International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 11 (2020), pp. 3287-3291
 https://dx.doi.org/10.37624/IJERT/13.11.2020.3287-3291

A Study on Comparative Evaluation of Software Reliability Model using Exponential-exponential and Burr-Hatke-exponential Life Distribution

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Abstract

In this study, the characteristics of reliability model of software using the non-homogeneous Poisson process with a finite failure were applied. The life distributions were presented the exponential-exponential distribution and Burr–Hatke-exponential model widely used in the software field. The results in this study, all the comparison models have the non-decreasing form in the mean value function for the pattern comparison. And, in terms of the forecast width difference of the mean value estimation about the true pattern, Burr-Hatke-exponential distribution model than the exponential-exponential distribution model can be seen that have the smallest characteristic in term of the comparison. Thus Burr-Hatke-exponential distribution model than the exponential-exponential distribution model in the comparison can be regarded as an efficient model in terms of the relative accuracy. Because of the mean square error estimation value is also smaller the case of the exponential-exponential distribution model than Burr–Hatke-exponential model, the exponential-exponential distribution model than Burr–Hatke-exponential model can be considered as an efficient model. In the case of the determination coefficient estimation value is also larger Burr–Hatke-exponential model than the exponential-exponential distribution model, so Burr–Hatke-exponential model can be considered as an efficient model in terms of goodness-of-fit. In terms of the reliability function, gradually seems as a non-increasing pattern as the mission time passes. Therefore, in terms of the reliability function, gradually seems as a non-increasing pattern as the mission time elapses. Therefore, in terms of the reliability, the exponential-exponential higher model shows than the Burr–Hatke--exponential model. From this study, the software operators identified the types of software failures using the life distribution effect feature for the exponential-exponential distribution and the Burr–Hatke-exponential distribution. Thus, software developers can be used as a basic guideline to identify depending on the life distribution following the exponential-exponential distribution and Burr–Hatke-exponential model and to investigate the causes of software failure transition.

Keywords: Non-homogeneous Poisson process, Exponential-exponential distribution, Burr-Hatke-exponential, Mission time

I. INTRODUCTION

Software reliability systems can be look upon as basic elements of significant intellectual issues of industrial management. Such a system can offer high-excellence package to software users if it can accomplish correctness and reliability. Therefore, software maintenance is to support the accuracy and trustworthiness of the software system. The software reliability growth model, which is an engineering study connected to such maintenance, has been proposed. These studies were advanced a software reliability model [1] that follows the non-homogeneous Poisson process (NHPP), which predicts the failure intensity function and the mean value function using the reliability attribute factors such as the number of remaining failures and the total cost of software development. Under these situations, Yamada and Osaki [2] was accentuated that the patterns of the mean value function can be foreseen using the maximum likelihood estimation method. Also, and the reliability appearances of the mean value function were obtainable by explaining the graph of the confidence interval for the mean value function [1, 3]. The defect detection rate was developed using the exponential distribution, which is the basic model in this field, consists of the intensity function which is a constant (hazard function) [4, 5]. Kim [6] also was studied the reliability of life distribution using Burr-XII and Type-2 Gumbel distributions. In addition, Kim [7] was presented the study on comparative evaluation of software reliability model applying modified exponential distribution.

In this study, the reliability model characteristics of the software using the non-homogeneous Poisson process with finite failure was analysed. The life distributions were presented the exponential-exponential distribution and Burr–Hatke-exponential model widely used in the software field.

II. RELATED RESEARCH

II.I Exponential-exponential distribution

Among the models widely used in the field of software reliability, a special form of the Weibull exponential distribution is the exponential-exponential distribution. The probability density function (PDF) and the cumulative distribution function (CDF) of the distribution are as follows [8].

$$f(t|a,b) = ab\,exp\,(bt - ae^{bt} + a) \tag{1}$$

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$$F(t|a,b) = 1 - exp(-ae^{bt} + a) \tag{2}$$

Note that $t \in (0,\infty]$ and $a > 0$ is the shape parameter and $b > 0$ is the scale parameter. In finite failure NHPP model, θ was the specified expected value of faults that would be discovered observing time $(0,t]$. Thus, the intensity function and the mean value function of NHPP can be detailed as follows [5].

