

Probability of Position and Motion Parameters Estimation for a Radio Beacon in Passive Search and Rescue Systems

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Abstract

In this article, the method to estimate the range to the beacon in passive search and rescue systems is proposed. The range to the beacon is determined by applying a set of Kalman filters to the bearing measurements with different hypotheses of the beacon motion and subsequent selection of the hypothesis with the minimum sum of residual errors in target range estimation. The expressions for the expectation and standard deviation of the prediction errors are derived and the probability of selection the correct hypothesis is given. The proposed method allows to evaluate the measurement time, required to determine the range to the beacon with probability 0.9...0.95, which agrees well with statistical analysis of multiple data sets.

Keywords: (Multiple) Hypotheses testing, Kalman filter, Probability, Passive radar, Ranging.

1. INTRODUCTION

Recent developments in passive or semi-active radars have expanded their application to target location along with active radars. The modern element base has significantly improved the accuracy of determining both the direction of the beacon and improve the accuracy of determining the characteristics of its radiation (carrier frequency, duration and muls and its period). The computing power of the equipment has also increased.

Improving the accuracy of determining the radio characteristics of the signal allows you to perform the identification of the signal at two points of reception with a higher probability, which gave more development for the methods of determining the location of the target by existing methods requires either two passive radars placed at different

positions [1; Kevin, et. El Republic., 2010; 23; 3; 26; 9]. Improved performance of processors and integrated circuits made it possible to perform more complex signal processing, which made it possible to perform target detection by an external signal (GSM or broadcast signal) [2; 7; 16; 9; 15; 18]. However, if the target emits, it is possible to determine its range by measuring the target bearing and its radiation parameters using only one mobile passive locator [10; 4; 22].

The determination of the range must be carried out with some specified accuracy. In these methods, acceptable location accuracy is not achieved after one measurement, but after a while, and is therefore unknown. As data accumulate, the accuracy of the range measurement will increase, and there is a need to determine at what point in time the estimated target location is accurate with a given accuracy.

This article displays the probability of the correct location of the target (emitting beacon) using the algorithm described below [8; 6].

2. MODEL AND METHOD

To determine the location of a mobile beacon using only one mobile direction finder, the hypothesis testing algorithm has been proposed [12; 20]. The direction finder moves at speed V_1 and course Ψ_1 when the radiation of the beacon is detected. After the time T_{cm} the finder changes the course to Ψ_2 and continue its movement at speed V_2 [14; 13; 17]. Multilevel prognosis of logistics chains in case of uncertainty: Journal of Open Innovation: Technology, Market, and Complexity, 4(1) doi:10.1186/s40852-018-0081-8. The beacon during the measurement time moves rectilinearly with a constant speed. It is required to determine the range to the beacon with the required relative accuracy δD . The geometry of the problem is shown in Figure 1.

