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Lehmer-3 Mean Labeling of Some Disconnected Graphs

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Abstract

A graph G=(V,E) with p vertices and q edges is called Lehmer-3 mean graph, if it is possible to label vertices x∈V with distinct label f(x) from 1,2,3,………….q+1 in such a way that when each edge e=uv is labeled with f(e=uv)= $\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer-3 mean labeling of G. In this paper we investigate Lehmer-3 mean labeling of some standard graphs

Keywords: Graph, Path, Cycle, Comb.

1. INTRODUCTION

A graph considered here are finite undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. For detailed survey Gallian survey [1] is referred and standard terminologies and notations are followed from Harary [2]. We will find the brief summary of definitions and information necessary for the present investigation.

Definition 1.1

A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph, if it is possible to label vertices x ЄV with distinct label f(x) from 1,2,3,………….q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G.

Definition 1.2

A path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \leq i \leq n$

Definition 1.3

Comb is a graph obtained by joining a single pendant edge to each vertex of a path

Definition 1.4

A closed path is called a cycle of G.

Definition 1.5

$P_n \odot K_{1,2}$ is a graph obtained by attaching $K_{1,2}$ to each vertex of P_n

Definition 1.6

$P_n \odot K_{1,3}$ is a graph obtained from the path attaching $K_{1,3}$ to each of its vetices

Definition 1.7

$P_n \odot K_3$ is a graph connected by a complete graph K_3 in its each vertex

2. MAIN RESULTS

Theorem: 2.1

mC_n is a Lehmer -3 mean graph for $n \geq 3$ and $m \geq 1$

Proof:

Let the vertex set of mC_n be $V=V_1 \cup V_2 \cup \ldots\ldots\ldots\ldots.. \cup Vm$

where $V_i=\{v_i^1, v_i^2, \ldots\ldots\ldots v_i^m\}$ and the edge set of mCn is $E=E_1 \cup E_2 \cup \ldots\ldots\ldots. \cup E_m$, where $E_i=\{e_i^1, e_i^2, \ldots\ldots\ldots.. e_i^n\}$.

A function $f:V(mC_n)\rightarrow\{1,2,\ldots\ldots.q+1\}$ is defined as $f(u_i^j)=n(i-1)+j$;$1\leq i\leq m,1\leq j\leq n$.
Then the set of labels of the edges of mC_n are $\{1,2,\ldots\ldots.mn\}$
Hence mC_n is a lahmer -3 mean graph.

Example: 2.2

The Lehmer -3 mean labeling of $3C_6$ is shown below.

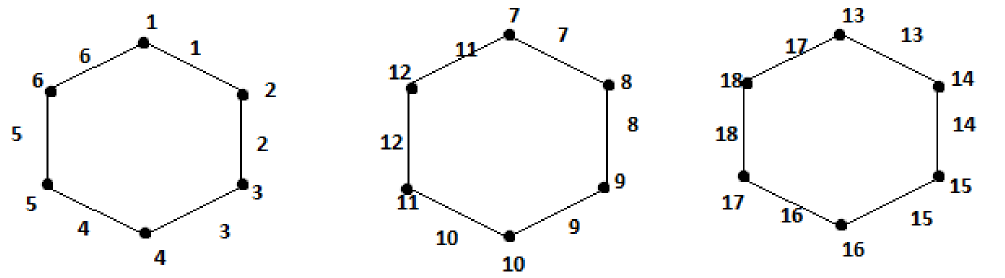

Figure:1

Theorem: 2.3

$mC_n \cup P_k$ is a Lehmer -3 mean graph for $m,K\geq 1$ and $n\geq 3$

Proof:

let mC_n be the m copies of C_n and P_k be the path of length k, the vertex set of mC_n be $V=V_1\cup V_2\cup\ldots\ldots\cup V_m$ where $V_i=\{v_i^1,v_i^2,\ldots\ldots v_i^n\}$ and the edge set of mC_n is $E=E_1\cup E_2\cup\ldots\ldots.\cup E_m$. where $E_i=\{e_i^1,e_i^2\ldots..e_i^n\}$

Let $U_1,U_2, \ldots..U_k$ be the vertices of P_k.

