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A Study on Lattice Ordered Fuzzy Soft Group

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Abstract

Soft set theory has a rich potential application in several fields. A soft group is a parameterized family of subgroups and a fuzzy soft soft group is a parameterized family of fuzzy subgroups. The concept of fuzzy soft group is generalization of soft group. Ab-dulkadir Aygunoglu and HalisAygun introduced the notion of fuzzy soft groups in 2009. In this paper, the concept of lattice ordered fuzzy soft group is introduced and some of its properties are discussed and studied. Further more, the definition of fuzzy soft group chain and complete lattice ordered fuzzy soft groups are discussed.

Keywords: Fuzzy set, Soft set, Fuzzy soft set, Soft group, Fuzzy soft group, Lattice ordered fuzzy soft group.

MSC[2010] Classifications: 06D72, 06B23

1 INTRODUCTION

The concept of lattices arose primarily for the study of certain distinguished subsets of algebraic structures such as groups and rings. The theory of lattices was first defined by Richard Dedikind and attracted the attention of mathematicians in 1930s. Fuzzy set is a generalization of a classical set and the membership function is a generalization of the characteristic function. Fuzziness can be used in many areas of daily life, such as engineering, medicine, meteorology, manufacturing and others. The notion of fuzzy set was defined by Lotfi A. Zadeh [28] in 1965. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodsov in [19]. After Molodsov's work, some different applications of soft sets were studied in [5; 10]. Further more, Maji Biswas and Roy worked on soft set theory in [17]. Also Maji et al. presented the definition of fuzzy soft set[15] and Roy et al. presented some applications of this notion to decision making problems in [16]. Also AbdulKadir Aygunoglu and Halis Aygun introduced the notion of fuzzy soft groups in [1], which is a generalization of soft groups. The concept of soft group was introduced by Aktas and Cagman[2]. J.Vimala introduced the notion of fuzzy lattice ordered group[26] and discussed some of their properties. The concept of soft lattices was proposed in [9, 6] and soft lattice operations were studied in [10]. Faruk Karaaslan and Naim Cagman defined the concept of modular fuzzy soft lattice and distributive fuzzy soft lattice and some of their basic properties were studied in fuzzy soft lattice theory [7]. Also Muhammad Irfan Ali et al. introduced a new concept of soft sets with some order among the parameters and some properties of lattice ordered soft set were studied in [3].

In this paper, we define the notion of the lattice ordered fuzzy soft group and studied some of its algebraic properties .
This paper is organized as follows: In sec 2, some preliminary definitions are given. In sec 3, the definition of lattice ordered fuzzy soft group is given and some of its algebraic properties are studied.

2 PRELIMINARIES

In this section, we have presented the basic definitions and results of fuzzy sets, soft sets, soft group and fuzzy soft group theory which are useful for subsequent discussion.
A poset $(L; \leq)$ is said to form a lattice if for every $a, b \in L$, Sup{a, b} and Inf{a,b} exist in L. Then we write Sup{a,b} = $a \vee b$ and Inf{a,b} = $a \wedge b$. If P is a poset in which every two members are comparable, it is called a chain. A finite chain with n elements is said to have length n - 1. (i.e., length is the number of links that the chain has). Every chain is a lattice. Since any two elements a, b of a chain are comparable, say $a \leq b$, then $a \wedge b$ = Inf {a,b}=a and $a \vee b$ = Sup{a, b} = b. A poset $(L, \leq)$ is a lattice iff every non-empty finite subset of L has Sup and Inf.

Dual of a lattice is a lattice. Product of two lattices is a lattice. A lattice L is called a complete lattice if every non-empty subset of L has its Sup and Inf in L. A lattice L is called a distributive lattice if $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$, for all $a, b, c \in L$.

Let G be a group. A fuzzy subset of a group G is called a fuzzy subgroup of the group G if (i)$\mu(xy) \geq \text{Min}\{\mu(x),\mu(y)\}$, for every x, y ϵ G and (ii)$\mu(x^{-1}) = \mu(x)$, for every x ϵ G.

Definition 2.1 [28]:
Let X be a non-empty set, then a fuzzy set over X is a function from X into I = [0; 1]. ie., $\mu: X \rightarrow I$.

