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Effect Of Rivlin-Ericksen Fluid On Mhd Fluctuating Flow With Heat And Mass Transfer Through A Porous Medium Bounded By A Porous Plate

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Abstract

In the present problem, a theoretical analysis of two dimensional flow of viscoelastic fluid (Rivlin-Ericksen type) fluid with heat and mass transfer through a porous medium bounded by a porous plate has been presented in the presence of transverse magnetic field. The suction velocity perpendicular to the plate fluctuates in magnitude but not in the direction about a nonzero constant mean and the free stream velocity fluctuates in time about nonzero constant mean.
The effects of Magnetic parameter (M), Porosity parameter (k) and Visco-elastic parameter (□) on the velocity and temperature of fluid are discussed with the help of Graphs.

Keywords: Visco-elastic (Rivlin-Ericksen) Fluid, Porous medium, MHD Flow, Heat transfer, Mass transfer, Fluctuating flow.

INTRODUCTION:

Many researchers have worked on fluctuating flows of viscous incompressible fluids past an infinite plate. Lighthill (1954) initiated the work on fluctuating flows, and studied an important class of two dimensional time dependent flow problems dealing with the response of the boundary layer to extend unsteady fluctuations about a mean value. After that Stuart (1955), SuryaprakashRao (1962, 63), Reddy (1964), Messiha (1966), Siddappa and Chetty (1975) and Om Prakash et al (1978) have appeared in the literature.
Flows through porous media are very much prevalent in nature and therefore the study of flows through porous media has become of principal interest in many

scientific and engineering applications. This type of the flows have shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoir; in chemical engineering for filtration and water purification processes. Further to study the underground water resources, seepage of water in river beds also one needs to investigate the flows of fluid through porous media. Thus, there are a numerous practical uses of fluid flows through porous media.

The modern theory of flow through porous media is mainly based on a simple experiment first performed by Darcy (1956). The very nature of heuristic derivation of Darcy's law suggests that it is probably valid only in a certain limited seepage velocity domain outside which different and more general laws would hold. Recently Ahmadi and Manvi (1971) have studied some general flow problem based on drag theory of permeability initiated by Emersleben (1924-25) and a modern exposition of which has been given by Brinkman (1947-48); Iberall (1960) and by Happel (1959).

In the drag theory of permeability the walls of the pores are treated as obstacles to an otherwise straight flow of viscous fluids. The drag of the fluid on each portion of the walls is estimated from the navier stokes equations, and the sum of the all the drags is thought to be equal to the porous medium to flow (i.e. equal to μ/K, according to Darcy Law). It is to be expected that drag theory would give the good results in the case of highly porous medium in which the porosity is very close to unity. Gulab Ram and Mishra (1977) extended the work of Ahmadi and Manvi (1971) for MHD flow problem. Naval Kishore (1979) extended the work of Gulab Ram and Mishra (1977) for fluctuating free stream flow. Kumari and Varshney (2006) have analysed MHD fluctuating flow of viscous fluid with mass transfer through a porous medium bounded by a porous plate. Recently, Varshney and Sharma (2010) have discussed effect of heat transfer on MHD fluctuating flow of viscous fluid with mass transfer through a porous medium bounded by a porous plate.

Our problem under study is an extension of the problem of Varshney and Sharma (2010) with viscoelastic fluid (Rivlin-Ericksen type) fluid. The object of this study is to find the effects of magnetic field (M), permeability of the porous medium (k) and Visco-elastic parameter (λ)on the velocity, temperature and concentration of fluid.