The function $f:V(mC_n\cup P_k)\rightarrow\{1,2,\ldots\ldots q+1\}$ is defined as $f(v_i^j)=n(i-1)+j;1\leq i\leq m,1\leq j\leq n$

and $f(u_i)=mn+i$;$1\leq i\leq k$.

Then the set of labels of edges of mC_n are distinct $\{1,2,\ldots..mn\}$

The set of labels of edges of P_k is $\{mn+1,mn+2,\ldots\ldots mn+k-1\}$

Thus $mC_n \cup P_k$ forms a Lehmer -3 mean graph.

Example: 2.4

Lehmer -3 mean labeling of $3C_6 \cup P_5$ is given below.

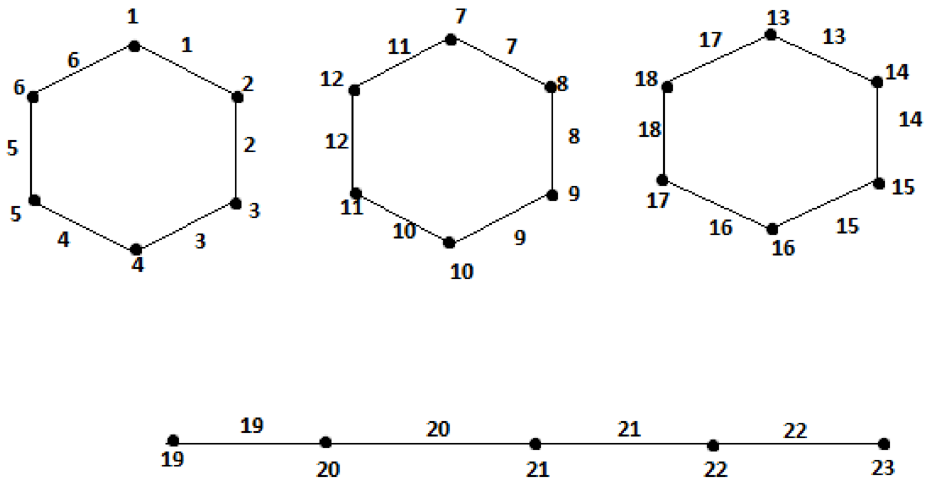

Figure:2

Theorem: 2.5

$mC_n \cup C_k$ is a Lehmer -3 mean graph for $m \geq 1$ and $n,k \geq 3$

Proof:

Let mC_n be the m copies of C_n and C_k be any cycle with K vertices. The graph has mn+k number of vertices and edges. The vertex set of mC_n be $V=V_1 \cup V_2 \cup \ldots\ldots \cup V_m$,where $V_i=\{V_i^1,v_i^2,\ldots\ldots..V_i^n\}$ and the edge set of mC_n is $E=E_1 \cup E_2 \cup \ldots\ldots \cup E_m$ where $E_i=\{e_i^1,e_i^2,\ldots..e_i^n\}$

Let $u_1,u_2,\ldots.u_n$ be the cycle C_k .

Define a function $f:V(mC_n \cup C_k) \rightarrow [1,2,\ldots.q+1\}$ as $f(v_i^j)=n(i-1)+j$; $1 \leq i \leq m$, $1 \leq j \leq n$,

$f(u_i)=mn+i$; $1 \leq i \leq k$

Then the edges of mC_n and C_k are$\{1,2,\ldots.mn\}$ and $\{mn+1,mn+2,\ldots\ldots..mn+k\}$respectively

Hence $mC_n \cup C_k$ is a Lehmer-3 mean graph.

Example: 2.6

The graph $3C_4 \cup C_6$ has 18 vertices and the same number of edges. The pattern is given below.