Definition 2.2 [19]:
Let X be an initial universe set and E a set of parameters with respect to X. Let P(X) denote the power set of X and $A \subseteq E$. A pair (F,A) is called a soft set over X, where F is a mapping given by $F:A \rightarrow P(X)$.
A soft set over X is a parameterized family of subsets of the universe X.

Definition 2.3 [1] :
Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f, A) is called a fuzzy soft set over X, where f is a mapping from A into I^X . That is, for each a ϵ A, $f(a) = f_a : X \rightarrow I$ is a fuzzy set on X.

Definition 2.4 [1]:
Let A be the fuzzy set of the universe X. Take the parameter set E=[0,1], and define the mapping $F : E \rightarrow P(X)$ as follows:
$F(\alpha)=\{x \epsilon X : \quad \mu_A(x)\geq\alpha\}, \quad \alpha \epsilon [0, 1]$. Then F ($\alpha$) is an α -level set of A.

Definition 2.5 [1]:
For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \sqsubseteq (g,B)$ if
(i) $A \subset B$, and
(ii) For each a ϵA, $f_a \leq g_a$, that is, f_a is fuzzy subset of g_a.
Note that for all a ϵ A, f_a and g_a are identical approximations.

Definition 2.6 [1] :

Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be equal if $(f, A) \sqsubseteq (g, B)$ and $(g, B) \sqsupseteq (f, A)$.

Definition 2.7 [1] :

Let (f, A) be a fuzzy soft set over X. The soft set $(f, A)_\alpha = \{(f_a)_\alpha : a \epsilon A\}$, for each $\alpha \epsilon (0,1]$, is called an α-level soft set of the fuzzy soft set (f, A), where $(f_a)_\alpha$ is an α-level set of the fuzzy set f_a. Here, for each $\alpha \epsilon (0,1]$, $(f, A)_\alpha$ is a soft set.

Definition 2.8 [2]:
Let X be a group and (F,A) be a soft set over X. Then(F,A) is said to be a soft group over X iff F (a) is a subgroup of X, for each $a \in A$.
A soft group is a parameterized family of subgroups of X.

Definition 2.9 [1] :

Let X be a group and (f, A) be a fuzzy soft set over X. Then (f,A) is said to be a fuzzy soft group over X if for each $a \in A$ and $x,y \in X$,

(i) $f_a(x,y) \geq \min\{f_a(x),f_a(y)\}$
(ii) $f_a(x^{-1}) \geq \quad f_a(x)$.

That is, for each $a \in A$, f_a is a fuzzy subgroup.

Example 2.10 [1]:

Let N be the set of all natural numbers and define $f : N \to I^R$ by $f(n) = f_n : R \to I$, for each $n \in N$ where

$$f_n(x) = \begin{cases} 1/n & \text{if } x = k2^{n}, \ \exists k \in Z \text{ where Z is the set of all integers.} \\ 0 & \text{otherwise} \end{cases}$$

Then the pair (f, N) forms a fuzzy soft set over R, and the fuzzy soft set (f, N) is a fuzzy soft group over R.

Definition 2.11[20]:
Let X be a non- empty set and P(X) be bounded lattice with respect to operations of intersection and union, and set inclusion as a partial order. If the set of parameters E is also a lattice with respect to certain binary operations or partial order, then a non-empty subset A of E also inherits the partial order from the set E.

A soft set (F,A) is called a lattice ordered soft set if for the mapping $F : A \to P(X)$, $x \leq y$ implies $F(x) \subseteq F(y)$, for each $x, y \in A \subseteq E$.

Definition 2.12 [15] :
Union of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and

$$h(c) = \begin{cases} f_c & \text{if } c \in A - B \\ g_c & \text{if } c \in B - A, \quad \text{for all } c \in C \\ f_c \vee g_c & \text{if } c \in A \cap B \end{cases}$$

We write $(f, A) \sqcup (g, B) = (h, C)$.

Definition 2.13 [15] :

Intersection of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cap B$ and $h_c = f_c \wedge g_c$, for all $c \in C$.

We write $(f, A) \sqcap (g, B) = (h, C)$.

