

## Fixed point theorems in parametric metric Space

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### Abstract

In this paper we proved the some fixed point theorems in Parametric metric spaces.

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### 1. Introduction and preliminaries

Fixed point theorems are very important tool for proving the existence and eventually the uniqueness of the solutions to various mathematical models (integral and partial differential equations, variational inequalities). In last few years different types of generalized metric spaces have been developed by different authors in different approach. Some generalized metric spaces are D-metric space, Cone metric space [4] etc. The notion of parametric metric spaces being a natural generalization of metric spaces was recently introduced and studied by Hussain et al. [2]. In this paper, we present some fixed point theorems under various expansive conditions in parametric metric spaces. These results improve and generalize some important known results in [6,7,8].

### 2. Preliminaries

**Definition 2.1.** Let  $X$  be a nonempty set and a function

$$\rho : X \times X \times (0, +\infty) \rightarrow [0, +\infty)$$

is said to be a parametric metric on  $X$  if,

- (1)  $\rho(x, y, t) = 0$  for all  $t > 0$  if and only if  $x = y$ ,

$$(2) \rho(x, y, t) = \rho(y, x, t) \text{ for all } t > 0,$$

$$(3) \rho(x, y, t) \leq \rho(x, z, t) + \rho(z, y, t) \text{ for all } x, y, z \in X \text{ and all } t > 0.$$

and the pair  $(X, \rho)$  is called parametric metric space.

**Definition 2.2.** Let  $\{x_n\}_{n=1}^{\infty}$  be a sequence in a parametric metric space  $(X, \rho)$ .

(1)  $\{x_n\}_{n=1}^{\infty}$  is said to be convergent to  $x \in X$ , if

$$\lim_{n \rightarrow \infty} \rho(x_n, x, t) = 0.$$

written as  $\lim_{n \rightarrow \infty} x_n = x$ , for all  $t > 0$ ,

(2)  $\{x_n\}_{n=1}^{\infty}$  is said to be a Cauchy sequence in  $X$  if for all  $t > 0$ , if

$$\lim_{n, m \rightarrow \infty} \rho(x_n, x_m, t) = 0.$$

(3)  $(X, \rho)$  is said to be complete if every Cauchy sequence is a convergent sequence.

**Definition 2.3.** Let  $(X, \rho)$  be a parametric metric space and a function  $T : X \rightarrow X$  is continuous at  $x \in X$ , if for any sequence  $\{x_n\}_{n=1}^{\infty}$  in  $X$  such that  $\lim_{n \rightarrow \infty} x_n = x$ , then  $\lim_{n \rightarrow \infty} Tx_n = Tx$ .

**Example 2.4.** Let  $X = \{f/f : (0, +\infty) \rightarrow R\}$ . And define the function  $\rho : X \times X \times (0, +\infty) \rightarrow [0, +\infty)$  by  $\rho(f, g, t) = |f(t) - g(t)|, \forall f, g \in X$  and all  $t > 0$ . Then  $\rho$  is a parametric metric on  $X$  and the pair  $(X, \rho)$  is a parametric metric space.

**Lemma 2.5.** Let  $\{x_n\}_{n=1}^{\infty}$  be a sequence in a parametric metric space  $(X, \rho)$  such that

$$\rho(x_n, x_{n+1}, t) \leq \lambda \rho(x_{n-1}, x_n, t)$$

where  $\lambda \in [0, 1)$  and  $n = 1, 2, \dots$ . Then  $\{x_n\}_{n=1}^{\infty}$  is a Cauchy sequence in  $(X, \rho)$ .

**Lemma 2.6.** Let  $(X, \rho, s)$  be a parametric metric space with the coefficient  $s = 1$ . Let  $\{x_n\}_{n=1}^{\infty}$  be a sequence of points of  $X$  such that

$$\rho(x_n, x_{n+1}, t) \leq \lambda \rho(x_{n-1}, x_n, t)$$

where  $\lambda \in [0, \frac{1}{s})$  and  $n = 1, 2, \dots$ . Then  $\{x_n\}_{n=1}^{\infty}$  is a Cauchy sequence in  $(X, \rho, s)$ .