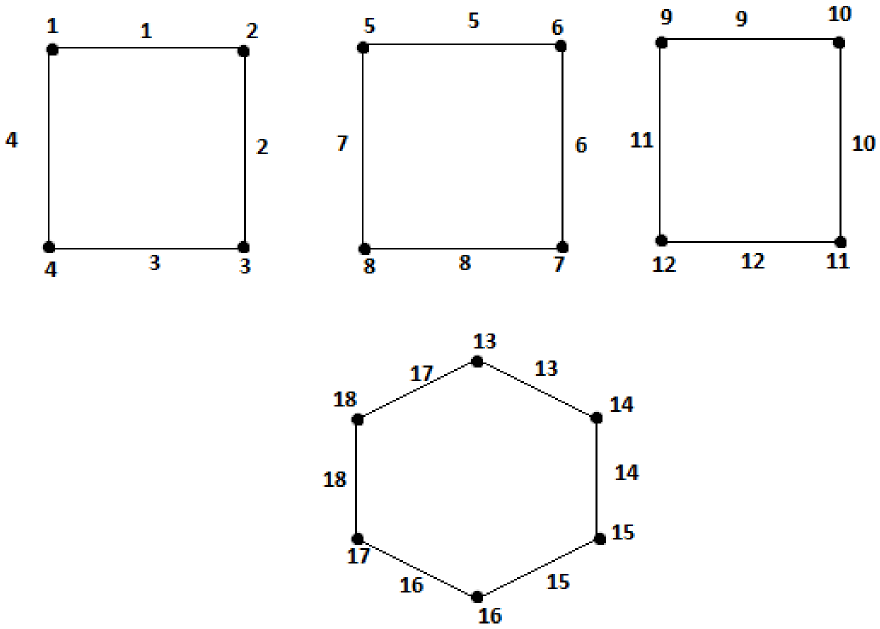

Figure: 3

Theorem: 2.7

m $C_n \cup (P_l \odot K_1)$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of m times C_n and $(P_l \odot K_1)$

C_n be a cycle with n vertices $u_1, u_2, \ldots u_n$ respectively

Let $(P_l \odot K_1)$ be a comb with vertices as $v_1, v_2 \ldots v_l$; $w_1, w_2 \ldots w_l$

Define a function f: V(G) → {1,2,… q+1} defined by

$f(u^j_i) = n(i-1) + j$; $1 \leq i \leq m,\ 1 \leq j \leq n$

$f(v_k) = mn + (2k-1)$; $1 \leq k \leq l$

$f(w_k) = mn + 2k$; $1 \leq k \leq l$

Thus we obtain distinct edge labelings.