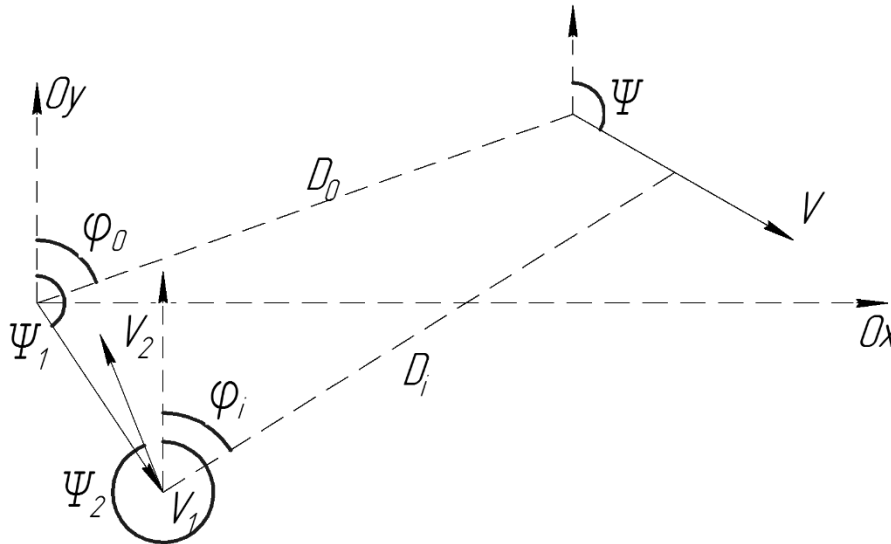


Figure 1: The geometry of the mutual motion of the direction finder and beacon

The notations in Figure 1 are the following: V_1, V_2 – the speed of the finder, Ψ_1, Ψ_2 – the course of the finder movement, V, Ψ – speed and heading of the beacon, D_0, φ_0 – range and bearing to the beacon at the moment of its detection, D_i, φ_i – the distance and bearing to the beacon at time i . Oy, Ox – axes of coordinates, the axe Oy directed to the North, the axe Ox – directed to the East.

To determine the range to the beacon, a set of one-dimensional Kalman filters is formed [24], each filter is determined by a set of hypotheses about the initial range to the

beacon, the speed and the course of its movement. Each Kalman filter is denoted by three indices abc (a – the index of initial range hypotheses, b – the index of speed hypotheses, c – the index of course hypotheses).

The Kalman filter with indices abc calculates:

- 1) Prediction of the beacon bearing:

$$\varphi_{i/i-1}^{abc} = \hat{\varphi}_{i-1}^{abc} + \frac{V^b \cdot dt \cdot \sin(\Psi^c - \hat{\varphi}_{i-1}^{abc}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \sin\left(\arg\left(\frac{\Delta x_i}{\Delta y_i}\right) - \hat{\varphi}_{i-1}^{abc}\right)}{\hat{D}_{i-1}^{abc} + V^b \cdot dt \cdot \cos(\Psi^c - \hat{\varphi}_{i-1}^{abc}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \cos\left(\arg\left(\frac{\Delta x_i}{\Delta y_i}\right) - \hat{\varphi}_{i-1}^{abc}\right)}$$

- 2) Predicted MSE error: $p_{i/i-1} = p_{i-1}$
- 3) Kalman Gain: $K_i = \frac{p_{i/i-1}}{p_{i/i-1} + \sigma_\varphi^2}$
- 4) Correction of the beacon bearing: $\hat{\varphi}_i^{abc} = \varphi_{i/i-1}^{abc} + K_i \cdot (\tilde{\varphi}_i - \varphi_{i/i-1}^{abc})$
- 5) MSE: $p_i = (1 - K_i) \cdot p_{i/i-1}$

V^b – the hypothesis on the beacon speed, Ψ^c – the hypothesis on the beacon course, $\hat{D}_0^{abc} = D^a$ – the hypothesis on the initial range to the beacon, V beacon speed, Ψ – the rate of motion of the beacon, dt – the time between bearing measurements, Δx_i – movement of the direction finder along the Ox axis during the time dt at step i , Δy_i – movement of the direction finder along the Oy axis during the time dt at step i .

For each filter, the sum of squared residual errors is calculated

$$S_i = \sum_{k=1}^i (\tilde{\varphi}_j - \hat{\varphi}_j)^2.$$

The estimation of the beacon range and motion parameters is obtained as the hypothesis, for which the corresponding Kalman has the minimal sum of squared residual errors.

2.1. Probability of correct range hypothesis selection with a given relative accuracy.

The parameter that determines the choice of the hypothesis with the current range to the target and the parameters of the target movement is the sum of squares of residuals S_i at time i . Thus, the probability of correct range hypothesis selection at step i of the algorithm is the probability that the sum of the squares of the residuals S_i^0 for the hypothesis with indices abc at the current range to the beacon with the sum of squared residual errors at the step i is less than the sum of squared residual errors S_i^1 for the hypothesis with indices $a'b'c'$. Here the hypothesis with indices $a'b'c'$ has the minimum sum of squared residual errors out of all the hypotheses with the current range estimation outside of the interval $[D_i^{abc} \cdot (1 - 2 \cdot \delta D); D_i^{abc} \cdot (1 + 2 \cdot \delta D)]$. The probability of correct range hypothesis selection becomes:

$$p(S_i^0 < S_i^1) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_0(x) \cdot dx \cdot f_1(h) \cdot dh = \int_{-\infty}^{+\infty} F_0(h) \cdot f_1(h) \cdot dh$$

$f_0(x)$ – probability density function (PDF) of the sum of squared residual errors S_i^0 , $f_1(h)$ – PDF of the sum of squared residual errors S_i^1 , $F_0(h)$ – cumulative density function (CDF) of the sum of squared residual errors S_i^0 .

