International Journal of Mathematics Research.
ISSN 0976-5840 Volume 8, Number 2 (2016), pp. 85-92

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# Fuzzy Join - Semidistributive Lattice

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**Abstract**

In this Paper, Fuzzy Join- Semidistributive Lattice – Definition of Fuzzy Join-Semidistributive Lattice - Characterization theorem are given.

**Keywords**: Fuzzy Lattice, Fuzzy Modular Lattice, Fuzzy Distributive Lattice, Fuzzy Join-Semidistributive Lattice and Fuzzy Meet-Semidistributive Lattice.

## INTRODUCTION

The Concept of Fuzzy Lattice was already introduced by Ajmal,N[1], S.Nanda[4] and WilCox,L.R [5] explained modularity in the theory of Lattices, G.Gratzer[2], BarDalo, G.H and Rodrigues,E[3] Stern,m[6] explained semimodular Lattices, M.Mullai and B.Chellappa[8] explained Fuzzy L-ideal and V.Vinoba and K.Nithya[7] Explained fuzzy modular pairs in Fuzzy Distributive Lattice. A few definitions and results are listed that the fuzzy Join-semidistributive lattice using in this paper we explain fuzzy Join-semidistributive lattice, Definition of fuzzy Join-semidistributive lattice, Characterization theorem of Fuzzy Join-Semidistributive lattice and some examples are given.

**Definition: 1.1**

A Fuzzy lattice L is called a Fuzzy join-semi distributive if
$\mu(a \vee b) = \mu(a \vee c) \Rightarrow \mu(a \vee b) = \mu(a) \vee \mu(b \vee c)$, for all $\mu(a), \mu(b), \mu(c) \in L$.

**Theorem: 1.1**

Every Fuzzy join – semi distributive lattice is fuzzy lattice and converse is not true.

**Proof:**

Given L is a Fuzzy join – semi distributive lattice

$\Rightarrow \mu(a \vee b) = \mu(a) \vee \mu(b \vee c)$, for all $\mu(a), \mu(b), \mu(c) \in L$.

To prove L is a Fuzzy Lattice.

That is to prove $\mu(a \vee b) = \mu(a \vee c)$ , for all $\mu(a), \mu(b), \mu(c) \in L$.

Let $\mu(a), \mu(b), \mu(c)$ be arbitrary.

Then $\mu(a \vee b) = \mu(a) \vee \mu(b \vee c)$

$\geq \min\{ \mu(a), \mu(b \wedge c) \}$

$\geq \min \{ \mu(a), \min \{ \mu(b) , \mu(c)\}\}$

$\geq \min \{ \mu(a), \min \{ \mu(c) , \mu(b)\}\}$, by commutative law

$\geq \min \{ \mu(a) , \mu(c \wedge b)\}$

$= \mu(a) \vee (c \wedge b)$

$= \mu(a \vee c)$ , for all $\mu(a), \mu(b), \mu(c) \in L$.

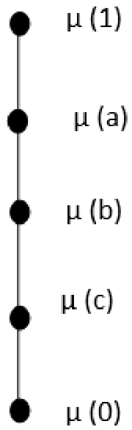
Hence L is a Fuzzy Lattice.

The converse need not be true.

(i.e) Every Fuzzy lattice need not be Fuzzy join- semi distributive.

We shall verify it by the following example.

Consider the Fuzzy lattice of following figure.




This Fuzzy lattice is not Fuzzy join-semi distributive.