MATHEMATICAL ANALYSIS:

Let us consider the flow of the fluid flow density ρ and viscosity μ through a porous medium of permeability k' occupying a semi infinite region of the space bounded by a porous plate. Let u' and v' be the components of velocity in x' and y' directions respectively taken along and perpendicular to the plate. Let $u'(t')$ be the free stream velocity parallel to the plate. Let the velocity components be independent of x'. A constant magnetic field of strength H_o is applied in y' direction. The equation of the flow of viscous and incompressible electrically conducting fluid through a highly porous medium are,

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} + \mu\left(1 + \lambda'\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2}$$

$$-(\sigma B_o^2+\frac{\mu}{k'})u'+g\beta(T'-T'_\infty)+g\beta'(C'-C'_\infty) \quad (1)$$

$$\rho\frac{\partial v'}{\partial t'}=-\frac{\partial p'}{\partial y'}-\frac{\mu}{k'}v' \quad (2)$$

$$\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\frac{\lambda}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\left(\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}\right)=D\frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

$$\frac{\partial v'}{\partial y'}=0 \quad (5)$$

for free stream

$$\rho\frac{dU'}{dt'}=-\frac{\partial p'}{\partial x'}-\sigma B_o^2 U'-\frac{\mu U'}{k'} \quad (6)$$

where all symbols have their usual meanings.
Let the fluctuating free stream and suction velocities be

$$U'=U_o(1+\varepsilon e^{in't'})\quad v'=-v_o(1+A\varepsilon e^{in't'}) \quad (7)$$

where U_o and v_o are the mean free stream and mean suction velocities respectively $\varepsilon << 1$ and A is such that $A\varepsilon < 1$, n' is the frequency of the fluctuations.
The boundary conditions are,

$u'=0,\ T'=T_w',\ C'=C_w'$ at $y'=0$ (8)

$u' \to U'(t'),\ T' \to \infty,\ C' \to \infty$ as $y' \to \infty$

Eliminating between (1) and (6), we have

$$\frac{\partial p'}{\partial x'}$$

$$\rho\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=-\rho\frac{\partial U'}{\partial t'}+\mu(1+\lambda'\frac{\partial}{\partial t'})\frac{\partial^2 u'}{\partial y'^2}$$

$$+(\sigma B_o^2+\frac{\mu}{k'})(U'-u')+g\beta(T'-T'_\infty)+g\beta'(C'-C'_\infty) \quad (9)$$

Let us introduce the following non-dimensional quantities

$$n = \frac{n'\nu}{v_o^2} \quad t = \frac{t'v_o^2}{\nu} \quad y = \frac{y'v_o}{\nu} \quad U = \frac{U'}{U_o} \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty} \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$$

$$u = \frac{u'}{U_o} \quad v = \frac{v'}{v_o}$$

$$M = \frac{\sigma\nu}{\rho}\frac{B_0^2}{v_0^2}$$

(Magnetic parameter),

$$k = \frac{k'v_o^2}{\nu^2}$$

(Porosity parameter)

$$G = \frac{g\beta(T'_w - T'_\infty)\nu}{U_o v_o^2}$$

(Grashof Number),

$$G' = \frac{g\beta'(C'_w - C'_\infty)\nu}{U_o v_o^2}$$

(Modified Grashof Number),

$$P_r = \frac{\mu C_p}{\lambda}$$

(Prandtle Number),

$$S_c = \frac{\nu}{D}$$

(Schmidt Number),

$$\alpha = \frac{\lambda' v_o^2}{\nu}$$

(Visco-elastic parameter),
where

$\nu = \dfrac{\mu}{\rho}$ is the Kinematic viscosity of the fluid.

With the use of non-dimensional quantities the equation (9), (3) and (4) yield

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{dU}{dt}+(1+\alpha\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2}$$

$$+(M+\frac{1}{k})(U-u)+G\theta+G'\phi \qquad (10)$$

$$\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} \qquad (11)$$

$$\frac{\partial\phi}{\partial t}+v\frac{\partial\phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2} \qquad (12)$$

The non-dimensional form of eq. (7) is

$$U=(1+\varepsilon e^{int})$$

$$v=-(1+A\varepsilon e^{int}) \qquad (13)$$

The boundary conditions (8) become

$u=0,\ \theta=1,\ \phi=1$ at $y=0$

$u\to U,\ \theta=0,\ \phi=0$ as $y\to\infty$ (14)

To solve equation (10) to (12) with help of (13) and (14), let us follow Lighthill's method and we assume

