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Contra-Continuity Maps on Topological Spaces

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Abstract

In this paper we present new definition and give two examples which is clarified in this diagram of (**Supra-pre closed sets maps),** for that we support it with a theorem

Keywords: supra pre-closed set (SU-COD),Supra pre-open set"

INTRODUCTION

In[11] *Taha.H.Jasem and V. amarendra babu* the given one definition and theorem and two example also, in[10] M. E. El-Shafei M. Abo- elhamaye T. M. Al-shami On supra R-open sets and some applications on topological spaces He is proved a new class of generalized supra open sets called supra R-open set is introduced. The relationships between some generalized supra open sets and this class are investigated and illustrated with enough examples. Also

In [9] O. R. SAYED" he is introduce" a new type of continuous maps called a "supra pre-continuous" maps and obtain some of their "properties and "characterizations. Also he "introduce the concepts of supra pre-"continuous" maps, sp-pre-on maps and "sp-pre-closed maps and "investigate "several properties for this class of maps. In particular, we study the relation between supra" pre-continuous" maps and sp-pre-on" maps (sp-pre-cd maps).

Throughout his paper[10] simply $(\tilde{X},\tilde{T})(\tilde{Y},\tilde{\sigma}), and\,(\tilde{Z},\tilde{\upsilon}) or$ simply, X , Y,and Z) denote "topological spaces on which no "separation" axioms are Assumed" unless explicitly stated. All sets are assumed to be "subset of "topological spaces. The "closure and the "interior" of a set A. are denoted by $Cl_p^{\mu}(\tilde{A})\; andInt_p^{\mu}(\tilde{A})$, respectively". A "subcollection $\tilde{\mu}\subset 2^x$ is called a "supra "topology [5] on X if $X\in\mu$ and $\tilde{\mu}$ is closed under arbitraryunion. $(\tilde{X},\tilde{\mu})$ is called a sp-topological" space ."General "topology is important in many fields of" applied sciences as well as branches of mathematics. In "reality it is used in data mining,computational topology for geometric design and molecular design,computer-aided design, "computer-aided geometric design and engineering design (briey CAGD), digital topology, information systems,non-commutative geometry and its application to particle physics and"quantum" physics etc. We "recommend that the" reader should" refer tothe "following papers, "respectively: [2-4, 6, 7, 8]. One can observe the inuence made in these realms of "applied research by general "topological spaces, "properties and structures. Rosen and Peters [8] have used "topology as a body of" mathematics that could unify diverse areas of CAGD and "engineering design "research. They have presented several examples of the" application of "topology to "CAGD and design.

1. "Supra pre-open sets"

Denition 1.1. A set A is sp-pre-on if $\tilde{A}\; \tilde{Int}^{\mu}(\breve{Cl}^{\mu}(\breve{A}))$. The complement of sp-pre-on is called sp-pre-cd. Thus $\bar{A}$ is 'sp-pre-cd if and only if $\tilde{Cl}^{\mu}(\tilde{A}))\subseteq\tilde{A}$

Theorem 1.2. (1) "Every sp-$\tilde{\sigma}$ open is sp-pre-open"

(2) "Every sp-pre-on is sp- b-open.

Proof. Obvious.

The followes examples show that sp-pre-ON')) is placed strictly

Between" supra _ $\tilde{\alpha}$ open and supra b-open.

Example 1.3. Let ($\tilde{y}$, η) be a sp-tp –space". Where

$\tilde{y}=\{\breve{a},\breve{b},\breve{c}\} and\, \eta=\{Y\;,\Phi,\{\breve{a}\},\{\breve{a},\breve{b}\},\{\breve{b},\breve{c}\}\}$

Here $\left\{\breve{a},\breve{b}\right\}$ is sp-pre-on", but it is not "supra - $\tilde{\alpha}$ open.

Example 1.4. Let (Y, η) be a sp-top space, where Y $=\left\{\breve{a},\breve{b},\breve{c},\breve{d}\right\}$

And $\eta=\{Y,\breve{\phi},\{\breve{a}\},\{\breve{b}\},\{\breve{a},\breve{b}\}\}$ Here $\left\{\breve{b},\breve{c}\right\}$ is a supra b-open,

The following diagram show how sp-pre-on sets are related to some similar types of "supra-open sets.

$supra\tilde{\alpha}-open\rightarrow supra\tilde{\alpha}-open\rightarrow supra\tilde{\alpha}\ pre-open\rightarrow supra\tilde{\alpha}b-open$

Theorem 1.5. (i) Arbitrary union of sp-pre-on sets is always sp- pre-on.

(ii) Finite intersection of sp-pre-on' sets may fail to be surap-pre-open.

