International Journal of Mathematics Research.
ISSN 0976-5840 Volume 8, Number 2 (2016), pp. 93-98

http://www.irphouse.com

The Split Equitable Domination Number of a Fuzzy Graph

S. Revathi[*] C.V.R. Hari Narayanan[**], R. Muthuraj[***]

**Department of Mathematics, Kings College of Engineering. Thanjavur, India.*
***Department of Mathematics, Govt.Arts college. Paramakudi, India.*
****Department of Mathematics, H.H.The Rajah's College, Pudukkottai, India.*

Abstract

An equitable dominating set D of a fuzzy graph G=(σ, μ) is a split equitable dominating set if the induced subgraph $\langle V-S \rangle$ is disconnected. The split equitable domination number $\gamma_{fse}(G)$ of a fuzzy graph G in the minimum cardinality of a fuzzy split equitable dominating set. In this paper we, initiate the study of this new parameter and present some bounds and some exact values for $\gamma_{fse}(G)$.

Keywords: Fuzzy dominating set, equitable dominating set, split equitable dominating set, connected fuzzy graph

INTRODUCTION

Let $G = (\sigma, \mu)$ be a simple undirected fuzzy graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively.
A subset S of V is called a dominating set in G if every vertex in V-S , there exists a vertex $u \in S$ such that u dominates V. The domination number of G is the minimum cardinality taken overall dominating sets in G and is denoted by $\gamma(G)$ or simply γ_f .

A fuzzy dominating set S of a fuzzy graph G is called a minimal fuzzy dominating set of G, for every node $v \in S$, S-{v} is not a fuzzy dominating set.

A subset S of V is called an fuzzy equitable dominating set if for every $v \in V - S$ there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$.The minimum cardinality of such a dominating set is denoted by γ_{fe} and is called the equitable domination number of G.
A fuzzy graph G is connected if there is atleast one path between every pair of vertices in G there exists a strongest path between any two nodes of G.
An equitable dominating set S is said connected equitable dominating set if the subgraph $\langle S \rangle$ is induced by S is connected. The Minimum of the cardinalities of the connected equitable dominating sets of G is called the connected equitable domination by number and denoted by $\gamma_{fce}(G)$.
A dominating set S of a fuzzy graph $G = (\sigma \ \mu)$ is a split dominating set if the induced subgraph $\langle V - S \rangle$ is disconnected .The split dominating number $\gamma_{fs}(G)$ of a fuzzy graph G is the minimum cardinality of a split dominating set of G. Kulli and JanaKiram (1) introduced the concept of split domination in graphs. Analogously in this paper we now define the following concept. An equitable dominating set S of a fuzzy graph G is a split equitable dominating set if the induced subgraph $\langle V - S \rangle$ is disconnected. The split equitable dominating number $\gamma_{fse}(G)$ of a fuzzy graph G is the minimum cardinality of a split equitable dominating set.
We note that $\langle V - S \rangle$ if the fuzzy graph is not complete and either contains a non complete component or contains atleast two non-trivial components $\gamma_{se}(G)$ not exists , if the fuzzy graph totally equitable disconnected (all the vertices of G are equitable isolated) we also note that γ_{fse}-set not exists if the fuzzy graph G is total disconnected.

A vertex $u \in V$ is said to be degree equitable adjacent with a vertex $v \in V$ if u and v are adjacent and $|\deg(u) - \deg(v)| \leq 1$ .The split equitable dominating set S is said to be a minimal equitable dominating set if no proper subset of S is split equitable dominating set. Similarly as the standard dominating set every minimum equi dominating set is minimal but the converse not true some good examples.
If a vertex $u \in V$ be such that $|\deg(u) - \deg(v) \geq 2 \ | for\ all\ v \in N(u)$, then u is in every equitable dominating set such vertices are called equitable isolates.

Let $u \in V$ the equitable neighbourhood of u denoted by $N_{fe}(u)$ is defined as $N_{fe}(u) = \{v \in V : v \in N(u), |\deg(u) - \deg(v)| \leq 1\}$ the cardinality of $N_{fe}(u)$ is denoted by $\deg_{fe} G(u)$

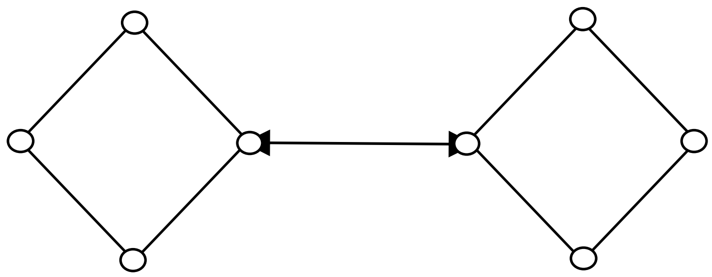
The maximum and minimum equitable degree of a vertex in G are denoted by $\Delta_{fe}(G)$ *and* $\delta_{fe}(G)$.