Definition 2.14 [15]:

If (f, A) and (g, B) are two fuzzy soft sets, then (f, A) AND (g, B) is denoted as $(f, A) \wedge (g, B)$. If $(f, A) \wedge (g, B)$ is defined as $(h, A \times B)$, where $h(a, b) = h_{a, b} = f_a \wedge g_b$, for all $(a, b) \in A \times B$.

Preposition 2.15 [1]:

Let (f, A) be a fuzzy soft group over X. Then (f, A) is a fuzzy soft group over X iff for all $a \in A$ and for arbitrary $\alpha \in (0; 1]$ with $(f_a)_\alpha \neq \Phi$, the α-level soft set $(f, A)_\alpha$ is a fuzzy soft group over X.

3 LATTICE ORDERED FUZZY SOFT GROUP

In this section, we introduce the definition of lattice ordered fuzzy soft group and discuss some fundamental properties of lattice ordered fuzzy soft group.

Throughout this section, Let X be a group and P(X) be the power set of X. If the set of parameters E is also a lattice with respect to certain binary operations or partial order, then a non-empty subset A of E also inherits the partial order from the set E.

Definition 3.1:

Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a lattice ordered fuzzy soft group over X if for each $a \in A$ and $x, y \in X$,

(i) $f_a(x,y) \geq \min\{f_a(x), f_a(y)\}$([1])

(ii) $f_a(x^{-1}) \geq f_a(x)$([1])

(iii) $a \leq b$ implies $f_a \subseteq f_b$ for all $a, b \in A$.

ie., for all $a, b \in A$, $f_a \vee f_b$ and $f_a \wedge f_b$ exist in (f, A).

Example 3.2 :

Let N be the set of all natural numbers and $(N, \leq)$ be a lattice. If $a, b \in N$, then $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$. Define $f : N \to I^R$ by $f(n) = f_n : R \to I$ for each $n \in N$ where

$$f_n(x) = \begin{cases} 1-1/n & \text{if } x = k2^{-n},\ k \in Z \text{ where Z is the set of all integers.} \\ 0 & \text{otherwise} \end{cases}$$

Here, for each $n_1, n_2 \in N$, $n_1 \le n_2$ implies $f_{n1} \subseteq f_{n2}$. Then the pair $((f, N), \vee, \wedge, \subseteq)$ forms

a lattice ordered fuzzy soft group over R.

Proposition 3.3:

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X then for all a, b, $c \in A$ and $x \in X$, it satisfies the following

(1) $f_{a\,.\,c}(x) \subseteq f_{b\,.\,c}(x)$

(2) $f_{a+c}(x) \subseteq f_{b+c}(x)$

(3) $f_{a\vee c}(x) \subseteq f_{b\vee c}(x)$

(4) $f_{a\wedge c}(x) \subseteq f_{b\wedge c}(x)$,

where $a \vee c = \max\{a, c\}$ and $a \wedge c = \min\{a, c\}$

Proposition 3.4 :

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X. Then the following statements are equivalent

(i) $a \le b$ implies $f_a \subseteq f_b$, for all $a, b \in A$

(ii) $f_a \vee f_b = f_b$, for all $a, b \in A$

(iii) $f_a \wedge f_b = f_a$, for all $a, b \in A$.

Proposition 3.5 :

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X. Then for all a , $b \in A$, $f_a \vee f_b$ and $f_a \wedge f_b$ are the least upper bound and the greatest lower bound of f_a and f_b respectively.

Proposition 3.6 :

A lattice ordered fuzzy soft group is a poset.

Proposition 3.7 :

A lattice ordered fuzzy soft group over X is a fuzzy soft group chain over X.

Proof:

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X. Then for all $a, b \in A$, $a \leq b$ implies $f_a \subseteq f_b$. Since any two elements of A are comparable, then any two fuzzy

subgroups of (f,A) are comparable.Then (f, A) is a fuzzy soft group chain over X.

The following proposition shows that the converse of the above theorem is true.

Proposition 3.8 :

Every fuzzy soft group chain over X is a lattice ordered fuzzy soft group over X.

Proof:

Let (f, A) be a fuzzy soft group chain over X. Then for all $a,b \in A$, any two fuzzy

subgroups are comparable, ie., $f_a \subseteq f_b$ or $f_b \subseteq f_a$. Then $f_a \vee f_b$ and $f_a \wedge f_b$ exist in (f, A).