### 3. Main results

**Theorem 3.1.** Let  $(X, \rho)$  be a complete parametric metric space and  $T$  a continuous mapping satisfying the following condition:

$$\rho(Tx, Ty, t) \leq \alpha \max[\rho(x, y, t), \rho(x, T(x), t), \rho(y, T(y), t), \\ \rho(x, T(y), t), \rho(T(x), y, t)]$$

for all  $x, y \in X$ ,  $x \neq y$ , and for all  $t > 0$ , where  $\alpha \in [0, 1)$ . Then  $T$  has a unique fixed point in  $X$ .

*Proof.* Choose  $x_0 \in X$  be arbitrary, to define the iterative sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows,  $Tx_n = x_{n+1}$  for  $n = 1, 2, 3, \dots$ . Taking  $x = x_n$  and  $y = x_{n+1}$  in (1), we obtain

$$\rho(Tx_n, Tx_{n+1}, t) \leq \alpha \max[\rho(x_n, x_{n+1}, t), \rho(x_n, Tx_n, t), \rho(x_{n+1}, Tx_{n+1}, t), \\ \rho(x_n, Tx_{n+1}, t), \rho(Tx_n, x_{n+1}, t)].$$

$$\rho(x_{n+1}, x_{n+2}, t) \leq \alpha \max[\rho(x_n, x_{n+1}, t), \rho(x_n, x_{n+1}, t), \rho(x_{n+1}, x_{n+2}, t), \\ \rho(x_n, x_{n+2}, t), \rho(x_{n+1}, x_{n+1}, t)].$$

$$\rho(x_{n+1}, x_{n+2}, t) \leq \alpha \max[\rho(x_n, x_{n+1}, t), \rho(x_n, x_{n+2}, t)].$$

**Case (i):** If  $\rho(x_{n+1}, x_{n+2}, t) \leq \alpha \rho(x_n, x_{n+1}, t)$ . Hence by induction, we obtain

$$\rho(x_{n+1}, x_{n+2}, t) \leq \alpha^{n+1} \rho(x_0, x_1, t), \quad \forall t > 0 \quad \text{and} \quad \alpha < 1.$$

By Lemma 2.5,  $\{x_n\}_{n \in \mathbb{N}}$  is a Cauchy sequence in  $X$ . But  $X$  is a complete parametric metric space; hence,  $\{x_n\}_{n \in \mathbb{N}}$  is converges. Call the limit  $x^* \in X$ . Then,  $x_n \rightarrow x^*$  as  $n \rightarrow +\infty$ . By continuity of  $T$  we have,

$$Tx^* = T(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} x_{n+1} = x^*.$$

That is,  $Tx^* = x^*$ ; thus,  $T$  has a fixed point in  $X$ .

**Case (ii):** If  $\rho(x_{n+1}, x_{n+2}, t) \leq \alpha \rho(x_n, x_{n+2}, t)$ .

$$\begin{aligned} \rho(x_{n+1}, x_{n+2}, t) &\leq \alpha (\rho(x_n, x_{n+1}, t) + \rho(x_{n+1}, x_{n+2}, t)) \\ &\leq \frac{\alpha}{1 - \alpha} \rho(x_n, x_{n+1}, t). \\ &\leq h \rho(x_n, x_{n+1}, t) \quad \text{where} \quad h = \frac{\alpha}{1 - \alpha} < 1 \end{aligned}$$

Hence by induction, we obtain

$$\rho(x_{n+1}, x_{n+2}, t) \leq h^{n+1} \rho(x_0, x_1, t)$$

By Lemma 2.5,  $\{x_n\}_{n \in \mathbb{N}}$  is a Cauchy sequence in  $X$ . But  $X$  is a complete parametric metric space; hence,  $\{x_n\}_{n \in \mathbb{N}}$  is converges. Call the limit  $x^* \in X$ . Then,  $x_n \rightarrow x^*$  as  $n \rightarrow +\infty$ . By continuity of  $T$  we have,

$$Tx^* = T(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} x_{n-1} = x^*.$$

That is,  $Tx^* = x^*$ ; thus,  $T$  has a fixed point in  $X$ .

