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On Some New Trilateral Generating Relations Involving I-Function of Two Variables

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Abstract

The aim of this research paper is to establish some new trilateral generating relations involving I-function of two variables.

Keywords: I-function of two variables, I-function of one variable, H-function of one variable, Trilateral, Generating Relations.

1. INTRODUCTION:

The I–function of two variables introduced by Sharma & Mishra [2], will be defined and represented as follows:

$$I\begin{bmatrix} x \\ y \end{bmatrix} = I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{matrix} x \\ y \end{matrix}\middle| \begin{matrix} [(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}] \\ [(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}] \end{matrix} \begin{matrix} :[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}] \\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}] \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\,x^{\xi}y^{\eta}d\xi d\eta, \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i=1}^{r}[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}[\prod_{j=m_1+1}^{q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=n_1+1}^{p_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}[\prod_{j=m_2+1}^{q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=n_2+1}^{p_{i''}}\Gamma(e_{ji''}-E_{ji''}\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, $p_{i'}$, $p_{i''}$, q_i, $q_{i'}$, $q_{i''}$, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i > 0$, $q_{i'} \geq 0$, $q_{i''} \geq 0$, (i = 1, …, r; i′ = 1, …, r′; i′′ = 1, …, r′′; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1-c_j+\gamma_j\xi)$ (j = 1, ..., n_1), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour.

The contour L_2 is in the η–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(f_j-F_j\eta)$ (j=1,....., n_2) lie to the right, and the poles of $\Gamma(1-e_j+E_j\eta)$ (j = 1, ..., m_2), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also

$$R' = \sum_{j=1}^{p_i}\alpha_{ji} + \sum_{j=1}^{p_{i'}}\gamma_{ji'} - \sum_{j=1}^{q_i}\beta_{ji} - \sum_{j=1}^{q_{i'}}\delta_{ji'} < 0,$$

$$S' = \sum_{j=1}^{p_i}A_{ji} + \sum_{j=1}^{p_{i''}}E_{ji''} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{q_{i''}}F\delta_{ji'} < 0,$$

$$U = \sum_{j=n+1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji} + \sum_{j=1}^{m_1}\delta_j - \sum_{j=m_1+1}^{q_{i'}}\delta_{ji'} + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_{i'}}\gamma_{ji'} > 0, \quad (2)$$

$$V = -\sum_{j=n+1}^{p_i}A_{ji} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{m_2}F_j - \sum_{j=m_{21}+1}^{q_{i''}}F_{ji''} + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_{i''}}E_{ji''} > 0, \quad (3)$$

and $|\arg x| < \frac{1}{2} U\pi$, $|\arg y| < \frac{1}{2} V\pi$.

In our investigation we shall need the following result [4]:

$$\sum_{r=0}^{\infty}\frac{(1-\lambda)^r}{r!}G_A(\lambda,\alpha,\beta;\lambda-r;xy(1-z),\frac{x(1-2y)}{1-z},\frac{x(1-2y)}{1-z})z^r$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(\alpha+\beta)_n}(x-1)^n P_n^{(-n,\alpha+\beta+n)}\left(\frac{1+x}{1-x}\right) \times P_n^{(\alpha+\beta-1,-\beta-n)}(1-2y),$$

$$|xy(1-z) < 1, \left|\frac{x(1-2y)}{1-z}\right| < 1; \tag{4}$$

$$\sum_{r=0}^{\infty} \frac{(1-\lambda)^r}{r!} G_B(\lambda,\alpha,\beta,\gamma;\lambda-r;xy(1-z),\frac{x(2y-1)}{1-z},\frac{x(2y-1)}{1-z})z^r$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(1-\beta-\gamma)_n}(x-1)^n P_n^{(-n,1-\beta-\gamma+n)}\left(\frac{1+x}{1-x}\right) \times P_n^{(-\beta-\gamma,\alpha+\beta+\gamma-1-n)}(1-2y),$$

$$|xy(1-z) < 1, \left|\frac{x(2y-1)}{1-z}\right| < 1; \tag{5}$$

where G_A and G_B are hypergeometric functions of three variables has been defined by Pandey [1].