Sum of squared residuals at the i -th step of the algorithm is

$$S_i = \sum_{k=1}^i (\tilde{\varphi}_k - \hat{\varphi}_k)^2 = \sum_{k=1}^i (\varphi_k + \Delta\tilde{\varphi}_k - \varphi_k - \Delta\hat{\varphi}_k)^2 = \sum_{k=1}^i (\Delta\tilde{\varphi}_k - \Delta\hat{\varphi}_k)^2$$

$$\hat{\varphi}_i = K_i \cdot \tilde{\varphi}_i + (1 - K_i) \cdot \hat{\varphi}_{i/i-1} = K_i \cdot \varphi_i + K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot (\hat{\varphi}_{i-1} + \hat{u}_i) =$$

$$= K_i \cdot \varphi_i + K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot (\varphi_{i-1} + \Delta\hat{\varphi}_{i-1} + \varphi_i - \varphi_{i-1} + \Delta u_i) =$$

$$= K_i \cdot \varphi_i + K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot \varphi_i + (1 - K_i) \cdot \Delta\hat{\varphi}_{i-1} + (1 - K_i) \cdot \Delta u_i =$$

$$= \varphi_i + K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot \Delta\hat{\varphi}_{i-1} + (1 - K_i) \cdot \Delta u_i$$

K_i – Kalman Gain, $\hat{\varphi}_{i/i-1} = \hat{\varphi}_{i-1} + \hat{u}_i$ – prediction of bearing to the beacon, $\hat{u}_i = \varphi_i - \varphi_{i-1} + \Delta u_i$ – control action, φ_i – the true value of bearing, Δu_i – prediction error.

Bearing estimation error is equal to:

$$\Delta\hat{\varphi}_i = \hat{\varphi}_i - \varphi_i = \varphi_i + K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot \Delta\hat{\varphi}_{i-1} + (1 - K_i) \cdot \Delta u_i - \varphi_i =$$

$$= K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot \Delta\hat{\varphi}_{i-1} + (1 - K_i) \cdot \Delta u_i = K_i \cdot \Delta\tilde{\varphi}_i + (1 - K_i) \cdot K_{i-1} \cdot \Delta\tilde{\varphi}_{i-1} +$$

$$+ \dots + (1 - K_i) \cdot \dots \cdot (1 - K_2) \cdot K_1 \cdot \Delta\tilde{\varphi}_1 + (1 - K_i) \cdot \dots \cdot (1 - K_1) \cdot \Delta\tilde{\varphi}_0 +$$

$$+ (1 - K_i) \cdot \Delta u_i + (1 - K_i) \cdot (1 - K_{i-1}) \cdot \Delta u_{i-1} + \dots + (1 - K_i) \cdot \dots \cdot (1 - K_1) \cdot \Delta u_1$$

If the variance of the noise after filtering is zero (Q), then Kalman Gain is equal to $K_i = (i + 1)^{-1}$, and the expression for the bearing estimation error takes the following form:

$$\Delta\hat{\varphi}_i = \sum_{k=0}^i \frac{\Delta\tilde{\varphi}_k}{i+1} + \sum_{k=1}^i \frac{k}{i+1} \cdot \Delta u_k = \sum_{k=0}^i \frac{\Delta\tilde{\varphi}_k}{i+1} + U_i$$

, $\tilde{\varphi}_j$ – where φ_j – the bearing of the radio beacon, $\tilde{\varphi}_j$ – the measured bearing of the beacon, $\hat{\varphi}_j$ – the estimated bearing of the beacon, $\Delta\tilde{\varphi}_j$ – the measurement error in the bearing of the radio beacon, $\Delta\hat{\varphi}_j$ – estimation error of the bearing to the beacon.

