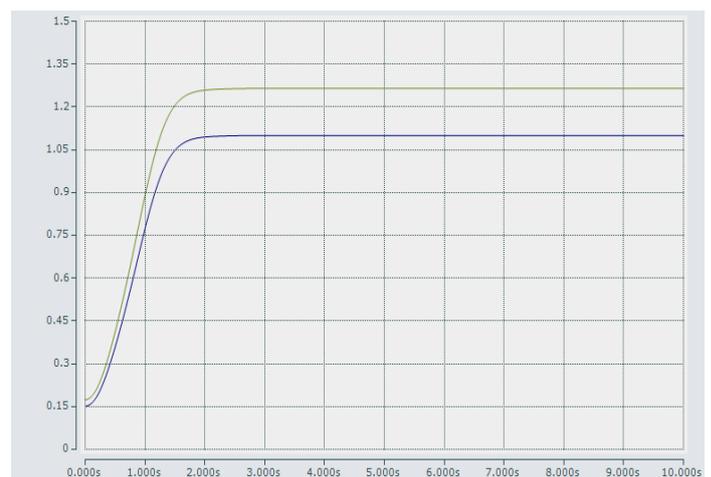


Figure 13: Simulation of the distributed system when the set-points are: $y_{Maglev1}^{SP1} = 1.1[m]$ and $y_{Maglev2}^{SP1} = 1.24[m]$

- Simulation when the set-points are: $y_{Maglev1}^{SP2} = 1.15[m]$ and $y_{Maglev2}^{SP2} = 1.45[m]$:

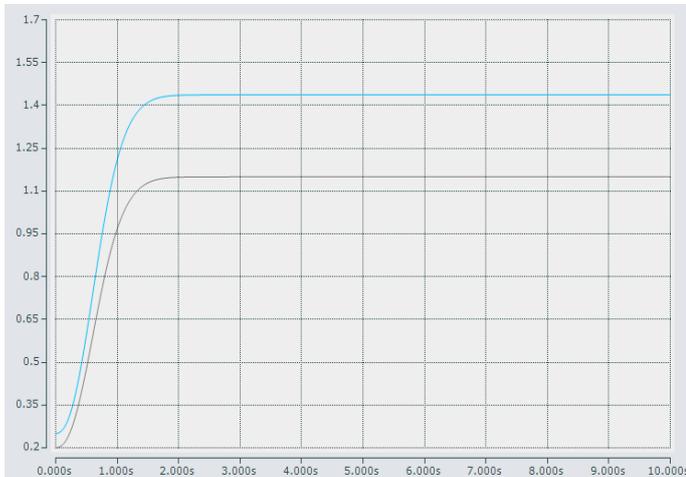


Figure 14: Simulation of the distributed system when the set-points are: $y_{Maglev1}^{SP2} = 1.15[m]$ and $y_{Maglev2}^{SP2} = 1.45[m]$

- Simulation when the set-points are: $y_{Maglev1}^{SP3} = 1.2[m]$ and $y_{Maglev2}^{SP3} = 1.4[m]$:



Figure 15: Simulation of the distributed system when the set-points are: $y_{Maglev1}^{SP3} = 1.2[m]$ and $y_{Maglev2}^{SP3} = 1.4[m]$

Table 4: Results of the simulation of the real-time distributed control

Set point	Characteristics	Magnetic levitation Real-time results
1.1m	Time Delay	0
	Overshoot	0
	Rising Time	2s
	Steady State Error	0.0014m
	Settling Time	2.2s
1.15m	Time Delay	0
	Overshoot	0
	Rising Time	1.6s
	Steady State Error	0.0085m
	Settling Time	2s
1.2m	Time Delay	0
	Overshoot	0
	Rising Time	1.1s
	Steady State Error	0.0001m
	Settling Time	1.5s
1.24m	Time Delay	0
	Overshoot	0
	Rising Time	1.8s
	Steady State Error	0.0005m
	Settling Time	2.2s
1.4m	Time Delay	0
	Overshoot	0
	Rising Time	1.05s
	Steady State Error	0.0009m
	Settling Time	1.2s
1.45m	Time Delay	0
	Overshoot	0
	Rising Time	1.75s
	Steady State Error	0.0013m
	Settling Time	2s

The simulation of the distributed implementation of the magnetic levitation systems shows satisfactory values of the trajectories specifications in Table 4. These results prove that it is possible to implement distributed control algorithms in PLC.