A vertex u of a equitable fuzzy graph is said to be an equitable isolated vertex if a vertex $u \in V$ be such that $|\deg(u) - \deg(v)| \geq 2$ *and if* $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ *for all* $v \in V - \{u\}$ $(i.e)\ N_{fe}(u) = \phi$

Let $u \in V$ the fuzzy equitable neighborhood of u denoted by $N_{fe}(u)$ is defined as $N_{fe}(u) = \{v \in V / v \in N(u), |\deg(u) - \deg(v)| < 1 \text{ and } \mu(uv) \leq \sigma(u) \wedge \sigma(v)\}$ and $u \in I_e \Leftrightarrow N_{fe}(u) = \phi$ The cardinality of $N_{fe}(u)$ is called fuzzy equitable degree of u and is denoted by $d_{fe(G)}(u)$.

The Maximum and minimum fuzzy equitable degree of a vertex in G are denoted by $\Delta_{fe}(G)$ *and* $\delta_{fe}(G)$ *that is* $\Delta_{fe}(G) = \max_{u \in V(G)} |N_{fe}(u)|$ *and* $\delta_{fe}(G) = \min_{u \in V(G)} |N_{fe}(u)|$

Example



S={a,c,d,f} is an equitable dominating set. $\langle S \rangle = \langle a, c, d, f \rangle$ is disconnected

S is an split equitable dominating set.

Proposition: 1 For any fuzzy graph G $(i)\ \gamma_{fe}(G) \leq \gamma_{fse}(G)$ $(ii)\ \gamma_{fs}(G) \leq \gamma_{fse}(G)$

Proof:

Let S be the minimum split equitable dominating set of G. Now, since S is a split equitable dominating set then S is equitable dominating set. Hence $\gamma_{fe}(G) \leq |S| = \gamma_{fse}(G)$

Theorem: 2

A split equitable dominating set S of G is minimal for each vertex $v \in S$ one of the following three conditions holds

(i) there exists a vertex $u \in V - S$ such that $N_e(u) \cap S = \{v\}$

(ii) v is an equitable isolated vertex in $\langle S \rangle$

(iii) $\langle (V - S) \cup \{v\} \rangle$ is disconnected.

Proof:
Suppose S is is minimal and there exists a vertex $v \in S$ such that v does not hold any of the above conditions. Then by conditions (i) and (ii) $S_1 = S - \{v\}$ is a equitable dominating set of G.Also by (iii) $\langle V - S \rangle$ is disconnected.This implies that S1 is a split equitable dominating set of G, a contradiction.

Theorem: 3
If G is regular fuzzy graph or (k,k+1) bi regular fuzzy graph for some k then $\gamma_{fe}(G) = \gamma_f(G)$

Theorem: 4
If G is regular or bi-regular fuzzy graph with atleast one equitable end vertex, then $\gamma_f(G) = \gamma_{fs}(G) = \gamma_{fse}(G) = \gamma_{fe}(G)$

Proof:
It is clear that if G is regular or bi-regular fuzzy graph and let S be an split equitable dominating set of G such that $|S| = \gamma_{fse}(G)$ .Then S is dominating set which intersect every maximum split dominating set of G and by theorem (3), S is an equitable dominating set intersect every maximum independent set of G. Therfore $\gamma_{fse}(G) \leq \gamma_{fs}(G)$, $\gamma_{fs}(G) \leq \gamma_{fse}(G)$

Hence if G is regular fuzzy graph then $\gamma_{fs}(G) \leq \gamma_{fse}(G)$ .Similarly we can prove that if G is (k,k+1) bi-regular fuzzy graph for some $k \geq 0$ then $\gamma_{fs}(G) \leq \gamma_{fse}(G)$

Theorem: 5
Let G be a fuzzy graph of order P then $(i)\ \gamma_{fe}(G) \leq \gamma_{fse}(G) \leq P - \Delta_{fe}(G)$

Proof:
Every fuzzy split equitable dominating set is equitable dominating set of G, $\gamma_{fe}(G) \leq \gamma_{fse}(G)$ .Let $u, v \in V.\ If\ d_{fe}(u) = \Delta_{fe}(G)$ and $d_{fe}(v) = \delta_{fe}(G)$ .clearly $V - N_{fe}(u)$ is a split fuzzy equitable dominating set. $\therefore \gamma_{fse}(G) \leq |V - N_e(u)|_{fe}$. (i.e) $\gamma_{fse}(G) \leq P - \Delta_{fe}(G)$.