The sum of the squares of residuals for the i -th reference is a random value in the form

$$S_i = \sum_{k=1}^i ((\Delta\tilde{\varphi}_k - \Delta\hat{\varphi}_k)^2) = \sum_{k=1}^i (D[(\Delta\tilde{\varphi}_k - \Delta\hat{\varphi}_k)^2] \cdot (N(0,1))^2) = \sum_{k=1}^i (D[(\Delta\tilde{\varphi}_k - \Delta\hat{\varphi}_k)^2] \cdot \chi^2(1))$$

$N(0,1)$ - random variable with standard normal distribution, $\chi^2(1)$ a random variable distributed by the Chi-square law with one degree of freedom.

To solve the problem of determining the location of the radio beacon, it is necessary to collect more than 100 measurements of the direction finder. Then, according to the Central limit theorem, the sum of the squares of residuals can be considered a normally distributed random variable [25; 21].

The probability density of the normal distribution is determined by the parameters of expectation and variance. We derive the expectation and variance of the sum of squares of residuals with the minimum sum of squares of residuals S_i^0 (no prediction errors) and prediction errors S_i^1 .

Kalman filter allows to use the following expression for the estimate of the bearing at the time i :

Control action U_i on step i is equal to:

$$u_i = \left\{ V \cdot dt \cdot \sin(\Psi - \varphi_{i-1}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \sin[\arg(\Delta x_i / \Delta y_i) - \varphi_{i-1}] \right\} / D_i = A_i / D_i, \text{ where}$$

$D_i = D_{i-1} + V \cdot dt \cdot \cos(\Psi - \varphi_{i-1}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \cos(\arg(\Delta x_i / \Delta y_i) - \varphi_{i-1})$ – the value of the current range to the beacon, D_0 – the initial range to the target.

The estimated control action (in case of errors) for the hypothesis with indices abc, provided $\cos \hat{\varphi}_i \approx \cos \varphi_i$ и $\sin \hat{\varphi}_i \approx \sin \varphi_i$, is equal:

$$\begin{aligned} \hat{u}_i^{abc} &= \left\{ V^b \cdot dt \cdot \sin(\Psi^c - \hat{\varphi}_{i-1}^{abc}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \sin[\arg(\Delta x_i / \Delta y_i) - \hat{\varphi}_{i-1}^{abc}] \right\} / \hat{D}_i^{abc} = \\ &= \left\{ V^b \cdot dt \cdot \sin(\Psi^c - \varphi_{i-1}) - \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \cdot \sin[\arg(\Delta x_i / \Delta y_i) - \varphi_{i-1}] \right\} / \hat{D}_i^{abc} = \hat{A}_i^{abc} / \hat{D}_i^{abc} \end{aligned}$$

The error of the control action is equal to:

$$\Delta u_i = \hat{A}_i^{abc} / \hat{D}_i^{abc} - A_i / D_i$$

In what follows, notation $\Delta \hat{\varphi}_i$ will be used for the error of bearing estimation in case of accurate prediction, i.e. $U_i=0$.

The bearing measurement error has a normal distribution with zero mean and variance σ_φ^2 ($M[\Delta \tilde{\varphi}_i] = 0, D[\Delta \tilde{\varphi}_i] = \sigma_\varphi^2$). The bearing measurement are considered to be uncorrelated.

2.2. The sum of squares of residuals without error prediction

The expectation and the variance of the bearing estimation error for accurate prediction are:

$$M[\Delta \hat{\varphi}_i] = M \left[\frac{1}{i+1} \cdot \Delta \tilde{\varphi}_i + \dots + \frac{1}{i+1} \cdot \Delta \tilde{\varphi}_0 \right] = M \left[\frac{1}{i+1} \cdot \Delta \tilde{\varphi}_i \right] + \dots + M \left[\frac{1}{i+1} \cdot \Delta \tilde{\varphi}_0 \right] = 0$$

$$D[\Delta \hat{\varphi}_i] = M \left[\left(\frac{1}{i+1} \cdot \Delta \tilde{\varphi}_i + \dots + \frac{1}{i+1} \cdot \Delta \tilde{\varphi}_0 \right)^2 \right] - M^2[\Delta \hat{\varphi}_i] = \frac{\sigma_\varphi^2}{i+1}$$