$$\lambda(t|\theta,a,b) = \theta f(t) = \theta[ab\, exp\ (bt - ae^{bt} + a)]) \tag{3}$$

$$m(t|\theta,a,b) = \theta F(t) = \theta\left[1 - exp\ (-ae^{-\lambda t} + a\right] \tag{4}$$

In Equation (3) and (4), x_n is replaced with the last failure time point and the log-likelihood function can be detailed as follows [9].

$$\begin{aligned} lnL_{NHPP}\ (\Theta|\underline{x}) &= -m(x_n) + (\textstyle\sum_{i=1}^{n} ln\, \lambda(x_i)) \\ &-\theta[1 - exp(-ae^{bx_n} + a)] \\ &+ \textstyle\sum_{i=1}^{n} ln\, [\theta(ab\, exp(bx_i - ae^{bx_i} + a))] \end{aligned} \tag{5}$$

Note that $\underline{x} = (x_1 \le x_2 \le x_3 \le \cdots \le x_n)$ and $\Theta = \{\theta, a, b\}$ is the parameter space.
When the shape parameter a is fixed, the estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ must be assessed the following construction using the equation (5).

$$\frac{\partial\, ln\, L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\theta} - [1 - exp\ (-ae^{bx_n} + a)] = 0) \tag{6}$$

$$\begin{aligned} \frac{\partial\, ln\, L_{NHPP}\ (\Theta|\underline{x})}{\partial b} &= \frac{n}{b} + \sum_{i=1}^{n} x_i - a\sum_{i=1}^{n} x_i e^{bx_i} \\ &- \theta a x_n\, exp(\, bx_n - ax_n e^{bx_n} + a) = 0 \end{aligned} \tag{7}$$

II. II Burr-Hatke-exponential distribution model

The probability density function and cumulative distribution function of this distribution of the Burr-Hatke-exponential distribution are defined as follows [10, 11].

$$F(t|b) = 1 - \frac{e^{-bt}}{1+bt}, f(t|b) = be^{-bt}\frac{2+bt}{(1+bt)^2} \tag{8}$$

Note that$t \in (0,\infty]$ and $b > 0$is the shape parameter. In finite failure NHPP model, $\theta$was specified the expected value of faults that would be discovered observing time$(0,t]$. Thus, the intensity function and the mean value function of NHPP are known as follows [7].

$$\lambda(t|\theta,b) = \theta f(t) = \theta b e^{-bt}\frac{2+bt}{(1+bt)^2} \tag{9}$$

$$m(t|\theta,b) = \theta F(t) = \theta\left[1 - \frac{e^{-bt}}{1+bt}\right] \tag{10}$$

The log-likelihood function by means of the Equation (9) and (10) can be detailed ensuing relation [2, 7].

$$\begin{aligned} lnL_{NHPP}\ (\Theta|\underline{x}) &= -m(x_n) + (\textstyle\sum_{i=1}^{n} ln\, \lambda(x_i)) \\ &= -\theta\left(1 - \frac{e^{-bx_n}}{1+bx_n}\right) + nln\, \theta + nlnb - b\textstyle\sum_{i=1}^{n} x_i \\ &+ \textstyle\sum_{i=1}^{n} ln(2+x_i) - 2\sum_{i=1}^{n} ln(1+bx_i) \end{aligned} \tag{11}$$

Note that $\underline{x} = (x_1 \le x_2 \le x_3 \le \cdots \le x_n)$ and $\Theta = \{\theta, b\}$ is the parameter space. The estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ must be assessed the following construction for the maximum likelihood estimation about all parameter using the Equation (11).

$$\frac{\partial\, ln\, L_{NHPP}\ (\Theta|\underline{x})}{\partial\theta} = \frac{n}{\theta} - \left(1 - \frac{e^{-bx_n}}{1+bx_n}\right) = 0 \tag{12}$$

$$\begin{aligned} \frac{\partial\, ln\, L_{NHPP}\ (\Theta|\underline{x})}{\partial b} &= \frac{n}{b} - \sum_{i=1}^{n} x_n + \sum_{i=1}^{n}\frac{x_i}{2+bx_i} \\ &- 2\sum_{i=1}^{n}\frac{x_i}{1+bx_i} - \theta x_n e^{-bx_n}\frac{(2+bx_i)}{(1+bx_i)^2} = 0 \end{aligned} \tag{13}$$