2. TRILATERAL GENERATING RELATIONS:

In this section we establish the following trilateral generating relations:

$$\sum_{r=0}^{\infty}\frac{z^r}{r!}G_A(\lambda,\alpha,\beta;\lambda-r;xy(1-z),\frac{x(1-2y)}{1-z},\frac{x(1-2y)}{1-z})$$

$$\times I^{0,\ n\ :m_1+1,n_1\ :m_2,n_2}_{p_i,q_i:r\ :p_{i'},q_{i'}+1:r':p_{i''},q_{i''}:r''}\left[\begin{matrix}z_1\\ z_2\end{matrix}\middle|\ \begin{matrix}\ldots,\ldots:\ldots\ldots\ldots..,\ldots\ldots.:\ldots\ldots\ldots,\ldots\ldots..\\ \ldots,\ldots:(1-\lambda+r;0),\ldots\ldots..:\ldots\ldots\ldots,\ldots\ldots.\end{matrix}\right]$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(\alpha+\beta)_n}(x-1)^n P_n^{(-n,\alpha+\beta+n)}\left(\frac{1+x}{1-x}\right)P_n^{(\alpha+\beta-1,-\beta-n)}(1-2y)$$
$$\times I^{0,\ n\ :m_1+1,n_1\ :m_2,n_2}_{p_i,q_i:r\ :p_{i'},q_{i'}+1:r':p_{i''},q_{i''}:r''}\left[\begin{matrix}z_1\\ z_2\end{matrix}\middle|\ \begin{matrix}\ldots,\ldots:\ldots\ldots\ldots..,\ldots\ldots.:\ldots\ldots\ldots,\ldots\ldots..\\ \ldots,\ldots:(1-\lambda;0),\ldots\ldots..:\ldots\ldots\ldots,\ldots\ldots.\end{matrix}\right], \tag{6}$$

where $|xy(1-z) < 1, \left|\frac{x(1-2y)}{1-z}\right| < 1, \lambda > 0, U > 0, V > 0, |\arg z_1| < \frac{1}{2}U\pi, |\arg z_2| < \frac{1}{2}V\pi$, where U and V are given in (2) and (3).

$$\sum_{r=0}^{\infty}\frac{z^r}{r!}G_B(\lambda,\alpha,\beta,\gamma;\lambda-r;xy(1-z),\frac{x(2y-1)}{1-z},\frac{x(2y-1)}{1-z})$$
$$\times I^{0,\ n\ :m_1+1,n_1\ :m_2,n_2}_{p_i,q_i:r\ :p_{i'},q_{i'}+1:r':p_{i''},q_{i''}:r''}\left[\begin{matrix}z_1\\ z_2\end{matrix}\middle|\ \begin{matrix}\ldots,\ldots:\ldots\ldots\ldots..,\ldots\ldots.:\ldots\ldots\ldots,\ldots\ldots..\\ \ldots,\ldots:(1-\lambda+r;0),\ldots\ldots..:\ldots\ldots\ldots,\ldots\ldots.\end{matrix}\right]$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(1-\beta-\gamma)_n}(x-1)^n P_n^{(-n,1-\beta-\gamma+n)}\left(\frac{1+x}{1-x}\right) \times P_n^{(-\beta-\gamma,\alpha+\beta+\gamma-1-n)}(1-2y)$$
$$\times I^{0,\ n\ :m_1+1,n_1\ :m_2,n_2}_{p_i,q_i:r\ :p_{i'},q_{i'}+1:r':p_{i''},q_{i''}:r''}\left[\begin{matrix}z_1\\ z_2\end{matrix}\middle|\ \begin{matrix}\ldots,\ldots:\ldots\ldots\ldots..,\ldots\ldots.:\ldots\ldots\ldots,\ldots\ldots..\\ \ldots,\ldots:(1-\lambda+r;0),\ldots\ldots..:\ldots\ldots\ldots,\ldots\ldots.\end{matrix}\right], \tag{7}$$

where $|xy(1-z) < 1, \left|\frac{x(2y-1)}{1-z}\right| < 1$ and $\lambda > 0, U > 0, V > 0, |\arg z_1| < \frac{1}{2}U\pi$, $|\arg z_2| < \frac{1}{2}V\pi$, where U and V are given in (2) and (3).

