

## Efficient Power of Brayton Heat Engine with Friction

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### Abstract

Efficient power of Brayton heat engine with friction is analyzed based on the first and second law of thermodynamics. The efficient power is the product of Brayton power output and Brayton efficiency. Hence, the proposed method considers not only Brayton power output but also Brayton efficiency. The work done against friction is also included into the analysis of Brayton heat engine. The efficient power of Brayton heat engine with friction is obtained and results are recovered known from finite time thermodynamics. Brayton heat engine with friction gives realistic prediction of engine efficiency and engine power than does the isentropic Brayton heat engine without friction.

**Keywords:** Brayton heat engine, finite time thermodynamics, efficient power, friction.

### 1. Introduction

Curzon and Ahlborn (1975) developed a theoretical model of a Carnot heat engine. They maximized the power with respect to temperature difference in both the hot and cold side heat exchangers and found the thermal efficiency at maximum power output as  $\eta_{C-A} = 1 - (T_L / T_H)^{1/2}$ . The analysis of Carnot engine at maximum power output has been applied to the Brayton cycle. Wu and Kiang (1990) optimized work & power of endoreversible Brayton cycle on the basis of finite time thermodynamics. They found that the power output of Brayton heat engine is a function of  $T_H$  and  $T_L$  only. Wu and

Kinag (1991) also extended their work by incorporating the non-isentropic nature of compressor and turbine and found lower power output as well as the engine efficiency when compared with an endoreversible Brayton heat engine under the same conditions. Wu et al (1995) analysed the performance of an endoreversible Brayton heat engine by incorporating a regenerator into Brayton cycle. Goktun and Yavuz (1998) analyzed the effect of an isothermal heat addition process in regenerative Brayton heat engine and found a considerable efficiency enhancement of the order of over 10% compared with reversible Brayton cycle. In past, many optimization studies for gas turbine engines based on endoreversible and irreversible mode have been carried out by number of researchers (Kaushik 2002; Tyagi 2003; Tyagi 2005; Kaushik 2006; Tyagi 2006). Yilmaz (2007) carried out performance optimization of a Joule Brayton engine based on the efficient power condition to consider the power output and the cycle efficiency together. Ebrahimi (2010) analyzed the performance of an endoreversible Atkinson cycle with variable specific heat ratio of the working fluid. Bizarro (2012) analyzed the efficiency of heat engine based on first and second law of thermodynamics by incorporating the presence of work done against friction.

None of the previous investigations included a friction and temperature dependent specific heat together. It is the purpose of this paper to incorporate friction and variable specific heat of the working fluid while calculating efficient power of Brayton heat engine with finite time thermodynamics.

### Nomenclature

1, 2, 3, 4	state points
$A_1, A_2$	constants
$C_p$	specific heat at constant pressure ( $\text{kJkg}^{-1}\text{K}$ )
$m$	mass flow ( $\text{kgs}^{-1}$ )
$P$	pressure (kPa)
$Q$	heat transfer
$T$	temperature (K)
$W$	power output (kW)

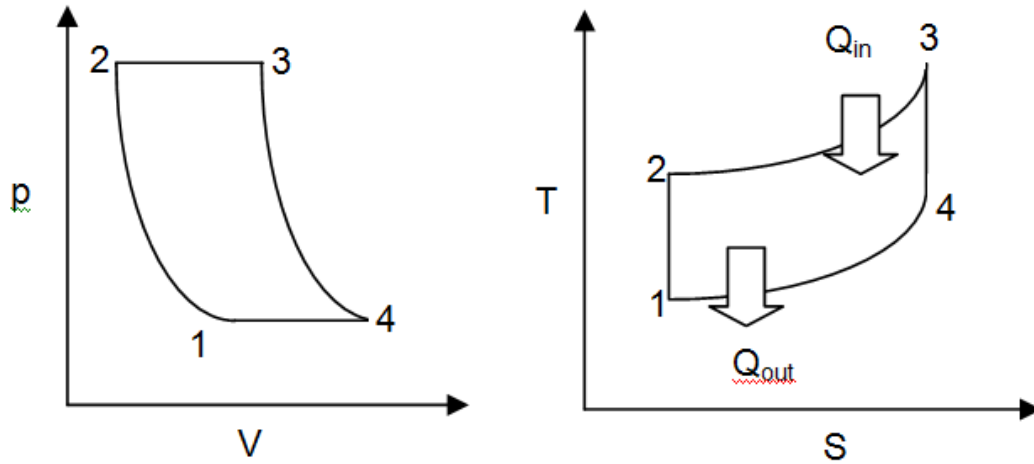
### Greek letters

$\beta$	maximum cycle temperature ratio
$\epsilon$	isentropic temperature ratio
$\gamma$	specific heat ratio
$\eta$	efficiency

## 2. Thermodynamic Analysis

Reversible gas engine cycle involving two constant pressure processes and two isentropic processes is shown in Fig. 1. The compression and expansion processes occur in quasi static manner and shown by process (1-2) and process (3-4) respectively

on p-V and T-S diagrams. The heat addition and heat rejection takes place at constant pressure as shown by process (2-3) and process (4-1) respectively in Fig. 1. The working temperature dependent specific heat representation assumes linear variation of specific heat with temperature.