$u(y, t)=f_1(y) + \varepsilon f_2(y)\, e^{int}$

$\theta(y, t)=h_1(y) + \varepsilon h_2(y)\, e^{int}$

And

$\phi(y, t)=g_1(y) +$ □ $g_2(y)\, e^{int}$........ (15)

Substituting (15) in equations (10) to (12) we obtain the following differential equations for the function $f_1(y)$, $f_2(y)$, $g_1(y)$, $g_2(y)$, $h_1(y)$ and $h_2(y)$ by neglecting squares of □ andseparating harmonic and non-harmonic terms,

$$f_1''+f_1'-(M+\frac{1}{k})f_1=-(M+\frac{1}{k})+Gh_1+G'g_1 \qquad (16)$$

$$(1+in\alpha)f_2''+f_2'-(M+in+\frac{1}{k})f_2=-(M+in+\frac{1}{k})$$

$$+Gh_2+G'g_2-Af_1' \qquad (17)$$

$$h_1''+P_r h_1'=0 \qquad (18)$$

$$h_2'' + P_r h_2' - inP_r h_2 = -AP_r h_1' \tag{19}$$

$$g_1'' + S_c g_1' = 0 \tag{20}$$

$$g_2'' + S_c g_2' - inS_c g_2 = -AS_c g_1' \tag{21}$$

with boundary conditions

$f_1=0$, $h_1=1$, $g_1=1$ at $y=0$ (22)

$f_1 \to 1$, $h_1 \to 0$, $g_1 \to 0$, as $y \to \infty$

$f_2=0$, $h_2=0$, $g_2=0$, at $y=0$

$f_2 \to 1$, $h_2 \to 0$, $g_2 \to 0$, as $y \to \infty$ (23)

We get the velocity, temperature and concentration as equation (15) by solving the equations from (16) to (21) with boundary conditions (22) and (23)

$$f_1 = 1 - e^{-Ry} + G_1(e^{-P_r y} - e^{-Ry}) + G_2(e^{-S_c y} - e^{-Ry}) \tag{24}$$

$$\begin{aligned} f_2 = [e^{-Hy}\{-1 + \frac{AR}{R^2 - R - M'} + AG_1(\frac{R}{R^2 - R - M'} - \frac{P_r}{P_r^2 - P_r - M'}) \\ + AG_2(\frac{R}{R^2 - R - M'} - \frac{S_c}{S_c^2 - S_c - M'}) \\ - \frac{AGP_r}{in}(\frac{1}{P^2 - P - M'} - \frac{1}{P_r^2 - P_r - M'}) \\ - \frac{AG'S_c}{in}(\frac{1}{S^2 - S - M'} - \frac{1}{S_c^2 - S_c - M'})\} + 1 - \frac{ARe^{-Ry}}{R^2 - R - M'} \\ - AG_1(\frac{Re^{-Ry}}{R^2 - R - M'} - \frac{P_r e^{-P_r y}}{P_r^2 - P_r - M'}) \\ - AG_2(\frac{Re^{-Ry}}{R^2 - R - M'} - \frac{S_c e^{-S_c y}}{S_c^2 - S_c - M'}) \\ - \frac{GAP_r}{in}(\frac{e^{-Py}}{P^2 - P - M'} - \frac{P_r e^{-P_r y}}{P_r^2 - P_r - M'})] \\ - \frac{G'AS_c}{in}(\frac{e^{-Sy}}{S^2 - S - M'} - \frac{S_c e^{-S_c y}}{S_c^2 - S_c - M'})] \end{aligned} \tag{25}$$

$$h_1 = e^{-P_r y} \tag{26}$$

$$h_2 = \frac{AP_r}{in}(e^{-Py} - e^{-P_r y}) \tag{27}$$

$$g_1 = e^{-S_c y} \tag{28}$$

$$g_2 = \frac{AS_c}{in}(e^{-Sy} - e^{-S_c y}) \tag{29}$$

where

$$R = \frac{1 + \sqrt{1 + 4(M + 1/k)}}{2}$$

$$G_1 = \frac{G}{P_r^2 - P_r - (M + 1/k)}$$

$$G_2 = \frac{G'}{S_c^2 - S_c - (M + 1/k)}$$

$$P = P_1 + iP_2 = \frac{P_r + \sqrt{P_r^2 + 4inP_r}}{2}$$

$$P_1 = \frac{1}{2}[P_r + \{\frac{\sqrt{P_r^4 + 16n^2P_r^2} + P_r^2}{2}\}^{1/2}],$$