II. III The hazard functions

The hazard function, which means the instantaneous failure rate for the specified failure time, was defined [5, 13]. Thus, the hazard function of the exponential-exponential distribution is derived as follows using Equations (1) and (2) [7, 12].

$$h(t) = \frac{f(t)}{1-F(t)} = abe^{bt} \tag{14}$$

The Burr-Hatke-exponential distribution in which these patterns of hazard functions display an increasing pattern or a decreasing pattern as follows using Equations (8) [10]. Thus, the hazard function can be detailed as follows.

$$h(t) = \frac{b(2+bt)}{1+bt} \tag{15}$$

III. SOFTWARE FAILURE TIME RELIABILITY ANALYSIS

In this chapter, we were used software failure time data [7, 13] to compare and analyze the reliability characteristics of reliability models that exponential-exponential distribution and Burr-Hatke-exponential distribution model. Software failure time data are summarized in Table 1 and Box-plot was used in this study for the trend test in order to detect the presence of extreme values [7, 12]. That is, in the result of Figure 1, the median value is 144.015, the first quartile is 80.9, and the third quartile is 277.87, so the upper and lower limits of the box plot are calculated as follows.

$$277.87 + 1.5 \times (277.87 - 80.89) = 573.325) \tag{15}$$

$$80.89 - 1.5 \times (277.87 - 80.89) = -214.555 \tag{16}$$

Therefore, this data is life data, so the lower limit is meaningless, but the upper limit is 573.325, so three data (28,

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29, 30th) outside the upper limit were considered as outliers (extreme values) and were excluded from the parameter estimation [7].

Table 1. Failure time data

Failure Number	*Failure Time (hours)*	*Failure Time* $\times 10^{-2}$	*Failure Number*	*Failure Time (hours)*	*Failure Time* $\times 10^{-2}$
1	30.02	0.3002	16	151.78	1.5178
2	31.46	0.3146	17	177.5	1.775
3	53.93	0.5393	18	180.29	1.8029
4	55.29	0.5529	19	182.21	1.8221
5	58.72	0.5872	20	186.34	1.8634
6	71.92	0.7192	21	256.81	2.5681
7	77.07	0.7707	22	273.88	2.7388
8	80.9	0.809	23	277.87	2.7787
9	101.9	1.019	24	453.93	4.5393
10	114.87	1.1487	25	535	5.35
11	115.34	1.1534	26	537.27	5.3727
12	121.57	1.2157	27	552.9	5.529
13	124.97	1.2497	**28**	**673.68**	**6.7368**
14	134.07	1.3407	**29**	**704.49**	**7.0449**
15	136.25	1.3625	**30**	**738.68**	**7.3868**

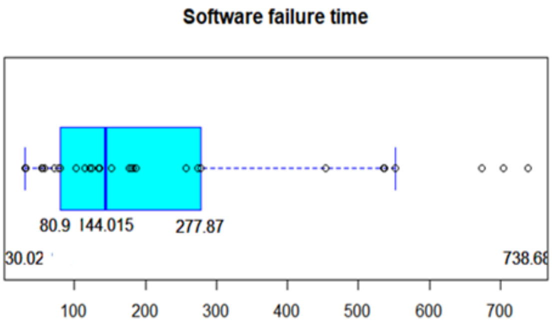


Fig. 1. Box plot test

Table 2. Basic statistics

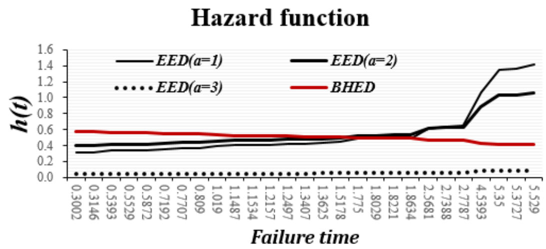
Mean	*0.1879*	*Skewness*	*1.4281*
Median	*0.1341*	*Range*	*0.5229*
standard deviation	*0.1568*	*Minimum*	*0.03*
Variance	*0.0246*	*Maximum*	*0.5529*
Kurtosis	*1.0371*	*Observed Number*	*27*