(iii) $\tilde{X}$ is a sp-pre-on set

Proof. (i) Let $\{\breve{A}_\lambda:\lambda\in\breve{A}\}$ be a family of supa-pre-open sets in a "topological space $\tilde{X}$. Then for any

$\lambda\in\breve{A}$ we have $\breve{A}_\lambda\subseteq Int^\mu(\breve{C}l^\mu(\breve{A}_\lambda)).Hence$

$$\bigcup_{\lambda\in A}\breve{A}_\lambda\subseteq\bigcup_{\lambda\in A}(I\breve{n}t^\mu(\breve{C}l^\mu(\breve{A}_\lambda)))\subseteq I\breve{n}t^\mu(\breve{A}_\lambda))$$

$$\subseteq I\hat{n}t^\mu(\bigcup_{\lambda\in A}(\hat{C}l^\mu(\hat{A}_\lambda))\subseteq I\hat{n}t(\tilde{C}l(\bigcup_{\lambda\in A}\hat{A}_\lambda)$$

Therefore $\bigcup_{\lambda\in A}\tilde{A}_\lambda$ is a sp-pre-on set.

(ii) In Example 2.1 both $\left\{\hat{a},\hat{b}\right\}and\left\{\hat{b},\hat{c}\right\}$

are sp-pre-on, but their intersection {e} is not sp- pre-on.

Theorem 1.6. (i) Arbitrary "intersection of sa pre-closed sets is always sp-pre-cd.

(ii) Finite union of sp-pre-cd' sets may fail to be 'sp- pre-cd.

Proof. (i) This follows immediately from Theorem 2.2.

(ii) In Example 2.1 both $\left\{\hat{a}\right\}and\left\{\hat{b}\right\}$ are sp-pre-closed", but their union {ab} is not sp-pre-cd.

Denition 1.7. The sp-pre-cd' of a set A, denoted by $\hat{C}l_p^\mu(\hat{A})$ is the "intersection" of the 'sp-pre-cd sets including A. The "supra pre-interior of a set A;denoted by $I\hat{n}t_p^\mu(\hat{A})$, is the union of the sp-pre-on sets "included in $\hat{A}$.

Remark 1.8. It is clear that $Int_p^{\mu}(\tilde{A})$,

is a sp-pre-on' set and $\widehat{Cl}_p^{\mu}(\widehat{A})$ is 'sp- pre-cd.

Theorem 1.9. $(i)\widehat{A} \subseteq \widehat{Cl}_p^{\mu}(\widehat{A}): and \widehat{A} = \widehat{Cl}_p^{\mu}(\widehat{A})$ iff $\widehat{A}$ is a 'sp-pre-cd set.

$(ii)\widehat{Int}_p^{\mu}(\widehat{A}) \subseteq \widehat{A}; and \widehat{Int}_p^{\mu}(\widehat{A}) = \widehat{A}$ iff A is a sp-pre-on set.

$(iii) Y - Int_p^{\mu}(\widehat{A}) = \widehat{Cl}_p^{\mu}(Y - \widehat{A}):$

$(iv) Y - \widehat{Cl}_p^{\mu}(\widehat{A}) = \widehat{Int}_p^{\mu}(Y - \widehat{A}).$

$(v)\ If \widehat{A} \subseteq \tilde{B},\ then\ "\widehat{Cl}_p^{\mu}(\widehat{A}) \subseteq \widehat{Cl}_p^{\mu}(\widehat{B}) and:$

$Int_p^{\mu}(\tilde{A}) \subseteq \widehat{Int}_p^{\mu}(\widehat{B})$

Proof. Obvious.

Theorem 1.10.

$(\mathbf{a})\widehat{Int}_p^{\mu}(\alpha) \bigcup \widehat{Int}_p^{\mu}(\beta) \subseteq \widehat{Int}_p^{\mu}(\alpha \bigcup \beta);$

$(b)\widehat{Cl}_p^{\mu}(\alpha \bigcap \beta) \subseteq \widehat{Cl}_p^{\mu}(\alpha) \bigcap \widehat{Cl}_p^{\mu}(\beta).$

Proof. obvious. The inclusions in (a) and (b) in Theorem 2.5 can not replaced by equalities by Example 2.1. Where, if α = {b} and β = {c};then

$Int_p^{\mu}(\alpha) = Int_p^{\mu}(\beta) = \Phi; and; Int_p^{\mu}$

$(\alpha \bigcup \beta) = \{\tilde{a}, \tilde{b}\}: Also,\ if\ \delta = \{\tilde{a}\ \tilde{b}\}$

$and, \gamma = \{a, c\}, then, Cl_p^{\mu}(\delta) = Cl_p^{\mu}\ (\gamma) = X, and,$

$Cl_p^{\mu}(\delta \bigcap \gamma) = \{\tilde{a}\}$

Proposition 1.11. (1) The "intersection of 'sp- on and sp-pre-on is 'sa pre-open.