Hence m $C_n \cup (P_l \odot K_1)$ be a Lehmer-3 mean graph

Example: 2.8

$3C_6 \cup (P_4 \odot K_1)$ is a Lehmer-3 mean graph

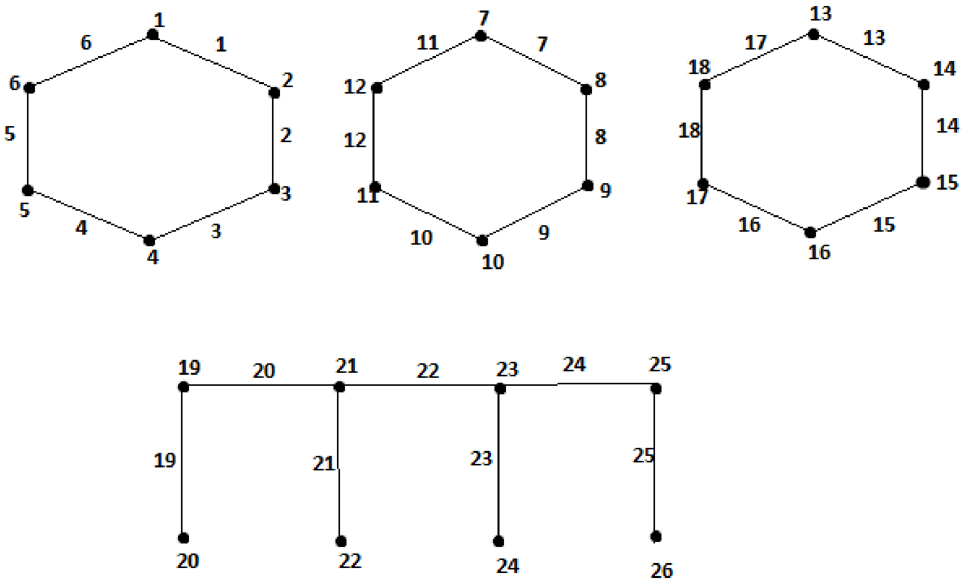

Figure: 4

Theorem: 2.9

$m\,C_n \cup (P_l \odot K_{1,2})$ be a Lehmer-3 mean graph

Proof:

Let G be a graph obtained from the union of mC_n and $(P_l \odot K_{1,2})$

Let mCn be the n copies of Cn and let $P_l \odot K_{1,2}$ be the graph with vertices $v_1, v_2 \ldots v_l$; $w_1, w_2 \ldots w_l$ and $z_1, z_2 .. z_l$.

Let the vertices of mC_n be $U = U_1 \cup U_2 \cup \ldots\ldots \cup U_n$ where $U_i = \{u_i^1, u_i^2, u_i^3 \ldots\ldots\ldots u_i^n\}$ and the edges of mC_n is $E = E_1 \cup E_2 U \ldots\ldots \cup E_n$, where $E_i = \{e_i^1, e_i^2, \ldots.. e_i^n\}$.

Let $V_1, V_2, \ldots\ldots V_l$ and $W_1, W_2, \ldots\ldots\ldots W_l, Z_1, Z_2, \ldots\ldots.. Z_l$ be the vertices of $(P_l \odot K_{1,2})$

Define a function $f: V(G) \rightarrow \{1, 2, \ldots\ldots q+1\}$ by

$f(u_i^j) = n(i-1)+j$; $1 \le i \le m$, $1 \le j \le n$

$f(v_k) = mn+(3k-2)$; $1 \le k \le l$

$f(w_k) = mn+(3k-1)$; $1 \le k \le l$

$f(z_k) = mn+(3k)$; $1 \leq k \leq l$

Thus the edge labelings are distinct .

Hence $mC_n \cup (P_l \Theta K_{1,2})$ is a Lehmer -3 mean graph.

Example: 2.10

$3C_4 \cup (P_5 \Theta K_{1,2})$ is a Lehmer -3 mean graph.

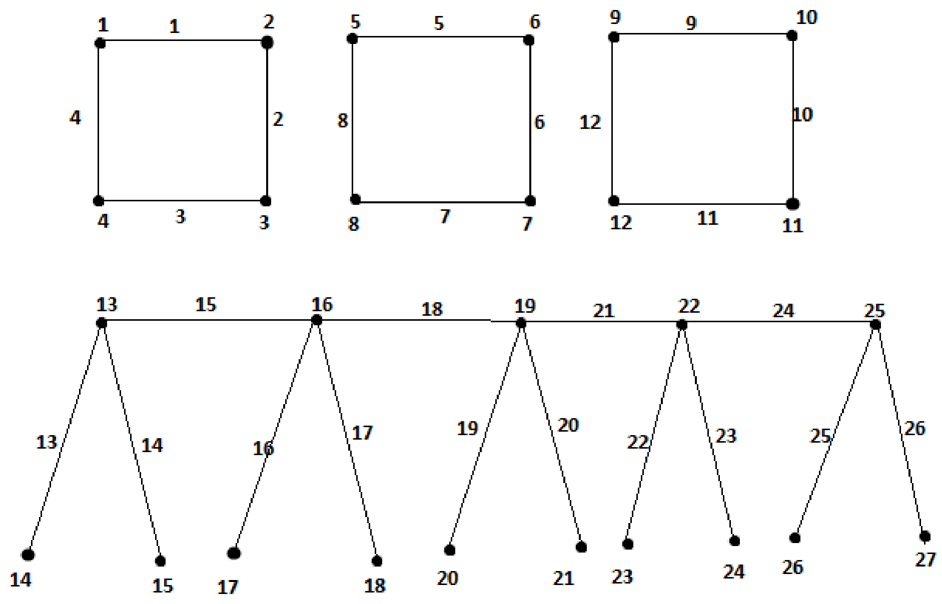

Figure: 5

Theorem: 2.11

$mC_n \cup (P_l \Theta K_{1,3})$ is a Lehmer -3 mean graph.

