

Solving Fuzzy Transportation Problem using Ranking of Trapezoidal Fuzzy Numbers

¹P.Maliniand²Dr. M.Ananthanarayanan

¹*Assistant Professor, Department of Mathematics, Jeppiaar Engineering College, Chennai- 119, Tamil Nadu, India.*

²*Associate Professor, Department of Mathematics, A.M.JAIN College, Chennai-600 114, Tamil Nadu, India.*

Abstract

Ranking fuzzy numbers play a vital role in decision making problems, data analysis and socio economics systems. Ranking fuzzy numbers is a necessary step in many mathematical models. Many of the methods proposed so far are non-discriminating. This paper presents a new ranking method and using which we convert the fuzzy transportation problem to a crisp valued transportation problem which then can be solved using MODI Method to find the fuzzy optimal solution. Numerical examples show that the fuzzy ranking method offers a impressive tool for handling the fuzzy transportation problems.

Keywords: Trapezoidal fuzzy numbers, Ranking function, Fuzzy Transportation Problem.

1. INTRODUCTION

Transportation problem is used globally in solving certain concrete world problems. A transportation problem plays a vital role in production industry and many other purposes. The transportation problem is a special case of Linear programming problem, which permit us to regulate the optimum shipping patterns between origins and destinations. The solution of the problem will empower us to determine the number of entities to be transported from a particular origin to a particular destination so that the cost obtained is minimum or the time taken is minimum or the profit obtained is maximum. Let a_i be the number of units of a product available at origin i and b_j be the number of units of the product required at destination j . Let C_{ij} be the

cost of transporting one unit from origin i to destination j and let x_{ij} be the amount of quantity carried or shipped from origin i to destination j . A fuzzy transportation problem is a transportation problem in which the transportation expenditures, supply and demand quantities are fuzzy quantities. Ranking fuzzy number is a necessary step in many mathematical models. The concepts of fuzzy sets were first introduced by Zadeh[14]. Since its inception several ranking procedure have been developed. There onwards many authors presented various approaches for solving the FTP problems[1],[2],[4]. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani[3]. Presently Chen and H Wang[5] reviewed the existing method for ranking fuzzy numbers and each approach has drawbacks in some aspects such as indiscrimination and finding not so easy to interpretate. As of now none of them is completely accepted.

Ranking normal fuzzy number were first introduced by Jain[7] for decision making in fuzzy situations. Chan stated that in many situations it is not possible to restrict the membership function to the general form and proposed the concept of generalized fuzzy numbers. Since then remarkable efforts are made on the development of numerous methodologies. The development in ordering fuzzy numbers can even be found in [8],[9],[11],[12],[13]. Fuzzy numbers must be ranked before a decision is taken by a decision maker.

In this paper a new method is presented for the ranking of generalized fuzzy trapezoidal numbers. To illustrate this proposed method, an example is discussed. As the proposed ranking method is very direct and simple it is very easy to understand and using which it is easy to find the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations.

This paper is organized as follows: Section 2 briefly introduced the basic definition of fuzzy numbers. In section 3, a new ranking procedure is proposed. In section 4, MODI method is adopted to solve Fuzzy transportation problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition

If X is a set of objects denoted generally by X , then a fuzzy set A in X is defined as a set of ordered pairs $A = \{(x, \mu_A(x)) \mid x \in X\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set A the membership function maps each element of X to a membership value between 0 and 1.

2.2 Definition

A fuzzy set A is defined on universal set of real numbers is said to be a generalized fuzzy number if its membership function has the following characteristics

- (i) $\mu_A(x) : \mathbb{R} \rightarrow [0,1]$ is continuous
- (ii) $\mu_A(x) = 0$ for all $x \in A (-\infty, a] \cup [d, \infty)$
- (iii) $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
- (iv) $\mu_A(x) = \omega$ for all $x \in [b, c]$, where $0 < \omega \leq 1$

2.3 Definition

A generalized fuzzy number $A = (a, b, c, d, \omega)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{\omega(x-a)}{b-a}, & a \leq x \leq b \\ \omega, & b \leq x \leq c \\ \frac{\omega(x-a)}{d-c}, & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

If $\omega = 1$, then $A = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and A is a generalized or non normal trapezoidal fuzzy number if $0 < \omega < 1$

As a particular case if $b=c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $A = (a, b, d; 1)$

2.4 Definition

Let $A_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $A_2 = (a_2, b_2, c_2, d_2; \omega_2)$ be generalized trapezoidal fuzzy number defined on real numbers \mathbb{R} then

- (i) $A_1 + A_2 = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; \min(\omega_1, \omega_2))$
- (ii) $A_1 - A_2 = (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2; \min(\omega_1, \omega_2))$

3. RANKING OF TRAPEZOIDAL FUZZY NUMBERS

In this section, a new approach for ranking of generalized trapezoidal number is proposed using trapezoid as reference point. Ranking methods map fuzzy number directly in to the real line. That is, $M : F \rightarrow \mathbb{R}$ which associate every fuzzy number with a real number and then use the ordering \geq on the real line.

Let $A = (a_1, b_1, c_1, d_1 ; \omega_1)$ be generalized trapezoidal fuzzy numbers then $R(A)$ is calculated as follows:

Step 1 :Find $\omega = \text{minimum}(\omega_1, \omega_2)$

Step2:Find $R(A) = \frac{\omega}{4} [k(a_1+d_1)+2(1-k)(b_1+c_1)]$,

where $k \in (0,1)$

4. NUMERICAL EXAMPLE

We shall present a solution to fuzzy transportation problem involving shipping cost, customer demand and availability of products using trapezoidal fuzzy numbers. Consider the following fuzzy transportation problem.

S O U R C E	DESTINATION				SUPPLY
	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
DEMAND	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Step 1: Now by using the ranking technique , we convert the given fuzzy problem in to a crisp valued problem. The problem is done by taking the value of k as 0.5 and $\omega=1$.

The FTP is

1.875	2.625	8.625	5.75	4.875
1.25	0.375	4.875	1.125	1.125
4.125	6.375	11.625	7.125	8.25
5.625	4.125	2.625	1.875	

Step 2: Using VAM procedure we obtain the initial solution as

0.75	4.125		
			1.125
4.875		2.625	0.75

Which is not an optimal solution

Step 3:Hence by using the MODI method we shall get the optimal solution as

	4.125	0.75	
		1.125	
5.625		0.75	1.875

The crisp value of the optimum fuzzy transportation for the given problem is Rs68

CONCLUSIONS

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers. This work proposed a new ranking method which is simple and efficient. As mentioned by Wang and Kerre, it is not possible to give a final answer to the question on which fuzzy ranking method is the best. Most of the time choosing a method rather than another is a matter of preference. However, believing that the results obtained in this paper gives us the optimum cost for the fuzzy transportation problems.

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