(2) The "intersection of sp-on and sp-pre-on is sp-pre-on.

" Supra pre-continuous maps"

Denition 1.12. Let $(\eta, \tilde{\tau}\ ) and (\nu, \tilde{\sigma}))$ be two topological spaces and be an associated supra topology with τ . A map $f : (\eta, \tilde{\tau}\ ) \to (\nu, \tilde{\sigma})$ is called" supra pre-continuous map if the inverse image of each open set in ν is a sp-pre-on' set in η .

Theorem 1.13. Every "continuous map is "supra pre-continuous.

Proof. Let $f : \tilde{\eta} \to \tilde{Y}$ be a "continuous map and A is open set in Y .

Then $f^{-1}(\tilde{A})$ is an "open set in $\tilde{\eta}$. Since μ associated with τ. then $\tau \in \mu$.

Therefore $f^{-1}(\tilde{A})$ is a "supra open set in $\tilde{\eta}$ which is a sp-pre-on set in $\tilde{\eta}$. Hence f is sp-pre-"continuous map.The converse of the above theorem is not true as shown in the

Example 1.14 $Let, Y = \{\tilde{a}, \tilde{b}, \tilde{c}\}; and; \tilde{\tau} = \{Y, \Phi, \{\tilde{a}, \tilde{b}\}\}; be$ a topology on Y. The"supra "topology μ is defined as follows:

$\mu = \{Y, \Phi, \{\tilde{a}\}, \{\tilde{a}, \tilde{b}\}\}; and;$
$let, f : (Y, \tilde{\tau}) \to (Y, \tilde{\tau})$
be a map defined as
$follows: f(\tilde{a}) = b; f(\tilde{b}) = c; f(\tilde{c}) = a.$

Since The inverse image of the open set f(a); g(b) is f(a); f(g) which is not an open set but it is a sp-

pre-on set. Then f is supra pre-continuous" map but not "continuous map

The following example shows that supra pre-continuous map need not be supra $\hat{\alpha}-$ continuous" map.

Example 1.15. Consider the set

$Y = \{\hat{a}, \hat{b}, \hat{c}, \hat{d}\}$ *with the topology*
$\hat{\tau} = \{Y, \Phi, \{\hat{a}, \hat{c}\}\{\hat{b}, \hat{d}\}\}$*and the supra*
topology
$\hat{\mu} = \{Y, \Phi\{\hat{a}, \hat{c}\}, \{\hat{b}, \hat{d}\}, \{\hat{a}, \hat{c}, \hat{d}\};$
$Also,\ let\ \beta = \{\hat{x}, \hat{y}, \hat{z}\}$

with the

topology $\sigma = \{Y, \Phi, \{\hat{z}\}, \{\hat{y}, \hat{z}\}.$
"*Define the map*
$f : (Y, \hat{\tau}) \to \{v, \hat{\sigma})$
$by f(\hat{a}) = y; f(\hat{b}) = f(\hat{c}) = z; f(\hat{d}) = x.$

Clearly, f is a 'supra pre continuous' map

Supra pre-continuous maps

Proposition 1.16.[11] Every supra neighborhood of any point in a supra topological space

(Y, μ) is a supra; R-open' set.

Proposition 1.17.[11] If *E* is a 'supra R-open set in supra topological space (Y, μ), then every proper superset of E is 'supra R-open. The converse of the above two proposition need not be true as shown in the following example

Example 1.18.[11] Let the supra topology $\mu = \{\emptyset, X, x, y, \{x, w, z\}\}$ on$X = x, y, w, z$. Then i x is supra R-open, but is not supra neighborhood of any point. (ii) For any proper superset A of , then A is supra R-open. But y is not supra R-open.

2- MAIN RESULT

Definition : 2.1. Let $(\eta,\delta)and(\zeta,\sigma))$ be two topological spaces and be an associated supra pre topology with τ . A map $f:(\eta,\delta) \to (\zeta,\sigma)$ is called contra-continuity map if a inverse of image of each Closed set in ζ is a Supra-pre closed set in η .