$$P_2 = \frac{1}{2}[\{\frac{\sqrt{P_r^4 + 16n^2P_r^2} - P_r^2}{2}\}^{1/2}]$$

$$S = S_r + iS_i = \frac{S_c + \sqrt{S_c^2 + 4inS_c}}{2}$$

$$S_r = \frac{1}{2}[S_c + \{\frac{\sqrt{S_c^4 + 16n^2S_c^2} + S_c^2}{2}\}^{1/2}]$$

$$S_i = \frac{1}{2}[\{\frac{\sqrt{S_c^4 + 16n^2S_c^2} - S_c^2}{2}\}^{1/2}]$$

$$H = H_r + iH_i = \frac{1+\sqrt{1+4M'}}{2(1+in\alpha)} = \frac{1+\sqrt{K_1+4in_1}}{2(1+in\alpha)}$$

$$M' = (M + \tfrac{1}{k} + in)(1+in\alpha)$$

$$K_1 = 1 + 4(M + \frac{1}{k} - n^2\alpha)$$

$$n_1 = n\alpha(M + \frac{1}{k}) + n$$

$$H_r = \frac{1 + H_r^* + H_i^* n\alpha}{2(1+n^2\alpha^2)}$$

$$H_i = \frac{H_i^* - (1+H_r^*)n\alpha}{2(1+n^2\alpha^2)}$$

$$H_r^* = \frac{1}{2}[1 + \{\frac{\sqrt{K_1^2+16n_1^2}+K_1}{2}\}^{1/2}]$$

$$H_i^* = \frac{1}{2}[\{\frac{\sqrt{K_1^2+16n_1^2}-K_1}{2}\}^{1/2}]$$

for physical significance we take only the real part of velocity

$$u = f_1(y) + \varepsilon(C_r \cos nt - C_i \sin nt) \tag{30}$$

where

$$C_r = e^{-H_r y}[-\cos H_i y + \frac{AR}{K_2^2+n^2}(K_2 \cos H_i y + n \sin H_i y)$$

$$+ AG_1\{\frac{R(K_2 \cos H_i y + n \sin H_i y)}{K_2^2+n^2} - \frac{P_r(K_6 \cos H_i y + n \sin H_i y)}{K_6^2+n^2}\}$$

$$- \frac{AGP_r}{n}\{\frac{(K_7 \cos H_i y + K_8 \sin H_i y)}{K_7^2+K_8^2} - \frac{(K_6 \sin H_i y - n \cos H_i y)}{K_6^2+n^2}\}$$

$$+AG_2\{\frac{R(K_2\cos H_iy+n\sin H_iy)}{K_2^2+n^2}-\frac{S_c(K_3\cos H_iy+n\sin H_iy)}{K_3^2+n^2}\}$$

$$-\frac{AG'S_c}{n}\{\frac{(K_5\cos H_iy+K_4\sin H_iy)}{K_4^2+K_5^2}-\frac{(K_3\sin H_iy-n\cos H_iy)}{K_3^2+n^2}\}]$$

$$+1-\frac{ARK_2e^{-Ry}}{K_2^2+n^2}-AG_1(\frac{RK_2e^{-Ry}}{K_2^2+n^2}-\frac{P_rK_6e^{-P_ry}}{K_6^2+n^2})$$

$$-AG_2(\frac{RK_2e^{-Ry}}{K_2^2+n^2}-\frac{S_cK_3e^{-S_cy}}{K_3^2+n^2})$$

$$-\frac{AGP_r}{n}\{\frac{e^{-P_1y}(K_7\cos P_2y+K_8\sin P_2y)}{K_7^2+K_8^2}-\frac{e^{-P_ry}(K_6\sin P_ry-n\cos P_ry)}{K_6^2+n^2}\}$$

