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Strong λ- Bi Near Subtraction Semigroups

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Abstract

In this paper we introduce the notion of Strong λ- *bi-near subtraction semigroup. Also we give characterizations of* Strong λ- *bi-near subtraction semigroup.*

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Key words: Strong λ bi-near subtraction semigroup, S_1- bi-near subtraction semigroup, S_2- bi-near subtraction semigroup, Nil near subtraction semigroup.

1. INTRODUCTION

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[4]. Zekiye Ciloglu, Yilmaz Ceven [5] gave the notation of Fuzzy Near Subtraction semigroups. Seydali Fathima et.al[2,3] introduced the notation of S_1-near subtraction semigroup and S_2-near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of Strong λ- Bi near subtraction semigroup .

2. PRELIMINARIES

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction semigroup If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X. A non-empty subset X together with two binary operations "–" and "." is said to be near subtraction semigroup if (i) (X,–) is a subtraction algebra (ii) (X,.) is a semi group and (iii) (x−y)z= xz−yz for every x, y, z∈X. A non-empty subset X is said to be **S_1-near subtraction semigroup** if for every a∈X there exists x∈X^* such that axa=xa. A non-empty subset X is said to be **S_2-near subtraction semigroup** if for every a∈X there exists x∈X^* such that axa=ax. A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer k>1 such that $a^k=0$ Which implies that xa=0 where $x=a^{k-1}$.

3. STRONG λ-BI NEAR SUBTRACTION SEMIGROUP

Definition 3.1

A non-empty subset X together with two binary operations"-" and "." Is said to be **Strong λ- bi near subtraction semigroup**. Then X is the both strong S_1 and strong S_2-near subtraction semigroup

Example 3.2

Let X={0,a,b,1} in which "-" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	0
c	c	0	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	0	b	b
c	0	a	b	c

Then X is a Strong λ- bi near-subtraction semi group

Result 3.3

Every Strong λ-bi near Subtraction Semigroup is a λ- bi near Subtraction Semigroup

Proof:

Let X be a Strong λ -bi near Subtraction Semigroup where X is the both Strong S_1 and Strong S_2 near subtraction semigroup ⇒ For a,b∈X, aba=ba and aba=ab ⇒ For a∈X, aba=ba and aba=ab for some b∈X. Therefore X is both S_1 and S_2 near subtractionsemigroup. Thus X is a λ- bi near Subtraction Semigroup.

Remark 3.4

The converse of the above result need not be true shown by a following example.

Let X={0,a,b,c} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	1	1	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	b	c
b	0	0	0	0
1	0	a	b	1

Thus X is an λ- bi near-subtraction semi group but not Strong λ - bi-near subtraction semi group

Hence, every λ- bi-near subtraction semi group need not be a Strong λ- bi-near subtraction semi group.

Theorem 3.5

Let X be a Boolean near subtraction semigroup. Each of the following statement implies that X is a Strong λ-bi near Subtraction Semigroup

1. X is commutative.
2. X is of Type I and Type II.
3. aXa=Xa and aX=aXa for all a∈X (That is, X is P_1' and P_1 near subtraction semigroup).
4. X is sub commutative.

Proof :

Let X be a Boolean near subtraction semigroup

Let X be a commutative near subtraction semigroup.and let a,b ∈ X. Now, aba = a(ba) = a(ab) (Since X be a commutative) =a^2b =ab(Since X be a Boolean)=ba

(Since X be a commutative) and let a,b $\in$ X. Now, aba = (ab)a = (ba)a (Since X be a commutative) =ba^2 =ba(Since X be a Boolean)=ab (Since X be a commutative). Thus X is a Strong λ-bi near subtraction semigroup.

Let X be of Type I and Type II near subtraction semigroup and let a,b$\in$X. Then aba=baa=ba^2=ba [Since X is Boolean] and aba=aab=a^{2}b=ab [Since X is Boolean]. That is, aba=ba and aba=ab. Thus X is a F*-bi near subtraction semigroup.

Let a$\in$X. Since Xa=aXa and aX=aXa, for every b$\in$X there exists y$\in$X such that ba=aya and ab=aya. Now aba=a(ba)=a(aya)=a^{2}ya=aya [Since X is Boolean]=ba and aba=(ab)a=(aya)a=aya^2=aya[Since X is Boolean]=ab. Thus X is a Strong λ-bi near subtraction semigroup.