Here

$\mu(a \vee b) = \mu(1)$

$\mu(a \vee c) = \mu(1)$

$\mu(b \wedge c) = \mu(0)$

$\mu(a) \vee \mu(b \wedge c) \geq \min\{ \mu(a), \mu(b \wedge c) \}$

$\geq \min\{ \mu(a), \mu(0) \}$

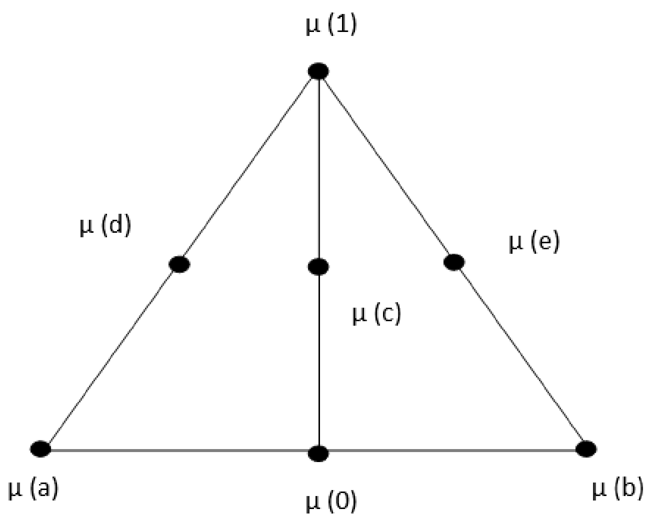
$= \mu(a)$

Thus $\mu(a \vee b) = \mu(a \vee c)$ , but $\mu(a \vee b) \neq \mu(a) \vee \mu(b \wedge c)$
for all $\mu(a), \mu(b), \mu(c) \in L$.
$\Rightarrow$ L is not a Fuzzy join-semi distributive lattice.

**Theorem: 1.2**
Every Fuzzy distributive lattice is Fuzzy join –semi distributive and the converse is not true.

**Proof** Given L is a Fuzzy distributive lattice.
$\Rightarrow \mu(a) \vee \mu(b \wedge c) = \mu(a \vee b) \wedge \mu(a \vee c)$ for all $\mu(a), \mu(b), \mu(c) \in L$. $\longrightarrow$ (1)

**To prove:** L is Fuzzy join-semi distributive Lattice.
For let $\mu(a), \mu(b), \mu(c) \in L$ be arbitrary and $\mu(a \vee b) = \mu(a \vee c)$
$\mu(a) \vee \mu(b \wedge c) = \mu(a \vee b) \wedge \mu(a \vee c)$, by (1)
$\geq \min\{ \mu(a \vee b), \mu(a \vee c)\}$
$\geq \min\{ \mu(a \vee b), \mu(a \vee b)\}$
$= \mu(a \vee b)$.
Thus $\mu(a \vee b) = \mu(a \vee c) \Rightarrow \mu(a \vee b) = \mu(a) \vee \mu(b \wedge c)$,
for all $\mu(a), \mu(b), \mu(c) \in L$.
$\Rightarrow$ L is a Fuzzy join-semi distributive lattice.
The converse need not be true.
(i.e) Every Fuzzy join-semi distributive lattice need not be Fuzzy distributive lattice.
We shall verify it by the following example.
Consider the lattice $S_7$ of following figure




This fuzzy lattice is Fuzzy join-semi distributive but not fuzzy distributive.

Here

$$\mu(a) \vee \mu(d \wedge b) \geq \min\{ \mu(a) , \mu(d \wedge b) \}$$
$$\geq \min\{ \mu(a) , \mu(a)\}$$
$$= \mu(a) \vee \mu(a)$$
$$= \mu(a)$$
$$\mu(a \vee d) \wedge \mu(a \vee b) \geq \min\{ \mu(a \vee d) , \mu(a \vee b)\}$$
$$\geq \min\{ \mu(d) , \mu(1)\}$$
$$= \mu(d) \wedge \mu(1)$$
$$= \mu(d)$$

Therefore $\mu(a) \vee \mu(d \wedge b) \neq \mu(a \vee d) \wedge \mu(a \vee b)$

$\Rightarrow$ $S_7$ is not Fuzzy distributive.

**Theorem: 1.3**

A Join- Fuzzy semi distributive lattice L is Fuzzy distributive if and only if L does not contain a Fuzzy sublattice isomorphic to $S_7$.

**Proof.** Assume that a Fuzzy join semi distributive lattice L is Fuzzy distributive lattice

**To prove:** L does not contain a Fuzzy sublattice isomorphic to $S_7$.
Suppose L contain a Fuzzy sublattice isomorphic to $S_7$
Then L is not Fuzzy distributive.
This is a contradiction.
Hence L does not contain a Fuzzy sublattice isomorphic to $S_7$.
Conversely, Assume that a Fuzzy Join-semi distributive L does not contain Fuzzy sublattice isomorphic to $S_7$.