In addition, the maximum likelihood estimation method is applied to estimate the parameters by a mathematical translation data (Failure Time $\times 10^{-2}$) to facilitate the parameter estimation. Basic statistical results for these data are summarized in Table 2. The result of parameter estimation was summarized in Table 3. The pattern of the hazard function using the parameter estimation results in Table 3 is summarized in Figure 2. In this Figure, the pattern of the exponential-exponential distribution model non-decreasing pattern of the hazard function, but Burr-Hatke-exponential distribution model shows non-increasing pattern.

Table 3. Parameter estimation

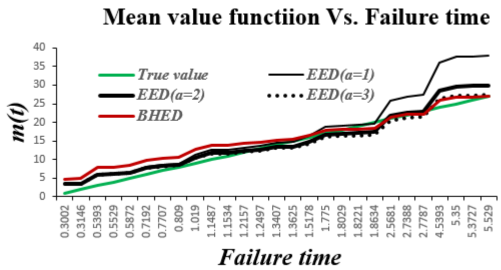
Model		MLE	*Model Comparison*	
			MSE	R^2
BHED		$\hat{\theta}_{MLE} = 29.0996$ $\hat{b}_{MLE} = 0.2991$	6.4349	0.9749
EED	$a = 1$	$\hat{\theta}_{MLE} = 38.5943$ $\hat{b}_{MLE} = 0.2885$	28.0709	0.5887
	$a = 2$	$\hat{\theta}_{MLE} = 30.6612$ $\hat{b}_{MLE} = 0.1879$	4.7840	0.9299
	$a = 3$	$\hat{\theta}_{MLE} = 28.2757$ $\hat{b}_{MLE} = 0.1397$	3.1607	0.9537

Note. BHED: *Burr-Hatke-exponential distribution model. EED: Exponential-exponential distribution. MLE: Maximum likelihood estimation. MSE : Mean square error.* R^2 *: Coefficient of determination.*



Note. *BHED: Burr-Hatke-exponential distribution model.*
EED: Exponential-exponential distribution.
a: Shape parameter.

Fig. 2. Hazard function for each model



Note. *BHED: Burr-Hatke-exponential distribution model.*
EED: Exponential-exponential distribution.
a: Shape parameter.

Fig. 3. Mean value function for each model

In Figure 3, all the comparison models are non-decreasing form in the mean value function pattern comparison, but Burr-Hatke-exponential distribution model than the exponential-exponential distribution model can be seen that the forecast difference for the width is the smallest. Thus Burr-Hatke-exponential distribution model than the exponential-exponential distribution model in the comparison can be regarded as an efficient model in terms of relative accuracy.

In addition, in terms of relative accuracy, the exponential-exponential distribution model for the case of the larger shape parameter(a=3) than the small shape parameter (a=1 and a=2) is the smallest error in forecasting value, in comparison from the true value, so it can be regarded as an efficient model in terms of a measure of accuracy. In addition, the statistics of the mean square error (MSE) [14, 15], which represent a measure of the forecast difference between the actual value and the forecasting value, are as follows.

$$MSE = \frac{\sum_{i=1}^{n}[m(x_i) - \hat{m}(x_i)]^2}{n-k} \tag{17}$$

Note that $m(x_i)$ is the cumulated true number of the faults perceived in $(0, x_i]$ and $\hat{m}(x_i)$ estimated number of the faults detected in$(0, x_i]$, n states the number of the realizing values and k is the number of the parameter. In Table 2, because the mean square error of the shape parameter $a = 2$ and $a = 3$ of the exponential-exponential distribution model is smaller than Burr–Hatke-exponential models, thus, the exponential distribution model than Burr–Hatke-exponential model is efficient. Specifically, because the mean square error of Burr–Hatke-exponential model is smaller than the shape parameter a=1 of the exponential-exponential distribution model small. Thus, the case of the shape parameter a=1 of the exponential-exponential distribution model, Burr–Hatke-exponential model than the exponential-exponential distribution model (a=1) is efficient. In addition, the coefficient of determination (R^2) is defined as an explanatory tools to explain the number of failures as the forecasting value. Thus, the model with a large coefficient of determination is stared as an efficient model in terms of the goodness-of-fit [13, 14]. The coefficient of determination (R^2) is as follows.