Let X be a Sub-commutative Let a $\in$ X, aX=Xa. Therefore for every b$\in$ X_1 there exists c$\in$ X_1 such that ba=ac and ab=ca. Now,aba a(ba) = a(ac) =ac^2=ac(Since X be a Boolean)= ba and aba=(ab)a = (ca)a=ca^2=ca(Since X be a Boolean)=ab. Thus X is a Strong λ-bi near Subtraction

Theorem 3.6

Any homomorphic image of a Strong λ-bi near Subtraction Semigroup is a Strong λ-bi near Subtraction Semigroup.

Proof :

Let f: X$\rightarrow$ Y be a homomorphism. Since X be a Strong λ--bi near subtraction semigroup $\Rightarrow$ aba =ba and aba = ab for all a,b$\in$ X. Let $y_1, y_2 \in$ Y then there exists $x_1,x_2 \in$ Y such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Clearly then $x_1y_1x_1 = y_1x_1$ and $x_1y_1x_1 = x_1y_1$. And the desired result now follows.

4. RESULTS ON STRONG λ-BI NEAR SUBTRACTION SEMIGROUP.

Proposition 4.1

If X is a Strong λ-bi near subtraction semigroup then E$\subset$ C(X).

Proof

For e$\in$E, eX = Xe [Since X is Sub-commutative] =Xe^2[X is left bipotent]. for e$\in$E, eX = Xe [Since X is Sub-commutative]$\Rightarrow$ eeX = eXe $\Rightarrow$ e^{2}X = eXe $\Rightarrow$ eX = eXe [since e^2=e].

Again Xe=eX⇒ Xee=eXe ⇒ Xe^2=eXe ⇒ Xe=eXe[since e^2=e].Clearly, Then for every x∈X, there exists u and v in y such that ex=eue and xe=eveexe=euee=eue^2=eue=ex and exe=eeve= e^2ve=eve=xe.. Thus ex=exe=xe for all x∈X. Consequently E⊂ C(X).

Theorem 4.2

Let X be a left self distributive S-near subtraction semigroup Then X is a Strong λ-bi near subtraction semigroup if and only if X is a GNF.

Proof

Assume that X is a GNF. Now for a∈X, Since X is a S-bi near subtraction semigroup.Let a∈ax =axa. Thus X is a regular... Let a,b∈ X. Then a = aba [Since X is regular] =abaa[Since X is self distributive] = aa [Since X is regular] =a^2. Therefore, X is Boolean and regular implies that X is Strong λ-bi near subtraction semigroup

Conversely, assume that X is a Strong λ-bi near subtraction semigroup. Since X is a S-bi near subtraction semigroup.Let a∈ax =axa. Thus X is a regular. Again by Proposition 4.1, E⊆ C(x). Therefore X is GNF.

Theorem 4.3

Let X be a S-near subtraction semigroup Then X is a Strong λ-bi near subtraction semigroup if and only if for every x∈X there exists a unique central idempotent e such that Xx = Xe and X is regular.

Proof

For the only if part let x∈X. Since X is a Strong λ-bi near subtraction semigroup and from by previous Theorem, X is a GNF. Therefore X is regular and E⊂C(x). Let a,x∈X. Since X is regular, x = xax, Now Xx = Xax = Xe, Where e = ax∈E. Since E∈ C(x), the idempotent e is central and Xx = Xe, for all x∈X. Let e_1∈E and Xe_1 = Xx, for some central idempotent e_1. Now $e_1 = e_1^2 \in Xe_1$ = Xx = Xe and so e_1= ne, for some n∈X. Consequently, e_1 = ne = ne^2 = (ne)e = e_1e and e = e^2∈Xe = X e_1 and so e = ue_1, for some u∈X. This implies that e = ue_1 = ue_1^2 = $(ue_1)e_1$ = ee_1. Since e is a central idempotent, ee_1 = e_1e and so e = e_1. Thus e is the unique central idempotent.

Conversely, since X is regular and idempotent central, X is a GNF. Therefore by Previous Theorem, X is a Strong λ-bi near subtraction semigroup.

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