The sum of squared residual errors for accurate forecasting ($U_j=0$) is equal:

$$S_i^0 = \sum_{j=1}^i (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j)^2$$

We define the mean of sum of squared residual errors:

$$M[S_i^0] = M \left[\sum_{j=1}^i (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j)^2 \right] = M[(\Delta \tilde{\varphi}_i - \Delta \hat{\varphi}_i)^2] + \dots + M[(\Delta \tilde{\varphi}_1 - \Delta \hat{\varphi}_1)^2] = \sigma_\varphi^2 \cdot \left(\sum_{j=1}^i \frac{j}{j+1} \right)$$

Determine the variance of the sum of squared residual errors:

$$D[S_i^0] = M \left[\left(\sum_{k=1}^i (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k)^2 \right)^2 \right] - (M[S_i^0])^2 = M \left[\sum_{k=1}^i (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k)^4 + \right. \\ \left. + 2 \cdot \sum_{k=2}^i \sum_{j=1}^{k-1} (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k)^2 \cdot (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j)^2 \right] - (M[S_i^0])^2 = \\ = \sigma_\varphi^4 \cdot \left(\sum_{k=1}^i \frac{4 \cdot k^2 - 2 \cdot k}{(k+1)^3} - 4 \cdot \sum_{k=2}^i \sum_{j=1}^{k-1} \frac{j}{(k+1)^2 \cdot (j+1)} \right)$$

To simplify the calculation of the variance, we approximate it as follows:

$$D[S_i^0] = \sigma_\varphi^4 \cdot \log_{1.645}^{0.294}(i)$$

2.3. Sum of squares of residuals with prediction errors

The sum of squares of residuals with prediction error:

$$S_i^1 = \sum_{j=1}^i (\tilde{\varphi}_j - \hat{\varphi}_j - U_j)^2 = \sum_{j=1}^i (\varphi_j + \Delta \tilde{\varphi}_j - \varphi_j - \Delta \hat{\varphi}_j - U_j)^2 = \sum_{j=1}^i (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j - U_j)^2$$

Define the expectation of the sum of squared residual errors with prediction error by:

$$M[S_i^1] = M \left[\sum_{j=1}^i (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j - U_j)^2 \right] = \sigma_\varphi^2 \cdot \sum_{j=1}^i \frac{j}{j+1} + \sum_{j=1}^i M[U_j^2]$$

Since the prediction error does not include measurement errors, the expectation of the prediction error is equal to:

$$M[U_j^2] = U_j^2$$

$$\text{Therefore: } M[S_i^1] = \sigma_\varphi^2 \cdot \sum_{j=1}^i \frac{j}{j+1} + \sum_{j=1}^i U_j^2.$$

The variance of the sum of squared residual errors in presence of prediction error is given by:

$$D[S_i^1] = M \left[\left(\sum_{k=1}^i (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k - U_k)^2 \right)^2 \right] - (M[S_i^1])^2 = M \left[\sum_{k=1}^i (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k - U_k)^4 + \right. \\ \left. + 2 \cdot \sum_{k=2}^i \sum_{j=1}^{k-1} (\Delta \tilde{\varphi}_k - \Delta \hat{\varphi}_k - U_k)^2 \cdot (\Delta \tilde{\varphi}_j - \Delta \hat{\varphi}_j - U_j)^2 \right] - (M[S_i^1])^2 = \sigma_\varphi^4 \cdot (\log_{1.645}^{0.147}(i))^2 + \\ + 2 \cdot \sigma_\varphi^2 \cdot \left(3 \cdot \sum_{k=1}^i \frac{U_k^2 \cdot k}{k+1} + \sum_{k=2}^i \sum_{j=1}^{k-1} \left(\frac{k \cdot U_j^2}{k+1} + \frac{j \cdot U_k^2}{j+1} - \frac{4 \cdot U_k \cdot U_j}{(k+1) \cdot (j+1)} \right) - \sum_{k=1}^i \frac{k}{k+1} \cdot \sum_{k=1}^i U_k^2 \right)$$