**Figure 1:** p-V and T-S diagrams for gas turbine cycle.

The variation of specific heat at constant pressure (kJ/kg K) with temperature is assumed under:

$$C_p = A_1 + A_2 T \quad (1)$$

Where  $A_1$ ,  $A_2$  are constants and  $T$  is temperature in K.

During process (2-3), heat addition ( $Q_H$ ) to the working fluid is given as:

$$Q_H = m \int_{T_2}^{T_3} C_p dT \quad (2)$$

During process (4-1), heat rejection ( $Q_L$ ) to the working fluid is given as:

$$Q_L = m \int_{T_1}^{T_4} C_p dT \quad (3)$$

$Q_H$  and  $Q_L$  can be written as:

$$Q_H = mC_p(T_3 - T_2) = m[A_1(T_3 - T_2) + \frac{A_2}{2}(T_3^2 - T_2^2)] \quad (4)$$

$$Q_L = mC_p(T_4 - T_1) = m[A_1(T_4 - T_1) + \frac{A_2}{2}(T_4^2 - T_1^2)] \quad (5)$$

Let the maximum cycle temperature ratio is  $\beta$  and isentropic temperature ratio is  $\epsilon$

$$\beta = \frac{T_3}{T_1} \quad (6)$$

$$\epsilon = \frac{T_2}{T_1} \quad (7)$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \text{ gives } T_4 = \frac{\beta T_1}{\epsilon} \quad (8)$$

$$W = Q_H - Q_L \quad (9)$$

$$= m \left[ A_1 (T_3 - T_2) + \frac{A_2}{2} (T_3^2 - T_2^2) - A_1 (T_4 - T_1) - \frac{A_2}{2} (T_4^2 - T_1^2) \right] \quad (10)$$

$$W = m \left[ A_1 (\beta T_1 - \epsilon T_1) + \frac{A_2}{2} (\beta^2 T_1^2 - \epsilon^2 T_1^2) - A_1 \left( \frac{\beta T_1}{\epsilon} - T_1 \right) - \frac{A_2}{2} \left( \frac{\beta^2 T_1^2}{\epsilon^2} - T_1^2 \right) \right] \quad (11)$$

$$\eta_{fric} = \frac{W - W_{fric}}{Q_{0,H}} = 1 - \frac{Q_{0,L}}{Q_{0,H}} \quad (12)$$

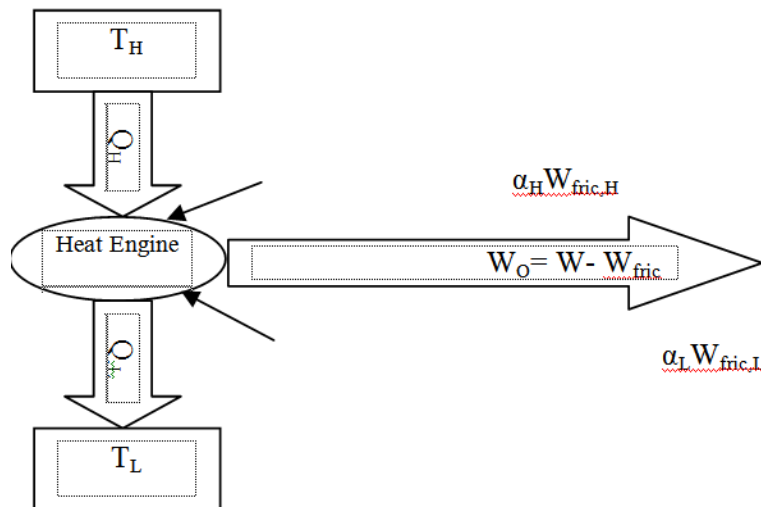
Where  $Q_{0,L}$  and  $Q_{0,H}$  are the net heat transfers from and into the cold and hot reservoirs, respectively

$$\text{We know that } \eta_{carnot} = 1 - \frac{T_L}{T_H}$$

According to Second law of thermodynamics, it can be written as

$$(\eta_{carnot} - \eta_{fric}) \frac{Q_{0,H}}{T_L} \geq 0$$

$\eta_{carnot} \geq \eta_{fric}$ , which satisfies the Carnot's theorem on the maximum efficiency.