$$-\frac{AG'S_c}{n}\{\frac{e^{-S_ry}(K_5\cos S_iy+K_4\sin S_iy)}{K_4^2+K_5^2}-\frac{e^{-S_cy}(K_3\sin S_cy-n\cos S_cy)}{K_3^2+n^2}\}$$

$$C_i=e^{-H_ry}[\sin H_iy+\frac{AR}{K_2^2+n^2}(n\cos H_iy-K_2\sin H_iy)$$

$$+AG_1\{\frac{R(n\cos H_iy-K_2\sin H_iy)}{K_2^2+n^2}-\frac{P_r(n\cos H_iy+K_6\sin H_iy)}{K_6^2+n^2}\}$$

$$-\frac{AGP_r}{n}\{\frac{(K_7\cos H_iy-K_8\sin H_iy)}{K_7^2+K_8^2}-\frac{(n\cos H_iy-K_6\sin H_iy)}{K_6^2+n^2}\}]$$

$$+AG_2\{\frac{R(n\cos H_iy-K_2\sin H_iy)}{K_2^2+n^2}-\frac{S_c(n\cos H_iy+K_3\sin H_iy)}{K_3^2+n^2}\}$$

$$-\frac{AG'S_c}{n}\{\frac{(K_5\cos H_iy-K_4\sin H_iy)}{K_4^2+K_5^2}-\frac{(n\cos H_iy-K_3\sin H_iy)}{K_3^2+n^2}\}]$$

$$-\frac{nARe^{-Ry}}{K_2^2+n^2}-AG_1(\frac{nRe^{-Ry}}{K_2^2+n^2}-\frac{nP_re^{-P_ry}}{K_6^2+n^2})$$

$$-\frac{AGP_r}{n}\{\frac{e^{-R_1 y}(K_8\cos P_2 y - K_7\sin P_2 y)}{K_7^2+K_8^2}-\frac{e^{-P_r y}(n\cos P_r y - K_6\sin P_r y)}{K_6^2+n^2}\}$$

$$-AG_2(\frac{n\,Re^{-Ry}}{K_2^2+n^2}-\frac{nS_c e^{-S_c y}}{K_3^2+n^2})$$

$$-\frac{AG'S_c}{n}\{\frac{e^{-S_r y}(K_5\cos S_i y - K_4\sin S_i y)}{K_4^2+K_5^2}-\frac{e^{-S_c y}(n\cos S_c y - K_3\sin S_c y)}{K_3^2+n^2}\}$$

and

$$K_2 = R^2 - R - (M+\frac{1}{k})$$

$$K_3 = S_c^2 - S_c - (M+\frac{1}{k})$$

$$K_4 = S_r^2 - S_i^2 - S_r - (M+\frac{1}{k})$$

$$K_5 = 2S_r S_i - S_i - n$$

$$K_6 = P_r^2 - P_r - (M+\frac{1}{k})$$

$$K_7 = P_1^2 - P_2^2 - P_1 - (M+\frac{1}{k})$$

$$K_8 = 2P_1P_2 - P_2 - n$$

At nt=π/2, the velocity equation will be as

$$u = 1 - e^{-Ry} + G_1(e^{-P_r y} - e^{-Ry}) + G_2(e^{-S_c y} - e^{-Ry}) - \varepsilon C_i \qquad (31)$$

RESULT AND DISCUSSION:

The Velocity Profile is plotted in Fig.-I having Graph-I to V at ε=0.2, A=0.5, n=1, t= □ /2,G=5,, S_c=0.5, P_r=0.5 and following different values of M, k and α.

	M	k	α
For Graph-I	2	0.2	1
For Graph-II	6	0.2	1
For Graph-III	2	0.8	1
For Graph-IV	2	0.2	4

From all the Graphs of Fig.-I it is observed that the velocity increases with the increase in y. It is also observed that the velocity increases with the increase in M, but it decreases with the increase in k and α.
The temperature and concentration do not changed with the change in above parameters.

PARTICULAR CASE:
When α is equal to zero, this problem reduces to the problem of Varshney and Sharma [2010].