Proof:

To prove (6), consider

$$\Delta = \sum_{r=0}^{\infty} \frac{z^r}{r!} G_A(\lambda, \alpha, \beta; \lambda - r; xy(1-z), \frac{x(1-2y)}{1-z}, \frac{x(1-2y)}{1-z})$$
$$\times I^{0,\ n\ :m_1,n_1\ :m_2,n_2}_{p_i,q_i:r\ :p_i'+1,q_i':r':p_i'',q_i'':r''} \left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} \ldots, \ldots : \ldots\ldots\ldots.., \ldots\ldots.: \ldots\ldots\ldots, \ldots\ldots.. \\ \ldots, \ldots : (1-\lambda+r; 0), \ldots\ldots..: \ldots\ldots\ldots, \ldots\ldots. \end{matrix}\right]$$

On expressing I-function of two variables in contour integral form as given in (1), we get

$$\Delta = \sum_{l=0}^{\infty} \frac{z^r \Gamma[1-\lambda+r]}{r!} \left[\frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\, \theta_2(\xi)\theta_3(\eta) z_1^{\xi} z_2^{\eta} d\xi d\eta\right]$$
$$\times G_A(\lambda, \alpha, \beta; \lambda - r; xy(1-z), \tfrac{x(1-2y)}{1-z}, \tfrac{x(1-2y)}{1-z})$$

On changing the order of summation and integration, we have

$$\Delta = \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\, \theta_2(\xi)\theta_3(\eta) z_1^{\xi} z_2^{\eta}$$
$$\times \left[\sum_{l=0}^{\infty} \tfrac{z^r \Gamma[1-\lambda+r]}{r!} G_A(\lambda, \alpha, \beta; \lambda - r; xy(1-z), \tfrac{x(1-2y)}{1-z}, \tfrac{x(1-2y)}{1-z})\right] d\xi d\eta$$

Now using the results

$$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}$$

and (4), we get

$$\Delta = \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\, \theta_2(\xi)\theta_3(\eta) z_1^{\xi} z_2^{\eta} \Gamma[1-\lambda]$$
$$\times \left[(1-z)^{\lambda-1} \sum_{n=0}^{\infty} \tfrac{n!}{(\alpha+\beta)_n} (x-1)^n P_n^{(-n,\alpha+\beta+n)} \left(\tfrac{1+x}{1-x}\right)\right.$$
$$\left.\times P_n^{(\alpha+\beta-1,-\beta-n)}(1-2y)\right] d\xi d\eta$$

which in view of (1), provides (6).

Proceeding on similar lines as above, the result (7) can be derived.

3. PARTICULAR CASES:

I. On specializing the parameters in main formulae, we get following generating relations in terms of I-function of one variable:

$$\sum_{r=0}^{\infty}\frac{z^r}{r!}G_A(\lambda,\alpha,\beta;\lambda-r;xy(1-z),\frac{x(1-2y)}{1-z},\frac{x(1-2y)}{1-z})$$

$$\times I^{m+1,n}_{p_i,q_i+1:r}[z|^{\dots\dots,\dots}_{(1-\lambda+r;0),\dots\dots}]$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(\alpha+\beta)_n}(x-1)^n P_n^{(-n,\alpha+\beta+n)}\left(\frac{1+x}{1-x}\right)P_n^{(\alpha+\beta-1,-\beta-n)}(1-2y)$$
$$\times I^{m+1,n}_{p_i,q_i+1:r}[z|^{\dots\dots,\dots}_{(1-\lambda;0),\dots\dots}], \quad (8)$$