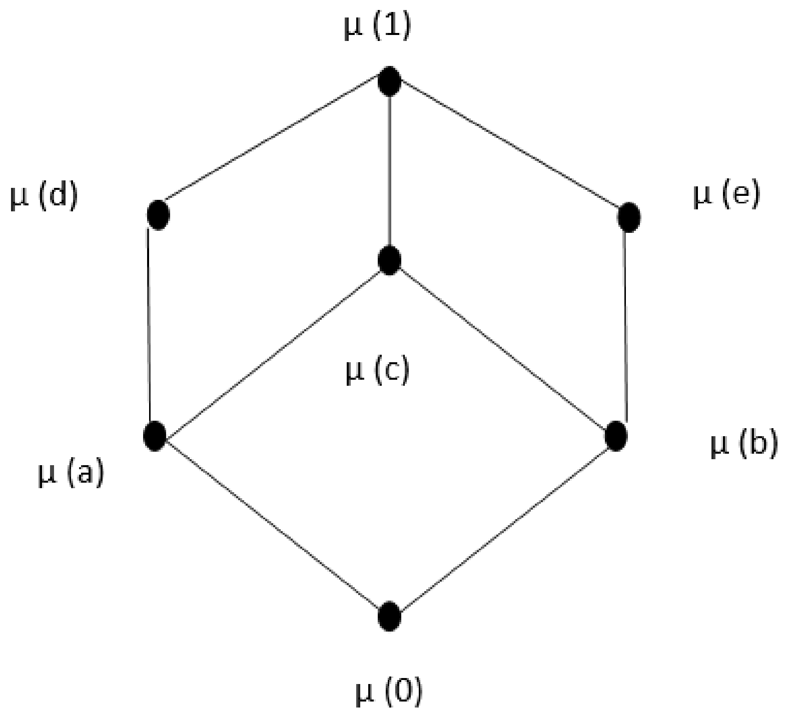
**To Prove:** L is Fuzzy distributive.
Suppose L is not Fuzzy distributive.
Then L contain a Fuzzy sublattice isomorphic to $S_7$.
This is Contradiction.
Hence L is a Fuzzy distributive lattice.

**Theorem: 1.4**

Every Fuzzy meet-semi distributive lattice need not be Fuzzy join-semi distributive lattice.

**Proof.** By an example,

Consider the Fuzzy lattice $S_7$ of following figure.




This Fuzzy lattice is Fuzzy meet – semi distributive but not Fuzzy join-Semi distributive.

Here $\mu(c \vee d) \geq \min\{ \mu(c) , \mu(d) \}$

$\geq \min\{ \mu(c) , \mu(e) \}$

$= \mu(c) \vee \mu(e)$

$= \mu(1)$

$\mu(c) \vee \mu(d \wedge e) \geq \min\{ \mu(c) , \mu(d \wedge e) \}$

$\geq \min\{ \mu(c) , \mu(o) \}$

$= \mu(c) \vee \mu(0)$

$= \mu(c)$

Therefore $\mu(c) \vee \mu(d \wedge e) \neq \mu(c \vee d)$

$\Rightarrow S_7$ is not Fuzzy Join- Semi distributive Lattice.

**Definition : 1.2**

A Fuzzy Lattice satisfying the above theorem is called upper locally Fuzzy distributive.