CONCLUSION
Velocity of fluid decreases with the increase in α.

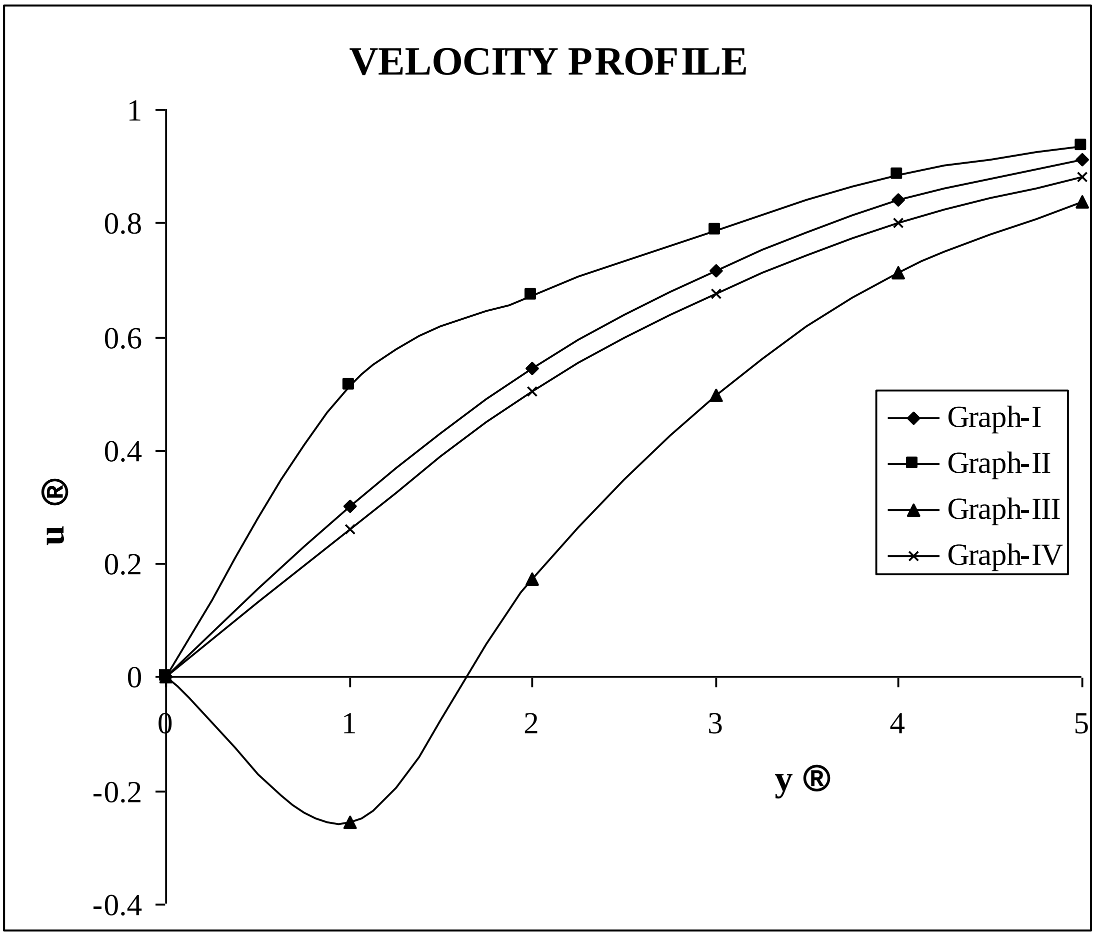

Figure 1

Table I: Values of velocity at ε=0.2, A=0.5, n=1, t=π/2, G=5, P_r=0.7, S_c=0.5 and $G' = 3$ different values of M, k and α.

y	Graph-I	Graph-II	Graph-III	Graph-IV
0	0	0	0	0
1	0.301999	0.515315	-0.25381	0.261999
2	0.545227	0.672413	0.173132	0.505227
3	0.71584	0.789089	0.496665	0.67584
4	0.841687	0.8841	0.714272	0.801687
5	0.91098	0.935703	0.836658	0.88098

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