2.4. Probability of correct hypothesis selection

The hypothesis with the minimum sum of squared residual errors for the case of no prediction error has mathematical expectation and standard deviation equal to:

$$M[S_i^0] = \sigma_\varphi^2 \cdot \sum_{j=2}^{i+1} \frac{j}{j+1}$$

$$\sigma_{S_i^0} = \sigma_\varphi^2 \cdot \log_{1.645}^{0.147}(i)$$

Mathematical expectation and RMS of the hypothesis with the prediction error are respectively equal to:

$$M[S_i^1] = \sigma_\varphi^2 \cdot \sum_{j=1}^i \frac{j}{j+1} + \sum_{j=1}^i U_j^2$$

$$\sigma_{S_i^1} = \left(\sigma_\varphi^4 \cdot (\log_{1.645}^{0.147}(i))^2 + 2 \cdot \sigma_\varphi^2 \cdot \left(3 \cdot \sum_{k=1}^i \frac{U_k^2 \cdot k}{k+1} + \sum_{k=2}^i \sum_{j=1}^{k-1} \left(\frac{k \cdot U_j^2}{k+1} + \frac{j \cdot U_k^2}{j+1} - \frac{4 \cdot U_k \cdot U_j}{(k+1) \cdot (j+1)} \right) - \sum_{k=1}^i \frac{k}{k+1} \cdot \sum_{k=1}^i U_k^2 \right) \right)^{0.5}$$

To obtain the sum of squares of residuals without prediction errors to the standard normal value, we subtract the value from both sums $M[S_i^0]$ and then divide by $\sigma_{S_i^0}$. As a result, we get:

$$M[S_i^0] = 0$$

$$\sigma_{S_i^0} = 1$$

$$M[S_i^1] = \frac{1}{\log_{1.645}^{0.147}(i)} \cdot \sum_{j=1}^i U_j^2$$

$$\sigma_{S_i^1} = \sqrt{1 + \frac{2 \cdot \sigma_\varphi^2 \cdot \left(3 \cdot \sum_{k=1}^i \frac{U_k^2 \cdot k}{k+1} + \sum_{k=2}^i \sum_{j=1}^{k-1} \left(\frac{k \cdot U_j^2}{k+1} + \frac{j \cdot U_k^2}{j+1} - \frac{4 \cdot U_k \cdot U_j}{(k+1) \cdot (j+1)} \right) - \sum_{k=1}^i \frac{k}{k+1} \cdot \sum_{k=1}^i U_k^2 \right)}{(\log_{1.645}^{0.147}(i))^2}}$$

Thus, the expression for calculating the probability of choosing a hypothesis without prediction errors can be written as follows:

$$p(S_i^0 < S_i^1) = \frac{1}{2} \cdot \int_{-\infty}^{+\infty} \left(1 + \operatorname{erf} \left(\frac{h}{\sqrt{2}} \right) \right) \cdot \frac{\exp \left(-\frac{(h - M[S_i^1])^2}{2 \cdot \sigma_{S_i^1}^2} \right)}{\sigma_{S_i^1} \cdot \sqrt{2 \cdot \pi}} \cdot dh \quad (1)$$

2.5. Simulations

To check the efficiency of the proposed method of calculating the probability of correctly determining the range calculated by the formula (1), 20 realizations for each set of parameters were simulated. For each realization, the following parameters of the targets location and movement were set: the speed of movement of the direction finder is 14 m/s, the initial course of the direction finder is 0°, the changed course – 180°, the time of course change is 600 seconds, the initial bearing of the beacon 90°, the initial ranges to the beacon are 150, 200 and 250 km, the speed of the beacon movement is 10 m/s, its course is 0°, the bearings are 0.2° and 0.3°. The number of hypotheses about the initial range is 40, the minimum range is 40 km, the step of hypotheses is 10 km, the number of hypotheses for the beacon speed is 21, the initial speed is 0 m/s, the step of hypotheses is 1 m/s, the number of hypotheses on the course of the beacon is 72, the initial value of the course is 0°, the step between hypotheses is 5°.