$$F:\eta \to \zeta$$

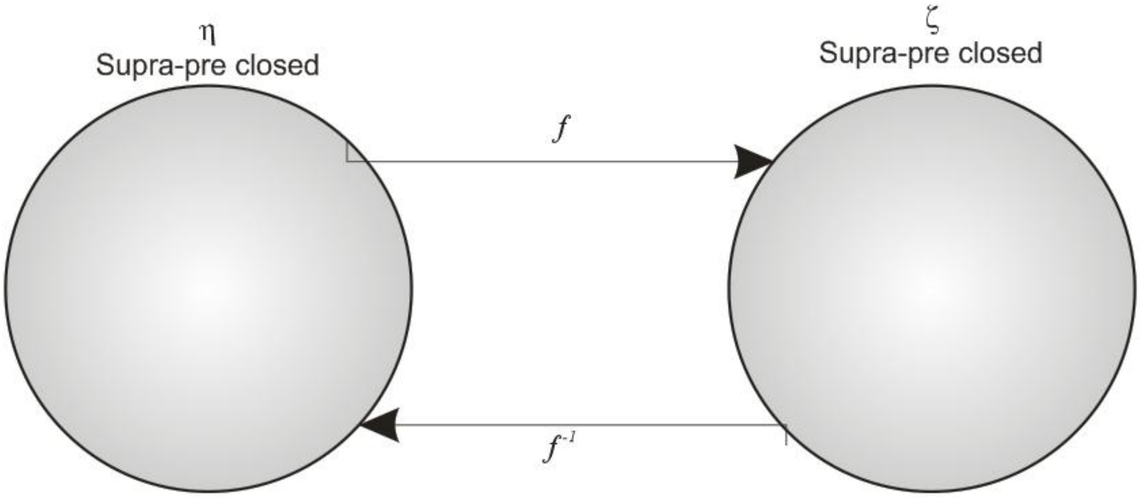

Figure 1

F is called **(contra-continuity Map)**

Theorem 2.2. for each contra-continuous' map is (supra-pre-closed set).

Proof. Let $F:\eta \to \zeta$ be a contra-continuous' map and A is supra-pre closed set in ζ . Then $f^{-1}(A)$ is an (SU-COD)set in η . Since associated with then $f^{-1}(A)$ is a supra-pre-closed set in η . which is a SU-COD in η . Hence f is SU-COD 'continues maps.The converse of the above theorem is not true we need to show by exame

Example :2.3

$Let, \alpha = \{x, y, x, v\}; and; \tau = \{\alpha, \Phi, \{x, y\}\}; be$ a topology on α . The "supra topology μ is defined as following:

$$\mu = \{\alpha, \Phi, \{x\}, \{x, y\}\}; and;$$
$$let, f : (\alpha, \eta) \to (\alpha, \zeta)$$
$$be\ a\ map\ defined"\ as$$
$$follows : f(x) = y; f(y) = z; f(z) = v.$$
$$f(v) = x$$

Since The inverse image of The closed set {x,y} is {x,z}which is not an supra-pre closed but it is a contra-continuity Maps. Then f is SU-COD set and continuous map but not supra-pre open continuous" map

Example: 2.4 Consider the set

$$\alpha = \{x, y, z, v\}\ with\ the\ topology$$
$$\alpha = \{\alpha, \Phi, \{x, z\}\{y, v\}\} and\ the\ supra\ topology$$
$$\mu = \{\alpha, \Phi\{x, z\}, \{y, v\}, \{x, z, v\}\};$$
$$Also,\ let Y = \{\ell, v, \vartheta\}$$
$$topology\ \zeta = \{Y, \Phi, \{v\}, \{y, v\}.$$

with the $Define\ the\ map, f : (X, \eta) \to (\tilde{X}, \zeta)$

$$by f(x) = y; f(y) = f(z) = v; f(v) = \ell.$$

Clearly, f is a is supra-closed set continues map

REFERENCES

[1] R. Devi, S. Sampathkumar and M. Caldas, On supra _-open sets and s_-continuous maps, General Mathematics, 16 (2) (2008), 77 - - 84.

[2] E. Khalimsky, R. Kopperman and P. Meyer, Computer graphics and connected topologies on _nite ordered sets, Topol. Appl., 36 (1990), 1--17.

[3] T. Kong, R. Kopperman and P. Meyer, A topological approach to digital topology, Am.Math.Monthly, 98 (1991), 901- -917.

[4] G. Landi, An introduction to non commutative spaces and thier geo- metrics. Lecture notes in Physics. New York: Springer-Verlag; 1997.

[5] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr,On supra topological spaces, Indian J. Pure and Appl. Math., 14 (4) (1983), 502-510.

[6] E. Moore and T. Peters, Computational topology for geometric design and molecular design. In: D. Ferguson and T. Peters, editors, Mathematics in industry challenges and frontiers 2003, SIAM; 2005.

[7] Z. Pawlak, Rough sets: Theoretical aspects of reasoning about data. System theory, knowldge engineering and problem solving, vol. 9. Dordrecht: Kluwer;1991

[8] D. Rosen and T. Peters, The role of the topology in engineering design research, Res. Eng. Des., 2 (1996), 81- -98.

[9] O. R. Sayed and T. Noiri, On supra b-open sets and supra b-continuity, Eur. J. Appl. Math., 3 (2) (2010), 295- 302

[10] M. E. El-Shafei M. Abo- elhamaye T. M. Al-shami On supra R-open sets and some applications on topological spaces July 04, 2016 .Volume 8 797-3080-3

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