**Uniqueness:**

Let  $y^*$  be another fixed point of  $T$  in  $X$ ; then  $Ty^* = y^*$  and  $Tx^* = x^*$ . Now,

$$\begin{aligned} \rho(Tx^*, Ty^*, t) &\leq \alpha \max[\rho(x^*, y^*, t), \rho(x^*, Tx^*, t), \rho(y^*, Ty^*, t), \\ &\quad \rho(x^*, Ty^*, t), \rho(Tx^*, y^*, t)]. \end{aligned}$$

This implies that

$$\rho(x^*, y^*, t) \leq \alpha \rho(x^*, y^*, t)$$

This is true only when  $\rho(x^*, y^*, t) = 0$ . So  $x^* = y^*$ . Hence  $T$  has a unique fixed point in  $X$ . ■

**Corollary 3.2.** Let  $(X, \rho)$  be a complete parametric metric space and  $T$  a continuous mapping satisfying the following condition:

$$\rho(Tx, Ty, t) \leq \alpha \max[\rho(x, y, t), \rho(x, T(x), t), \rho(y, T(y), t)]$$

for all  $x, y \in X$ ,  $x \neq y$ , and  $t > 0$ ,  $\alpha \in [0, 1)$ . Then  $T$  has a unique fixed point in  $X$ .

*Proof.* The proof of the corollary immediately follows since

$$\begin{aligned} &\max[\rho(x, y, t), \rho(x, T(x), t), \rho(y, T(y), t)] \\ &\leq \max[\rho(x, y, t), \rho(x, T(x), t), \rho(y, T(y), t), \\ &\quad \rho(x, T(y), t), \rho(T(x), y, t)] \end{aligned}$$

■

**Theorem 3.3.** Let  $(X, \rho)$  be a complete parametric metric space and  $T$  a continuous mapping satisfying the following condition:

$$\rho(Tx, Ty, t) \leq \alpha[\rho(x, T(x), t), \rho(y, T(y), t)] + \beta[\rho(x, T(y), t), \rho(T(x), y, t)]$$

for all  $x, y \in X$ , and  $\alpha + \beta < \frac{1}{2}$ ,  $\alpha, \beta \in \left[0, \frac{1}{2}\right)$ . Then  $T$  has a fixed point in  $X$ .

*Proof.* Choose  $x_0 \in X$  be arbitrary, to define the iterative sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows,  $Tx_n = x_{n+1}$  for  $n = 1, 2, 3, \dots$ . Taking  $x = x_n$  and  $y = x_{n+1}$  in (1), we obtain

$$\begin{aligned} \rho(Tx_n, Tx_{n+1}, t) &\leq \alpha[\rho(x_n, T(x_n), t) + \rho(x_{n+1}, T(x_{n+1}), t)] \\ &\quad + \beta[\rho(x_n, T(x_{n+1}), t) + \rho(T(x_n), x_{n+1}, t)] \\ &\leq \alpha[\rho(x_n, x_{n+1}, t) + \rho(x_{n+1}, x_{n+2}, t)] \\ &\quad + \beta[\rho(x_n, x_{n+2}, t) + \rho(x_{n+1}, x_{n+1}, t)] \\ &\leq \alpha[\rho(x_n, x_{n+1}, t) + \rho(x_{n+1}, x_{n+2}, t)] \\ &\quad + \beta[\rho(x_n, x_{n+2}, t)] \\ \rho(x_{n+1}, x_{n+2}, t) &\leq \alpha[\rho(x_n, x_{n+1}, t) + \rho(x_{n+1}, x_{n+2}, t)] \\ &\quad + \beta[\rho(x_{n+1}, x_n, t) + \rho(x_{n+1}, x_{n+2}, t)] \\ \rho(x_{n+1}, x_{n+2}, t) &\leq (\alpha + \beta)[\rho(x_n, x_{n+1}, t) + \rho(x_{n+1}, x_{n+2}, t)] \\ \rho(x_{n+1}, x_{n+2}, t) &\leq L\rho(x_n, x_{n+1}, t) \quad \text{where } L = \frac{(\alpha + \beta)}{(1 - (\alpha + \beta))} \end{aligned}$$