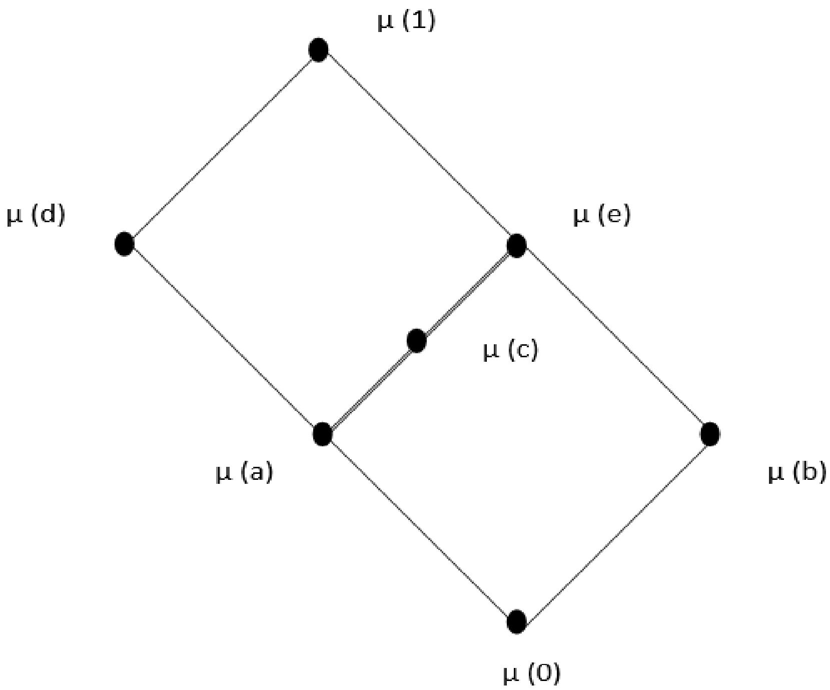
**Theorem: 1.5**

Every Fuzzy join-semi distributive lattice need not be Fuzzy meet-semi distributive lattice.

**Proof**

By an example

Consider the Fuzzy lattice $S_7$ of following figure.




This Fuzzy lattice is Fuzzy Join-Semi distributive but not Fuzzy meet-Semi distributive.

Here $\mu(c \wedge a) \geq \min\{ \mu(c), \mu(a) \}$

$\geq \min\{ \mu(c), \mu(b) \}$

$= \mu(c \wedge b)$

$= \mu(0)$

$\mu(c) \wedge \mu(a \vee b) \geq \min\{ \mu(c), \mu(a \vee b) \}$

$\geq \min\{ \mu(c), \mu(e) \}$

$= \mu(c) \wedge \mu(e)$

$= \mu(c)$

Therefore $\mu(c \wedge a) \neq \mu(c) \wedge \mu(a \vee b)$

$\Rightarrow S_7$ is not Fuzzy meet- semi distributive.

**Definition: 1.3**
A Fuzzy lattice satisfying above theorem is called lower locally Fuzzy distributive.

**Theorem: 1.6**
Fuzzy Dual of Fuzzy meet-semi distributive lattice is a Fuzzy join-semi distributive lattice.

**Proof:** Given L is a Fuzzy meet-semi distributive lattice.

$\Rightarrow \mu(a \wedge b) = \mu(a \wedge c)$ implies $\mu(a \wedge b) = \mu(a) \wedge \mu(b \vee c)$,
for all $\mu(a), \mu(b), \mu(c) \in L$.

$\Rightarrow$Fuzzy dual of above is $\mu(a \vee b) = \mu(a \vee c)$ implies $\mu(a \vee b) = \mu(a) \vee \mu(b \wedge c)$,
for all $\mu(a), \mu(b), \mu(c) \in \bar{L}$, the Fuzzy dual of L.

$\Rightarrow \bar{L}$ is a Fuzzy Join-Semi distributive.

**Theorem: 1.7**
Fuzzy dual of Fuzzy Join-semi distributive lattice is a Fuzzy meet-semi distributive lattice.

**Proof:**
Given L is a Fuzzy join-semi distributive lattice.

$\Rightarrow \mu(a \vee b) = \mu(a \vee c)$ implies $\mu(a \vee b) = \mu(a) \vee \mu(b \wedge c)$,
for all $\mu(a), \mu(b), \mu(c) \in L$,

$\Rightarrow$ Fuzzy dual of above $\mu(a \wedge b) = \mu(a \wedge c)$ implies $\mu(a \wedge b) = \mu(a) \wedge \mu(b \vee c)$,
for all $\mu(a), \mu(b), \mu(c) \in \bar{L}$ the Fuzzy dual of L.

$\Rightarrow \bar{L}$ is a Fuzzy meet- semi distributive lattice.

## CONCLUSION

This paper is proved that Every Fuzzy join – semi distributive lattice is fuzzy lattice and converse is not true, Every Fuzzy distributive lattice is Fuzzy join –semi distributive and the converse is not true, A Join- Fuzzy semi distributive lattice L is Fuzzy distributive if and only if L does not contain a Fuzzy sublattice isomorphic to $S_7$, Every Fuzzy meet-semi distributive lattice need not be Fuzzy join-semi distributive lattice, Every Fuzzy join-semi distributive lattice need not be Fuzzy meet-semi distributive lattice, Fuzzy Dual of Fuzzy meet-semi distributive lattice is a Fuzzy join-semi distributive lattice and Fuzzy dual of Fuzzy Join-semi distributive lattice is a Fuzzy meet-semi distributive lattice.

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