Therefore, (f, A) is a lattice ordered fuzzy soft group over X.

Definition 3.9:

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X. If every subset of (f, A) have both a greatest lower bound and a least upper bound, then it is called complete lattice ordered fuzzy soft group over X.

Proposition 3.10 :

Let $((f, A), \vee, \wedge, \subseteq)$ be a lattice ordered fuzzy soft group over X. Then it holds the following properties:

(i)Idempotent:

$f_a \vee f_a = f_a$ and $f_a \wedge f_a = f_a$, for all $a \in A$

(ii) Commutative:

$f_a \vee f_b = f_b \vee f_a$ and $f_a \wedge f_b = f_b \wedge f_a$, for all $a, b \in A$.

(iii) Associative:

$(f_a \vee f_b) \vee f_c = f_a \vee (f_b \vee f_c)$ and

$(f_a \wedge f_b) \wedge f_c = f_a \wedge (f_b \wedge f_c)$ for all $a, b, c \in A$.

(iv) Absorption:

$f_a \vee (f_a \wedge f_b)=f_a$

$f_a \wedge (f_a \vee f_b)=f_a$, for all a, b ϵ A

(v) Isotone Property:

$f_a \subseteq f_b \Rightarrow f_a \vee f_c \subseteq f_b \vee f_c$ and $f_a \wedge f_c \subseteq f_b \wedge f_c$ for all a, b, c ϵ A

Proposition 3.11:

Let (f, A) be a fuzzy soft group over X. Then ((f, A), $\vee$, $\wedge$, $\subseteq$) is a lattice ordered

fuzzy soft group over X i ff for all a ϵ A and for arbitrary $\alpha \epsilon (0, 1]$ with $(f_a)_\alpha \neq \Phi$, the

α-level soft set $(f, A)_\alpha$ is a lattice ordered fuzzy soft group over X.

Proof:

Let ((f, A), $\vee$, $\wedge$, $\subseteq$) be a lattice ordered fuzzy soft group over X.

Then $a \leq b$ implies $f_a \subseteq f_b$ for all a, b ϵ A.

Let $(f_a)_\alpha$ be a α-level fuzzy soft group over X .

Given: $(f_a)_\alpha \neq \Phi$ for all a ϵ A and for arbitrary $\alpha \epsilon (0, 1]$.

Let $x \epsilon (f_a)_\alpha \Rightarrow f_a(x) \geq \alpha$, for all x ϵ X.

To prove that $(f, A)_\alpha$ is a lattice ordered fuzzy soft group over X.

ie., To prove that for all a, b ϵ A, $(f_a)_\alpha \subseteq (f_b)_\alpha$, for arbitrary $\alpha \epsilon (0, 1]$

Suppose that $(f_a)_\alpha \not\subseteq (f_b)_\alpha$, for all a, b ϵ A

Let $x \epsilon (f_a)_\alpha$ then $x \notin (f_b)_\alpha$

Therefore $f_a(x) \geq \alpha$ and $f_b(x) \leq \alpha$

Then we get $f_a(x) \supseteq f_b(x)$

Which is contradiction to our assumption.

Then we get for all a, b ϵ A, $(f_a)_\alpha \subseteq (f_b)_\alpha$, for arbitrary $\alpha \epsilon (0, 1]$. conversely,

Let $(f, A)_\alpha$ be a lattice ordered fuzzy soft group over X.

Then for all a, b ϵ A, $(f_a)_\alpha \subseteq (f_b)_\alpha$, for arbitrary $\alpha \epsilon (0, 1]$.

Let $x \epsilon (f_a)_\alpha$ then $x \epsilon (f_b)_\alpha \Rightarrow f_a(x) \geq \alpha$ and $f_b(x) \geq \alpha$

The only possibility for this is given by

$a \leq b$ implies $f_a(x) \subseteq f_b(x)$ for all $x \in X$.

Then $a \leq b$ implies $f_a \subseteq f_b$

Then $((f, A), \vee, \wedge, \subseteq)$ is a lattice ordered fuzzy soft group over X.

CONCLUSION

In this present paper, concept of lattice ordered fuzzy soft group has been introduced. This work focused on properties of lattice ordered fuzzy soft group. To extend this work, one can investigate the other algebraic properties.

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