Proof:

Let G be a graph obtained from the union of mC_n and $(P_l \Theta K_{1,3})$

Let mC_n be the m copies of C_n and let $P_l \Theta K_{1,3}$ be the graph with $v_1,v_2 \ldots v_l$; $w_1,w_2 \ldots w_l$; $x_1,x_2 \ldots . x_l$ and $y_1,y_2 \ldots . y_l$ etc.

Let the vertices of mC_n be $U=U_1 \cup U_2 \cup U_3 \ldots \ldots . U_n$ where $U_i=\{U_i^1,U_i^2, \ldots . U_i^n\}$

And the edges of mC_n be $E=E_1 \cup E_2 \cup \ldots . E_n$ where $E_i=\{e_i^1,e_i^2 \ldots \ldots \ldots e_i^n\}$.

Let $v_1,v_2, \ldots \ldots . v_l$; $w_1,w_2, \ldots \ldots . . w_l$; $x_1,x_2 \ldots \ldots . x_l$; $y_1,y_2 \ldots y_l$ be the vertices of $(P_l \Theta K_{1,3})$

Define a function $f:V(G) \rightarrow \{1,2, \ldots . . q+1\}$ by

$f(u_i^j) =n(i-1)+j$; $1 \leq i \leq n$

$f(v_k)=mn+(4k-3)$; $1\leq k\leq l$

$f(w_k)=mn+(4k-2)$; $1\leq k\leq l$

$f(x_k)=mn+(4k-1)$; $1\leq k\leq l$

$f(y_k)=mn+4k$; $1\leq k\leq l$

Thus the edge labels are distinct.

Hence $mC_n\cup (P_l \odot K_{1,3})$ is a Lehmer -3 mean graph.

Example: 2.12

$3C_4\cup (P_4 \odot K_{1,3})$ is a Lehmer -3 mean graph.