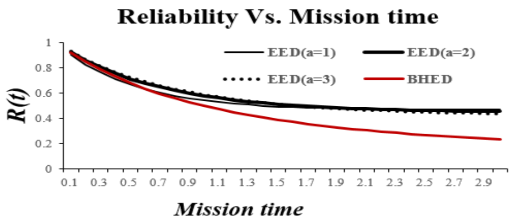
$$R^2 = 1 - \frac{\sum_{i=1}^{n}[m(x_i) - \hat{m}(x_i)]^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \tag{18}$$

Therefore, in Table 3, the estimated coefficient of determination is also larger when Burr–Hatke-exponential model than the exponential-exponential distribution model.

Thus, Burr–Hatke-exponential model than the exponential-exponential distribution model is stared as an efficient model in terms of goodness-of-fit [13, 14]. However, since all the proposed models have more than 50%, all models are judged to be efficient models [7, 14].

In the NHPP model, the software failure happens at last test failure time $x_{27} = 0.5529$ and reliability which is the probability that the software failure does not occur between 0.529 and $0.529 + t$ (where, t is the mission time) can be specified using the following construction [7, 14].

$$\hat{R}(t|x_{27} = 5.529) = e^{-\int_{5.529}^{5.529+t} \lambda(\eta)d\eta} \\ = exp[-\{m(t+5.529) - m(5.529)\}] \tag{19}$$



Note. BHED*: Burr-Hatke-exponential distribution model.*
EED*: Exponential-exponential distribution.*
a: Shape parameter.

Fig. 4. Transition of the reliability pattern

In the form of the reliability function in Figure 4 using the Equation (19), gradually seems as a non-increasing pattern as the mission time elapses. In terms of the reliability, the exponential-exponential higher than the Burr–Hatke-index distribution model shows.

V. CONCLUSION

In this study, the reliability model characteristics of the software using the non-homogeneous Poisson process with finite failure was analysed. The life distributions were presented the exponential-exponential distribution and Burr–Hatke-exponential model widely used in the software field.

By quantitatively modelling the trends of failures during the software development process or during the actual software operation, the efficiency and reliability can be evaluated by comparing and analysing the accuracy and reliability of the software. The reality is that defects can hardly be avoided in the course of modifications and changes made by large software.

The results of this study can be summarized as follows.

First, in the hazard function, which means the instantaneous failure rate for the specified failure time, the pattern of the exponential-exponential distribution model shows non-decreasing pattern, but Burr-Hatke-exponential distribution model shows non-increasing pattern.

Second, all the comparison models are non-decreasing form in the mean value function pattern comparison, but the exponential-exponential distribution model than Burr-Hatke-exponential distribution model can be seen that the forecast difference for the width is the smallest. Thus, the exponential-exponential distribution model than Burr-Hatke-exponential distribution model in the comparison can be regarded as an efficient model in terms of the relative accuracy.

Third, the mean square error estimation value is smaller in the case of the exponential-exponential distribution model (except when the shape parameter is 1) than Burr–Hatke-exponential model. Thus, the exponential-exponential distribution model than Burr–Hatke-exponential model can be considered as an efficient model.

Fourth, in the case of the determination coefficient estimation value is also larger Burr–Hatke-exponential model than the exponential-exponential distribution model, so Burr–Hatke-exponential model can be considered as an efficient model in terms of the goodness-of-fit. However, since all the proposed models have more than 50%, all models are judged to be efficient models.

Fifth, in the form of the reliability function, gradually seems as a non-increasing pattern as the mission time elapses. Therefore, in terms of reliability, the exponential-exponential model than the Burr–Hatke-exponential model shows higher.

Through this study, the software operators identified the types of software failures using the life distribution effect feature for the exponential-exponential distribution and Burr–Hatke-exponential distribution. Thus, software developers can be used as a basic guideline to identify depending on the life distribution following the exponential-exponential distribution and Burr–Hatke-exponential model and to investigate the causes of software failure transition.

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