**Figure 2:** Schematic diagram of a cyclic heat engine with friction.

Efficient Power is defined as the product of Power output and engine efficiency.

$$\text{Therefore, } P_{e,fric} = W_0 \times \eta_{fric} \quad (13)$$

$$W_0 = W - W_{fric} \text{ where } W_{fric} = W_{fric,H} + W_{fric,L}$$

Where  $W_0$  is the work output of Brayton heat engine with friction. (14)

$$W_{\text{fric,H}} = \alpha_H W_{\text{fric,H}} + (1 - \alpha_H) W_{\text{fric,H}} \text{ and } W_{\text{fric,L}} = \alpha_L W_{\text{fric,L}} + (1 - \alpha_L) W_{\text{fric,L}} \quad (15)$$

$$Q_{o,H} = Q_H - (1 - \alpha_H) W_{\text{fric,H}} \text{ and } Q_{o,L} = Q_L + (1 - \alpha_L) W_{\text{fric,L}} \quad (16)$$

$$W_0 = m \left[ A_1 (\beta T_1 - \epsilon T_1) + \frac{A_2}{2} (\beta^2 T_1^2 - \epsilon^2 T_1^2) - A_1 \left( \frac{\beta T_1}{\epsilon} - T_1 \right) - \frac{A_2}{2} \left( \frac{\beta^2 T_1^2}{\epsilon^2} - T_1^2 \right) \right] - \alpha_H W_{\text{fric,H}} - (1 - \alpha_H) W_{\text{fric,H}} - \alpha_L W_{\text{fric,L}} - (1 - \alpha_L) W_{\text{fric,L}} \quad (17)$$

$$P_{e,fric} = \left[ m \left[ A_1 (\beta T_1 - \epsilon T_1) + \frac{A_2}{2} (\beta^2 T_1^2 - \epsilon^2 T_1^2) - A_1 \left( \frac{\beta T_1}{\epsilon} - T_1 \right) - \frac{A_2}{2} \left( \frac{\beta^2 T_1^2}{\epsilon^2} - T_1^2 \right) \right] - \alpha_H W_{\text{fric,H}} - (1 - \alpha_H) W_{\text{fric,H}} - \alpha_L W_{\text{fric,L}} - (1 - \alpha_L) W_{\text{fric,L}} \right] X \left[ 1 - \frac{Q_L + (1 - \alpha_L) W_{\text{fric,L}}}{Q_H - (1 - \alpha_H) W_{\text{fric,H}}} \right] \quad (18)$$

This equation provides the frictional efficient power for Brayton heat engine. The equation is the replication of efficient power of reversible Brayton cycle as proved in corollary 1 (on vanishing the presence of work against friction) and corollary 2 (equating  $A_1 = C_p$  and  $A_2 = 0$ ).

**Corollary 1:**

If  $W_{\text{fric,H}} = 0$  and  $W_{\text{fric,L}} = 0$

$$P_{eb} = \left[ m \left[ A_1 (\tau T_1 - \theta T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta^2} - T_1^2 \right) \right] \right] X \left[ 1 - \frac{Q_L}{Q_H} \right] \quad (19)$$

This equation recovers the results of reversible Brayton heat engine cycle with variable specific heat of the working fluid.

**Corollary 2:**

If  $W_{\text{fric,H}} = 0$ ,  $W_{\text{fric,L}} = 0$ ,  $A_1 = C_p$  and  $A_2 = 0$ .

$$P_e = [Q_H - Q_L] X \left[ 1 - \frac{Q_L}{Q_H} \right] \quad (20)$$

The equation recovers the results of reversible Brayton heat engine cycle.

### 3. Conclusion

The efficient power of Brayton heat engine with friction has been addressed from a practical point of view. The temperature dependent specific heat of the working fluid is also included in the analysis. The results agree with that of reversible Brayton heat engine cycle without friction. The efficiency is expressed as the ratio of the useful work delivered to the surrounding, which is the actual work performed by the engine fluid minus the work done against friction, to the net energy transferred by the hot reservoir where as work output of Brayton heat engine per cycle is the difference between the ideal work,  $W$  and the total work,  $W_{\text{fric}}$  produced against friction, the latter being the sum of  $W_{\text{fric,H}}$  and  $W_{\text{fric,L}}$  which are the work done against friction for those parts of the cycle where the hot and cold reservoirs are present, respectively, and whose fractions  $\alpha_H$  and  $\alpha_L$  are dissipated in the fluid. The remaining fractions  $(1 - \alpha_H)$  and  $(1 - \alpha_L)$  are recirculated to the respective reservoirs.

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