Hence by induction, we obtain

$$\rho(x_{n+1}, x_{n+2}, t) \leq L^{n+1} \rho(x_0, x_1, t)$$

By Lemma 2.5,  $\{x_n\}_{n \in \mathbb{N}}$  is a Cauchy sequence in  $X$ . But  $X$  is a complete parametric metric space; hence,  $\{x_n\}_{n \in \mathbb{N}}$  is converges. Call the limit  $x^* \in X$ . Then,  $x_n \rightarrow x^*$  as  $n \rightarrow +\infty$ . By continuity of  $T$  we have,

$$Tx^* = T(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} x_{n+1} = x^*.$$

That is,  $Tx^* = x^*$ . Thus,  $T$  has a fixed point in  $X$ . ■

**Theorem 3.4.** Let  $(X, \rho)$  be a complete parametric metric space and  $T$  a continuous mapping satisfying the following condition:

$$\rho(Tx, Ty, t) + \alpha\rho(y, Tx, t) \geq \beta \frac{\rho(x, Tx, t)\rho(y, Ty, t)}{\rho(x, y, t)} + \gamma\rho(x, y, t)$$

for all  $x, y \in X$ ,  $x \neq y$ , and for all  $t > 0$ , where  $\alpha, \beta, \gamma \geq 0$  are real constants and  $\beta + \gamma - 2\alpha > 1$ ,  $\gamma - \alpha > 1$ . Then  $T$  has a unique fixed point in  $X$ .

*Proof.* Choose  $x_0 \in X$  be arbitrary, to define the iterative sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows,

$Tx_n = x_{n-1}$  for  $n = 1, 2, 3, \dots$ . Taking  $x = x_n$  and  $y = x_{n+1}$  in (1), we obtain

$$\begin{aligned} \rho(Tx_{n+1}, Tx_{n+2}, t) + \alpha\rho(x_{n+2}, Tx_{n+1}t) &\geq \beta \frac{\rho(x_{n+1}, Tx_{n+1}, t)\rho(x_{n+2}, Tx_{n+2}, t)}{\rho(x_{n+1}, x_{n+2}, t)} \\ &\quad + \gamma\rho(x_{n+1}, x_{n+2}, t). \\ \rho(x_n, x_{n+1}, t) + \alpha\rho(x_{n+2}, x_n, t) &\geq \beta \frac{\rho(x_{n+1}, x_n, t)\rho(x_{n+2}, x_{n+1}, t)}{\rho(x_{n+1}, x_{n+2}, t)} + \gamma\rho(x_{n+1}, x_{n+2}, t). \\ \rho(x_n, x_{n+1}, t) + \alpha\rho(x_{n+2}, x_n, t) &\geq \beta \frac{\rho(x_{n+1}, x_n, t)\rho(x_{n+1}, x_{n+2}, t)}{\rho(x_{n+1}, x_{n+2}, t)} + \gamma\rho(x_{n+1}, x_{n+2}, t). \\ \rho(x_n, x_{n+1}, t) + \alpha\rho(x_n, x_{n+2}, t) &\geq \beta\rho(x_n, x_{n+1}, t) + \gamma\rho(x_{n+1}, x_{n+2}, t) \\ \rho(x_n, x_{n+1}, t) + \alpha\rho(x_n, x_{n+1}, t) + \alpha\rho(x_{n+1}, x_{n+2}, t) &\geq \beta\rho(x_n, x_{n+1}, t) + \gamma\rho(x_{n+1}, x_{n+2}, t) \\ (1 + \alpha - \beta)\rho(x_n, x_{n+1}, t) &\geq (\gamma - \alpha)\rho(x_{n+1}, x_{n+2}, t) \end{aligned}$$