Theorem: 6
If S is a split equitable dominating set of a graph G then S is a both minimal fuzzy split equitable dominating set and a maximal fuzzy split equitable dominating set. Conversely any maximal fuzzy split equitable dominating set S in G is a fuzzy split equitable dominating set of G.

Proof:
If S is a fuzzy split equitable dominating set of G. $S_d = S - \{d\}$ is not a fuzzy split equitable dominating set for every $d \in S \quad and \; S \cup \{x\}$ is not a fuzzy split equitable dominating set for every $x \notin S$ so that S is a minimal fuzzy equitable dominating set and a maximal fuzzy split equitable dominating set.
Conversely, let S be a maximal fuzzy split equitable dominating set in G.Then for every $x \in V - S, \; S - \{x\}$ is not fuzzy split equitable dominating set and x is dominated by some element of S.Thus S is a fuzzy split equitable dominating set of G.

Theorem: 7
Let G be a fuzzy graph then $\gamma_{fse}(G) \leq P - \Delta_{fse}(G)$

Proof:
Let v be a vertex such that $d_{fse(G)}v = \Delta_{fse}. \; Then \;\; V - N_{sfe}(v)$ is a fuzzy split equitable dominating set of G so that $\gamma_{fse}(G) \leq \left|V - N_{sfe}(v)\right| = P - \Delta_{fse}(G)$.

Theorem: 8
For any fuzzy graph P_n with $n \geq 5$ vertices $\gamma_{fse}(\overline{p_n}) \leq P - 3$

Proof:
From the definition of the complete graph, it is clear that P_n is bi- regular fuzzy graph with degree n-2 or n-3, then the fuzzy split equitable dominating exists for $\overline{P_n}$ only if $n \geq 4. \; If \;\; n = 4 \;\; \overline{P_n} \cong P_n, \, hence \;\; \gamma_{fse}(P_n) = 2. When \;\; n \geq 5 \;\; , Let \;\; S = \{V_4, V_{5,} \ldots V_n\}$
Then $S \neq \phi$ and $V - S \;\; \{V_1, V_{2,} \, V_3\}$. $V(\overline{P_n}) - S = \{V_1 \, , V_2, V_3\},$ it is clear that all the vertices in $V(\overline{P_n}) - S$ are equitable dominating set of G and V_2 is isolated vertex in $\langle V(\overline{P_n}) - S \rangle$ that means $\langle V(\overline{P_n}) - S \rangle$ is disconnected. Hence S is fuzzy split equitable dominating set of $\overline{P_n}. \; Therefore \;\; \gamma_{fse}(\overline{P_n}) \leq \; P - 3. \, When \;\; P \geq 5.$

REFERENCES

[1] Kulli .V.R. and Janakiram.B, The split domination number of a graph, Graph Theory notes of NewYork, Newyork Academy of sciences XXXII (1997), (16-19)

[2] Nagoor Gani. A and Chandrasekaran.V.T. Domination in Fuzzy Graph Advances in Fuzzy sets and systems 1(1) (2006) (17-16)

[3] Puttaswamy, Murthy K.B " The Split Equitable domination number of a Graph"

[4] Rosenfeld .A Fuzzy Graphs in: Zadeh.L.a.Fu,K.S., Shimura,M(eds)."Fuzzy sets and their Applications",Academic Press, Newyork 1975.

[5] Sivakumar.S, Soner.N.D, Anwar Alwardi, Deepak, Pure Mathematical Sciences, Vol-1,2012,no.3,123-130.

[6] Somasundaram.A, Somasundaram.S. " Domination in Fuzzy Graphs-I"Elsevier Sciences (1998)787-791.

[7] Venkatasubramanian Swaminathan and kuppusamy Markandan Dharmalingam, Degree equitable domination on graphs,Kragujevac Journal of Mathematics Volume 35, Number 1,PP(177-183)2011.