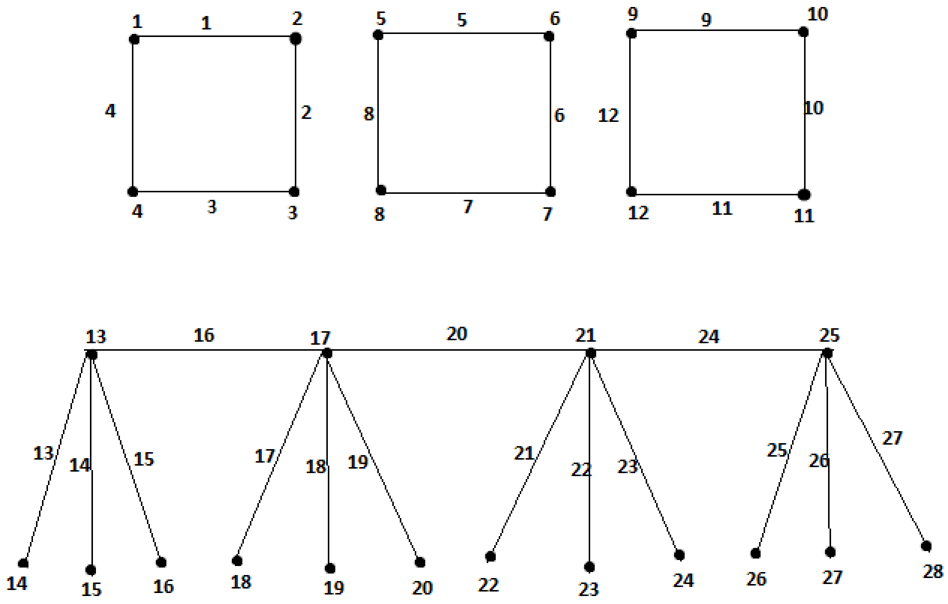

Figure : 6

Theorem:2.13

$m\,C_n \cup (P_l \odot K_3)$ be a Lehmer-3 mean graph

Proof

Let G be a graph obtained from the union of m times C_n and $(P_l \odot K_3)$

Let C_n be a graph with n vertices

Let $(P_l \odot K_3)$ be a graph with vertices as $v_1,v_2\ldots v_l$; $w_1,w_2\ldots w_l$ and $x_1,x_2..x_l$ respectively

Define a function f: V(G) → {1,2,… q+1} defined by

$f(u^j_i) = n(i-1) + j$; $1 \le i \le m,\ 1 \le j \le n$

$f(v_k) = mn + (4k - 3)$; $1 \le k \le l$

$f(w_k) = mn + (4k - 2)$; $1 \le k \le l$

$f(x_k) = mn + (4k - 1)$; $1 \le k \le l$

Thus the distinct edge labels are obtained

Hence $mC_n \cup (P_l \odot K_3)$ forms a Lehmer-3 mean graph

Example: 2.14

$3C_6$ A $(P_4 \odot K_3)$ is a Lehmer-3 mean graph

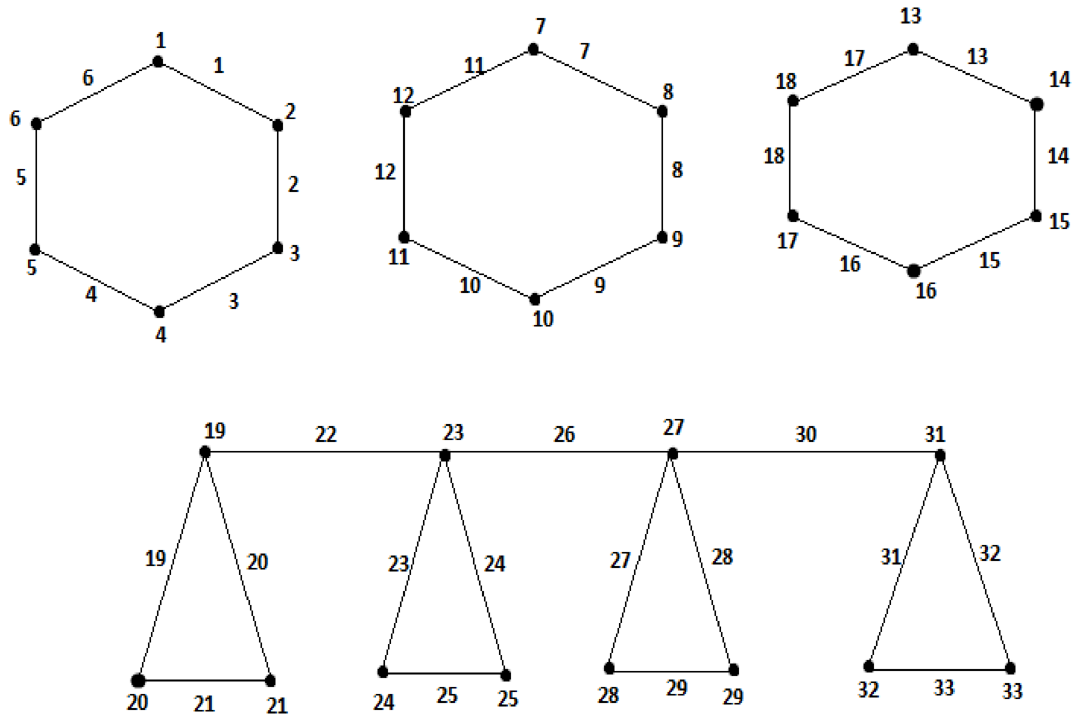

Figure :7

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