for all  $t > 0$ . The last inequality gives

$$\begin{aligned} \rho(x_{n+1}, x_{n+2}, t) &\leq \frac{1 + \alpha - \beta}{\gamma - \alpha} \rho(x_n, x_{n+1}, t) \\ &= k\rho(x_n, x_{n+1}, t) \quad \text{where} \quad k = \frac{1 + \alpha - \beta}{\gamma - \alpha} < 1. \end{aligned}$$

Hence by induction, we get

$$\rho(x_{n+1}, x_{n+2}, t) \leq k^{n+1} \rho(x_0, x_1, t)$$

By Lemma 2.5,  $\{x_n\}_{n \in \mathbb{N}}$  is a Cauchy sequence in  $X$ . But  $X$  is a complete parametric metric space; hence,  $\{x_n\}_{n \in \mathbb{N}}$  is converges. Call the limit  $x^* \in X$ . Then,  $x_n \rightarrow x^*$  as  $n \rightarrow +\infty$ . By continuity of  $T$  we have,

$$Tx^* = T(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} x_{n-1} = x^*.$$

That is,  $Tx^* = x^*$ ; thus,  $T$  has a fixed point in  $X$ .

### Uniqueness:

Let  $y^*$  be another fixed point of  $T$  in  $X$ ; then  $Ty^* = y^*$  and  $Tx^* = x^*$ . Now,

$$\begin{aligned} \rho(Tx^*, Ty^*, t) + \alpha\rho(y^*, Tx^*, t) &\geq \beta \frac{\rho(x^*, Tx^*, t)\rho(y^*, Ty^*, t)}{\rho(x^*, y^*, t)} + \gamma\rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) + \alpha\rho(x^*, y^*, t) &\geq \gamma\rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) &\geq (\gamma - \alpha)\rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) &\leq \frac{1}{\gamma - \alpha} \rho(x^*, y^*, t) \end{aligned}$$

This is true only when  $\rho(x^*, y^*, t) = 0$ .

$$\Rightarrow x^* = y^*.$$

Hence  $T$  has a unique fixed point in  $X$ . ■

**Theorem 3.5.** Let  $(X, \rho)$  be a complete parametric metric space and  $T$  a continuous mapping satisfying the following condition:

$$\begin{aligned} & \rho(Tx, Ty, t) + \alpha \min\{\rho(x, Ty, t), \rho(y, Tx, t)\} \\ & \geq \beta \frac{\rho(x, Tx, t)[\delta + \rho(y, Ty, t)]}{\delta + \rho(x, y, t)} + \gamma \rho(x, y, t) \end{aligned}$$

for all  $x, y \in X, x \neq y$ , and for all  $t > 0$ , where  $\delta \geq 1$ , and  $\alpha, \beta, \gamma \geq 0$  are real constants and  $\beta + \gamma - \alpha > 1 + \alpha, \gamma > 1 + \alpha$ . Then  $T$  has a fixed point in  $X$ .

*Proof.* Choose  $x_0 \in X$  be arbitrary, to define the iterative sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows,  $Tx_n = x_{n-1}$  for  $n = 1, 2, 3, \dots$ . Taking  $x = x_n$  and  $y = x_{n+1}$  in (1), we obtain