Simulation results are shown (see figures 2-4), where the graph "frequency" shows what part of the estimations at a time t does not exceed the limit of $\delta_D = 10\%$ of the true range.

The figures demonstrate that before the direction finder has changed the course, the probability to determine the range with a given accuracy is low while after the course has changed, the probability to determine the range with a given accuracy rises and approaches one.

The probability graph is calculated by the formula (1) from one realization of a data set. Since there is no a priori information about the range to the target at each step of the algorithm, the true value of the range is taken as the current range estimation of the Kalman filter with the smallest sum of squared residual errors. In this case, the probability of the correct hypothesis selection at any step of the algorithm is at least 0.5.

Despite the fact that the graphs are different for small number of measurements (before the time of the change of the direction finder's course), they have similar behavior for high probabilities of correct range estimation ($p > 0.9$).

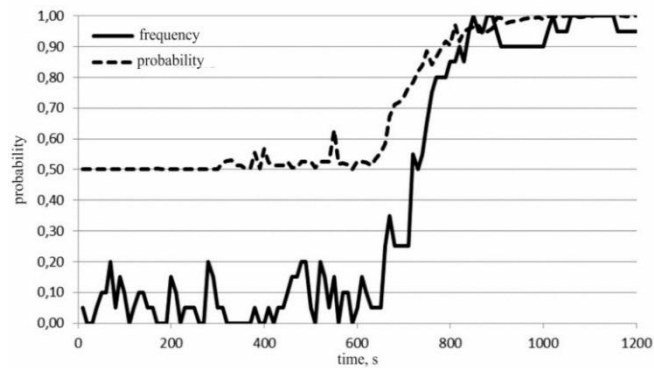


Figure 2: Probability of correct hypothesis selection, the range to the beacon 150 km, the standard deviation of bearing is 0.2

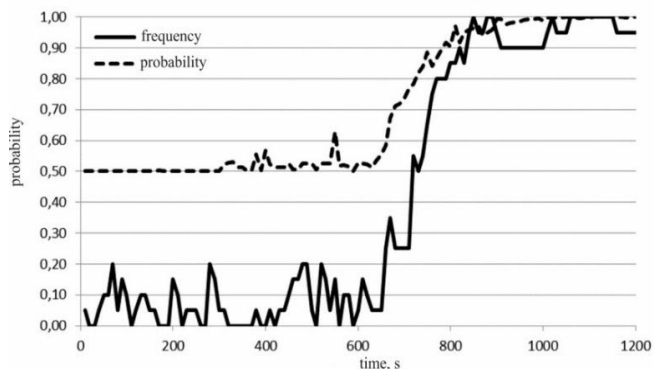


Figure 3: Probability of correct hypothesis selection, the range to the beacon 200 km, the standard deviation of bearing is 0.2)

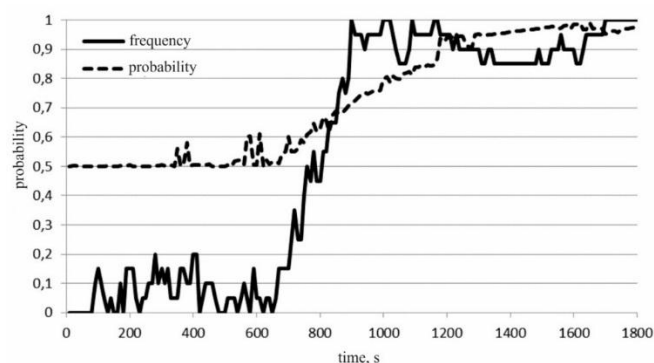


Figure 4: Probability of correct hypothesis selection, the distance to the beacon 250 km, the standard deviation of bearing 0.3

3. CONCLUSIONS

Simulation results demonstrate that the provided analytical expression for the probability of determining the range to the beacon with the given accuracy reaches a probability of

0.9..0.95 agrees well with numerical simulations. Therefore, the proposed technique can be considered for determining the location of the beacon with a given probability, in contrast to the methods with a fixed time required to perform the maneuver.

CONFLICT OF INTEREST

The authors confirm that there is no conflict of interest to declare for this publication

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