where $|xy(1-z) < 1, \left|\frac{x(1-2y)}{1-z}\right| < 1, \lambda > 0$, $|\arg z| < ½ B\pi$, where B is given by $B = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji} > 0$;

$$\sum_{r=0}^{\infty}\frac{z^r}{r!}G_B(\lambda,\alpha,\beta,\gamma;\lambda-r;xy(1-z),\frac{x(2y-1)}{1-z},\frac{x(2y-1)}{1-z})$$
$$\times I^{m+1,n}_{p_i,q_i+1:r}[z|^{\dots\dots,\dots}_{(1-\lambda+r;0),\dots\dots}]$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(1-\beta-\gamma)_n}(x-1)^n P_n^{(-n,1-\beta-\gamma+n)}\left(\frac{1+x}{1-x}\right)$$
$$\times P_n^{(-\beta-\gamma,\alpha+\beta+\gamma-1-n)}(1-2y)$$
$$\times I^{m+1,n}_{p_i,q_i+1:r}[z|^{\dots\dots,\dots}_{(1-\lambda;0),\dots\dots}], \quad (9)$$

where $|xy(1-z) < 1, \left|\frac{x(2y-1)}{1-z}\right| < 1$ and $\lambda > 0$, $|\arg z| < ½ B\pi$, where B is given by $B = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji} > 0$.

II. On choosing r = 1 in (8) and (9), we get following generating relations in terms of H-function of one variable, which are the results given by Shrivastava [3, p.98, (6) and (7)]::

$$\sum_{r=0}^{\infty}\frac{z^r}{r!}G_A(\lambda,\alpha,\beta;\lambda-r;xy(1-z),\frac{x(1-2y)}{1-z},\frac{x(1-2y)}{1-z})$$

$$\times H^{M+1,N}_{P,Q+1}[t|^{(a_j,\alpha_j)_{1,p},}_{(1-\lambda+r,0),(b_j,\beta_j)_{1,q}}]$$

$$= (1-z)^{\lambda-1}\sum_{n=0}^{\infty}\frac{n!}{(\alpha+\beta)_n}(x-1)^n P_n^{(-n,\alpha+\beta+n)}\left(\frac{1+x}{1-x}\right)P_n^{(\alpha+\beta-1,-\beta-n)}(1-2y)$$

$$\times H_{P,Q+1}^{M+1,N}\left[t\middle|{}^{(a_j,\alpha_j)_{1,p,}}_{(1-\lambda,0),(b_j,\beta_j)_{1,q}}\right], \tag{10}$$

where $|xy(1-z) < 1, \left|\frac{x(1-2y)}{1-z}\right| < 1, \lambda > 0$, |argz| < ½ Aπ, where A is given by

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0;$$

$$\sum_{r=0}^{\infty} \frac{z^r}{r!} G_B\left(\lambda, \alpha, \beta, \gamma; \lambda - r; xy(1-z), \frac{x(2y-1)}{1-z}, \frac{x(2y-1)}{1-z}\right) \times H_{P,Q+1}^{M+1,N}\left[t\middle|{}^{(a_j,\alpha_j)_{1,p,}}_{(1-\lambda+r,0),(b_j,\beta_j)_{1,q}}\right]$$

$$= (1-z)^{\lambda-1} \sum_{n=0}^{\infty} \frac{n!}{(1-\beta-\gamma)_n} (x-1)^n P_n^{(-n,1-\beta-\gamma+n)}\left(\frac{1+x}{1-x}\right) \times P_n^{(-\beta-\gamma,\alpha+\beta+\gamma-1-n)}(1-2y)\, H_{P,Q+1}^{M+1,N}\left[t\middle|{}^{(a_j,\alpha_j)_{1,p,}}_{(1-\lambda,0),(b_j,\beta_j)_{1,q}}\right], \tag{11}$$

where $|xy(1-z) < 1, \left|\frac{x(2y-1)}{1-z}\right| < 1$and |argz| < ½ Aπ, where A is given by

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0;$$

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