$$\begin{aligned} & \rho(Tx_{n+1}, Tx_{n+2}, t) + \alpha \min\{\rho(x_{n+1}, Tx_{n+2}, t), \rho(x_{n+2}, Tx_{n+1}t)\} \\ & \geq \beta \frac{\rho(x_{n+1}, Tx_{n+1}, t)[\delta + \rho(x_{n+2}, Tx_{n+2}, t)]}{\delta + \rho(x_{n+1}, x_{n+2}, t)} + \gamma \rho(x_{n+1}, x_{n+2}, t). \\ & \rho(x_n, x_{n+1}, t) + \alpha \min\{\rho(x_{n+1}, x_{n+1}, t), \rho(x_{n+2}, x_n, t)\} \\ & \geq \beta \frac{\rho(x_{n+1}, x_n, t)[\delta + \rho(x_{n+2}, x_{n+1}, t)]}{\delta + \rho(x_{n+1}, x_{n+2}, t)} + \gamma \rho(x_{n+1}, x_{n+2}, t). \\ & \rho(x_n, x_{n+1}, t) + \alpha \min\{\rho(x_{n+1}, x_{n+1}, t), \rho(x_{n+2}, x_n, t)\} \\ & \geq \beta \frac{\rho(x_{n+1}, x_n, t)[\delta + \rho(x_{n+1}, x_{n+2}, t)]}{\delta + \rho(x_{n+1}, x_{n+2}, t)} + \gamma \rho(x_{n+1}, x_{n+2}, t). \\ & \rho(x_n, x_{n+1}, t) + \alpha \rho(x_n, x_{n+2}, t) \geq \beta \rho(x_n, x_{n+1}, t) + \gamma \rho(x_{n+1}, x_{n+2}, t) \end{aligned}$$

$$\begin{aligned} & \rho(x_n, x_{n+1}, t) + \alpha \rho(x_n, x_{n+1}, t) + \alpha \rho(x_{n+1}, x_{n+2}, t) \geq \beta \rho(x_n, x_{n+1}, t) + \gamma \rho(x_{n+1}, x_{n+2}, t) \\ & (1 + \alpha - \beta) \rho(x_n, x_{n+1}, t) \geq (\gamma - \alpha) \rho(x_{n+1}, x_{n+2}, t) \end{aligned}$$

for all  $t > 0$ . The last inequality gives

$$\rho(x_{n+1}, x_{n+2}, t) \leq \frac{1 + \alpha - \beta}{\gamma - \alpha} \rho(x_n, x_{n+1}, t) = k \rho(x_n, x_{n+1}, t)$$

where  $k = \frac{1 + \alpha - \beta}{\gamma - \alpha} < 1$ . Hence by induction, we obtain

$$\rho(x_{n+1}, x_{n+2}, t) \leq k^{n+1} \rho(x_0, x_1, t)$$

By Lemma 2.5,  $\{x_n\}_{n \in \mathbb{N}}$  is a Cauchy sequence in  $X$ . But  $X$  is a complete parametric metric space; hence,  $\{x_n\}_{n \in \mathbb{N}}$  is converges. Call the limit  $x^* \in X$ . Then,  $x_n \rightarrow x^*$  as  $n \rightarrow +\infty$ . By continuity of  $T$  we have,

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That is,  $Tx^* = x^*$ ; thus,  $T$  has a fixed point in  $X$ .

**Uniqueness:**

Let  $y^*$  be another fixed point of  $T$  in  $X$ ; then  $Ty^* = y^*$  and  $Tx^* = x^*$ . Now,

$$\begin{aligned} \rho(Tx^*, Ty^*, t) + \alpha \min\{\rho(x^*, Ty^*, t), \rho(y^*, Tx^*, t)\} &\geq \beta \frac{\rho(x^*, Tx^*, t)[\delta + \rho(y^*, Ty^*, t)]}{\delta + \rho(x^*, y^*, t)} \\ &\quad + \gamma \rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) + \alpha \rho(x^*, y^*, t) &\geq \gamma \rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) &\geq (\gamma - \alpha) \rho(x^*, y^*, t) \\ \rho(x^*, y^*, t) &\leq \frac{1}{\gamma - \alpha} \rho(x^*, y^*, t) \end{aligned}$$

This is true only when  $\rho(x^*, y^*, t) = 0$ .

$$\Rightarrow x^* = y^*.$$

Hence  $T$  has a unique fixed point in  $X$ . ■

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