8. CONCLUSION

In this paper, exact input-state/input-output linearization and linear quadratic regulation of a nonlinear magnetic levitation system are proposed. The control algorithm and model of the

Table 4 gives the real-time trajectories specifications of the distributed system of the two closed-loop models of the magnetic levitation system.

plant are both implemented in MATLAB/Simulink. The model of the closed loop system was then transformed into TwinCAT3 function blocks language to perform real-time simulation with a Beckhoff CX5020 PLC. The output signal tracks the reference input; therefore, this nonlinear approach successfully controls the magnetic levitation system. These results show that nonlinear control algorithms can be implemented into modern PLC.

The future research is seen to be useful in the following directions:

- The function blocks in TwinCAT 3 can be improved to be event driven.
- A better algorithm translation tool can be developed to transform Matlab/Simulink code into language supported by the TwinCAT 3 function block development software.
- Model transformation technique can be tested with more industrial examples to boost the acceptance of the IEC 61499 standard in industry.
- Development of a hybrid system for distributed control and data exchange based on joint implementation of IEC 61499 and IEC 61850 standards respectively.

REFERENCES

- [1] Krener, A. and Respondek, W. 1985. "Nonlinear Observers with Linearizable Error Dynamics," *Siam J. Control and Optimization*, Vol. 23, No. 2, pp. 197-216.
- [2] Calvet, J. and Arkun, Y. 1988. Feedforward and feedback linearization of nonlinear systems and its implementation using internal model control (IMC). *Ind. Eng. Chem. Res.* 27, 1822.
- [3] Yang, C. and Vyatkin, V. 2009. "Automated Model Transformation between MATLAB Simulink/Stateflow and Function Blocks," in *INCOM*, Vol. 13, pp. 205-210.
- [4] Wong, T. 1986. Design of a magnetic levitation system: an undergraduate project. *IEEE Transactions on Education*, 29, 196-200.
- [5] Barie, W. and Chiasson, J. 1996 "Linear and Nonlinear state-space controllers of magnetic levitation," *International Journal of Systems Science*, vol. 27, number 11, pp. 1153-1163.
- [6] Zhang, F. and Suyama, K. 1995. "Nonlinear Feedback Control of Magnetic Levitating System by Exact Linearization Approach," Tokyo University of Mercantile Marine, Japan, *Proc. IEEE Conf. Contr. Appl.*, pp. 267-268.
- [7] Filho, M. and Munaro, E. 2001 "A Design Methodology of Tracking Controllers for Magnetic Levitation Systems," *Proceeding of the 2001 IEEE International Conference and Systems*, E84D (10):1398-1401.
- [8] Kuo, B. C., & Golnaraghi, M. F. (2003). *Automatic control systems* (8th ed.). New York: Kundli: John Wiley & Sons.
- [9] CE 152 Magnetic levitation model - user's manual, Humusoft s.r.o., Prague, Czech Republic, 2002.
- [10] Isidori and Ruberti, 1984. On the synthesis of linear input-output responses for nonlinear systems *Syst. Contr. Lett.*, 4, p. 17
- [11] Han, Y. 2009. "Development of nonlinear reconfigurable control of reconfigurable plants using the FPGA technology," Master Thesis, Cape Peninsula University of Technology.
- [12] Hedrick, J. K. and Girard, A. 2005 Control of Nonlinear Dynamic System: Theory and Applications. Columbia University Fall 2005.
- [13] Ogata, K. 2002. Modern Control Engineering Fourth Edition. Upper Saddle River, New Jersey, Prentice Hall.
- [14] Beckhoff et. al, "Getting started Welcome to TwinCAT 3." [Online]. Available: ftp://ftp.beckhoff.com/Software/TwinCAT/TwinCAT3/FirstSteps/TC3_Mini_Booklet_GB.PDF. [Accessed: 05-Jun-2017].
- [15] Mfoumboulou, Y. D. 2014. "Development of nonlinear control algorithms for implementation in distributed system," Master Thesis, Cape Peninsula University of Technology.
- [16] Pradhan, S. K. and Subudhi. B, " Nonlinear Control of a Magnetic Levitation System Using a New Input-Output Feedback Linearization ", *IFAC-Papers Online* 49-1 (2016) 332– 336.
- [17] Yadav, et al., 2015. "Design and Simulation of Nonlinear Control System for Magnetic Levitation of Steel Ball," *International Research Journal of Engineering and Technology*, Volume: 02, Issue: 08, November.
- [18] Zhang, Z.; Li, X. Real-Time Adaptive Control of a Magnetic Levitation System with a Large Range of Load Disturbance. *Sensors* 2018, 18, 1512.
- [19] Lafta, E. J. Alkurawy, 2019. Simulation of Robust Control of Magnetic Levitation System. *Journal of Engineering and Applied Sciences*, 14: 4523-4531.
- [20] Slotine J. 1991. Applied Nonlinear Control, Prentice Hall, New Jersey.